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Thermal stresses for frictional contact in wheel–rail systems

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Abstract

Based on a work by Knothe et al. (Wear 189 (1995) 91–99) a fully analytical solution for the temperature field, produced by sliding contact on the surface of a halfplane, is presented applying the Laplace transform technique. Further, the stress state in the halfplane under plane strain is analysed. A full solution is presented for the whole stress state as well as its extrema. All results are given in dimensionless form depending only on two parameters (dimensionless thermal penetration depth, Poisson's ratio).

Keywords: Wheel–rail system; Contact temperature; Sliding contact; Thermoelastic stress state

1. Introduction

Recently, Knothe and Liebelt published in Wear, [1], a detailed study on the temperature field due to the sliding contact in a wheel–rail system. They derived the following boundary value problem for the temperature T described with respect to an x–z coordinate system moving in the x-direction, positioned at the leading edge of the contact zone with a length of $2a$; see Fig. 1:

$$v\frac{\partial T}{\partial x}=\kappa\frac{\partial^2 T}{\partial z^2} \tag{1.1}$$

$$x=0:\quad T=0 \text{ for } 0\leq z\leq\infty \tag{1.2}$$

$$z=0:\quad -\lambda\frac{\partial T}{\partial z}=\dot{q}(x) \text{ for } 0\leq x\leq 2a \qquad \frac{\partial T}{\partial z}=0 \text{ for } x\geq 2a \tag{1.3}$$

Heat conduction is ignored in the longitudinal x-direction.

v is the forward velocity of the rolling wheel axis, κ the thermal diffusivity, λ the thermal conductivity. $\dot{q}$ is the heat flow rate due to the rolling/sliding contact being proportional to the contact pressure $p(x)$,

$$\dot{q}=\tilde{\alpha}\mu p(x)v_s \tag{1.4}$$

$\tilde{\alpha}$ is the heat partitioning factor, μ the coefficient of friction and v_s the sliding velocity.

Knothe and Liebelt demonstrated, that there exists only a small difference in the maximum surface temperature T_{max},

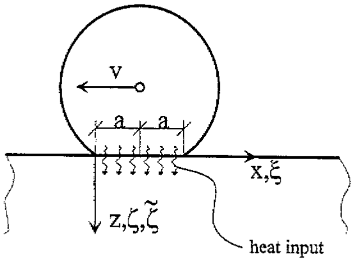

Fig. 1. Coordinate system fixed to the wheel at the contact interface.

if the contact pressure shows an ellipsoidal or constant distribution. Finally, they evaluated the temperature T_{max} as

$$T_{max}=1.25\sqrt{\frac{a\kappa}{v}}\frac{\tilde{\alpha}\mu v_s}{\lambda}p_o \tag{2}$$

p_o is the maximum contact pressure (the average pressure $p=\pi p_o/4$).

The surface temperature in the interval $[0,2a]$ is

$$T=T_{max}\sqrt{x/2a}\quad z=0\quad 0\leq x\leq 2a \tag{3.1}$$

Introducing a thermal penetration depth $\delta=\sqrt{\frac{2a\kappa}{v}}$ leads to

$$T_{max}=\frac{2\dot{q}}{\sqrt{\pi}}\frac{\delta}{\lambda} \tag{3.2}$$

Since the Peclet number $P_e=\frac{va}{2\kappa}$ is often used in the literature, P_e is related to the δ as

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$$\delta=\sqrt{\frac{2a\kappa}{v}}=\frac{a}{\sqrt{P_e}}\quad \tilde{\delta}=\frac{\delta}{2a}=\frac{0.5}{\sqrt{P_e}} \tag{3.3}$$

Knothe and Liebelt [1] gave neither explicit relations for the temperature field $T(x,z)$ nor expressions for the stresses due to this temperature field.

Therefore, it is the aim of this short paper to demonstrate that under the assumption of a constant surface contact pressure p = const, an analytical expression for the temperature field can be found. Furthermore, the according stress state will be evaluated.

A review on the literature published with respect to the temperature field due to friction can be found at the end of Section 2.

Finally, two theoretical aspects are discussed:

- Since the differential equation is of a parabolic type, an infinite velocity of the propagating thermal wave results. This may lead to an erroneous temperature distribution for a very high velocity v. In this case a temperature wave must be considered originating from an extended Fourier heat conduction law leading to an additional term $\partial^2 T/\partial x^2$.
- The problem formulation does not include thermoelastic coupling. In this case a dilatation rate term must be added to the right side of Eq. (1). However, thermoelastic coupling influences the temperature distribution only very little for metals like steel as it has been shown in a recent study by Tsuji et al. [2] for a moving heat source on a plate.

2. Temperature field

As in Ref. [1], the Laplace transform technique is used, however, for a dimensionless problem formulation. Introducing the reduced coordinates

$$x=2a\xi \quad z=\delta\zeta \tag{4.1}$$

the problem to be solved is

$$\frac{\partial T}{\partial \xi}=\frac{\partial^2 T}{\partial \zeta^2} \tag{4.2}$$

$$\xi\le 0:\quad T=0 \text{ for } 0\le\zeta\le\infty \tag{4.3}$$

$$\zeta=0:\quad \frac{\partial T}{\partial \zeta}=-\dot{q}\frac{\delta}{\lambda} \text{ for } 0\le\xi\le 1 \qquad \frac{\partial T}{\partial \zeta}=0 \text{ for } \xi>1 \tag{4.4}$$

The Laplace transform $L(T)=\tau(s)$ is applied with respect to the reduced x-coordinate ξ and yields

$$\tau=L\left(\dot{q}\frac{\delta}{\lambda}\right)\frac{1}{\sqrt{s}}e^{-\sqrt{s}\zeta} \tag{5.1}$$

$L(\dot{q}\delta/\lambda)$ is the Laplace transform of the function

$$\dot{q}\frac{\delta}{\lambda}[H(\xi)-H(\xi-1)] \tag{5.2}$$

$H(\xi-\xi_o)$ is the Heaviside step function with $H(\xi-\xi_o)=0$ for $\xi<\xi_o$ and $H(\xi-\xi_o)=1$ for $\xi\ge\xi_o$.

Employing the convolution theorem and taking the step function as the function with the shifted argument $\xi-u$, we obtain for the temperature field

$$T(\xi,\zeta)=\int_0^{\xi}\dot{q}\frac{\delta}{\lambda}\mathrm{Inv}\,L\left(\frac{1}{\sqrt{s}}e^{-\sqrt{s}\zeta}\right)du \text{ for } 0\le\xi\le 1 \tag{6.1}$$

$$T(\xi,\zeta)=\int_{\xi-1}^{\xi}\dot{q}\frac{\delta}{\lambda}\mathrm{Inv}\,L\left(\frac{1}{\sqrt{s}}e^{-\sqrt{s}\zeta}\right)du \text{ for } 1\le\xi\le\infty \tag{6.2}$$

The inverse of the Laplace transform of $(1/\sqrt{s})e^{-\sqrt{s}\zeta}$ is:

$$\mathrm{Inv}\,L\left(\frac{1}{\sqrt{s}}e^{-\sqrt{s}\zeta}\right)=\frac{1}{\sqrt{\pi u}}e^{-\zeta^2/4u} \tag{6.3}$$

It is possible to give analytical expressions for both the integrals Eqs. (6.1) and (6.2) by substituting the integration variable u by w via the relation $\xi^2/4u=w^2$. After some calculations one obtains

$$T(\xi,\zeta)=T_{max}\left\{\sqrt{\xi}\exp\left(\frac{-\zeta^2}{4\xi}\right)-\frac{\sqrt{\pi}}{2}\zeta\left[1-\mathrm{erf}\left(\frac{\zeta}{2\sqrt{\xi}}\right)\right]\right\}$$
$$\text{for } 0\le\xi\le 1 \tag{7.1}$$

$$T(\xi,\zeta)=T_{max}\left\{\sqrt{\xi}\exp\left(\frac{-\zeta^2}{4\xi}\right)-\sqrt{\xi-1}\exp\left(\frac{-\zeta^2}{4(\xi-1)}\right)-\frac{\sqrt{\pi}}{2}\zeta\left[\mathrm{erf}\left(\frac{\zeta}{2\sqrt{\xi-1}}\right)-\mathrm{erf}\left(\frac{\zeta}{2\sqrt{\xi}}\right)\right]\right\}$$
$$\text{for } 1<\xi\le\infty. \tag{7.2}$$

$\mathrm{erf}(\varepsilon)$ is the well-known error function

$$\mathrm{erf}(\varepsilon)=\frac{2}{\sqrt{\pi}}\int_0^{\varepsilon}e^{-t^2}dt$$

for details see e.g. Abramovitz and Stegun [3], chapter 7.

A simple expression exists for the surface temperature $T(\xi,\zeta=0)$:

$$T(\xi,\zeta=0)=T_{max}\sqrt{\xi} \quad \text{for } 0\le\xi\le 1 \tag{8.1}$$

$$T(\xi,\zeta=0)=T_{max}(\sqrt{\xi}-\sqrt{\xi-1}) \quad \text{for } 1\le\xi\le\infty \tag{8.2}$$

$$T(\xi,\zeta=0)=T_{max}\frac{1}{2\sqrt{\xi}} \quad \text{for } \xi\gg 1 \tag{8.3}$$

The temperature is continuous along the surface, although not differentiable at $\xi=1$. With respect to the variation of T with the depth one obtains for the temperature T at $\xi=1$ as

$$T_{\xi=1}=T_{max}\left\{\exp\left(\frac{-\zeta^2}{4}\right)-\frac{\sqrt{\pi}}{2}\zeta\left[1-\mathrm{erf}\left(\frac{\zeta}{2}\right)\right]\right\} \tag{8.4}$$

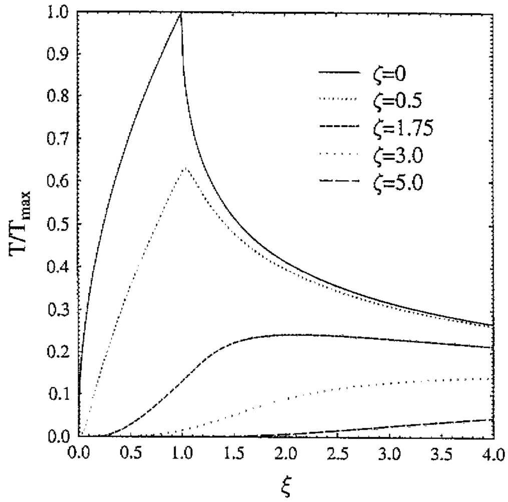

Fig. 2. Temperature profiles for various depths.

A graph of the reduced temperature $T(\xi,\zeta)/T_{max}$ can be taken from Fig. 2.

The solution Eqs. (7.1) and (7.2) can easily be handled for technical applications since all input parameters are "packed" into the maximum temperature T_{max}.

Carslaw and Jaeger presented in their famous book [4], §29 the same solution as Eqs. (7), although in a slightly different notation. Their results go back to the research by Jaeger done on heat flow involving moving sources of heat fifty years ago. However, they express the temperature field $T(\xi,\zeta)$ in an integral formulation by using the line source solution given in §10.7 of their book and refer with respect to the integral to a numerical evaluation. An integral formulation for Eqs. (7.1) and (7.2) can also be found in the surface mechanics literature; see Ref. [5], chapter 2A.

To the knowledge of the authors the first detailed discussion on the heat flow during wheel–rail contact was presented by Tanvir in 1980 in Wear [6]. However, this paper contains two shortcomings:

- In the first part of the paper [6], Tanvir analysed the temperature field for the interval $0 \le \xi \le 1$ (in the current notation). However, the "step" character of the heat input (see Eq. (5.2)) was omitted.
- In the second part of the paper [6] the "step" character of the heat input is considered for the interval $1 \le \xi \le \infty$. However, the inverse of the Laplace transform is obviously wrong, since the expression with the erf function (part of Eqs. (7.1) and (7.2) is missing). This error does not influence the surface temperature, which agrees fully with Eqs. (8.1) and (8.2).

Recently a Chinese group, [7], published the same results as Tanvir, [6], more or less copying Tanvir's results.

Yuen published in 1988 also an approximate solution for $T(\xi,\zeta)$ [8], however, losing the error function contribution in Eqs. (7.1) and (7.2).

The surface temperature Eqs. (8.1) and (8.2) can be found in the contact literature, too, also for various Peclet numbers; see K.L. Johnson's book, [9], chapter 12 and in the tribology literature; see e.g. Williams' book, [10], chapter 3.8.

Finally it should be noted that Mow and Cheng published a Fourier transform of the temperature field (in an analogous way to Eqs. (6.1) and (6.2)) already 30 years ago; see Ref. [11]. However, they did not present the temperature field itself as given in Eqs. (7.1) and (7.2).

The authors, however, did not find any publication, where the explicit temperature field is given as expressed by Eqs. (7.1) and (7.2).

3. Stress field

Thermoelastic contact problems with frictional heating are scarcely treated in the literature. For a short review the reader is referred to a recent paper by Yevtushenko and Kulchytsky–Zhyhailo [12], dealing with the frictional contact of a curved punch on a halfspace and a paper by Levytskyi and Onyshkevych [13], considering the problem of a frictional strip contact. One major difficulty for this kind of problems exists in the fact that the change in geometry of the actual configuration due to thermal expansion may influence significantly the contact area as well as the contact pressure. Very recently, Yevtushenko and coworkers published an integral equation concept; see Refs. [14–16], which allows the calculation of the unknown pressure distribution taking into account also the thermal deformation field. Such a difficulty does not appear in the case at hand, since a constant contact pressure is assumed and all parameters are hidden in one single parameter T_{max}. The problem at hand reduces to a thermoelastic one.

The heating of the rail is restricted to a small area in the railhead. A preliminary three-dimensional finite element study with an actual rail configuration has shown that the heated area is clamped by the surrounding head like by a pair of tongs. A comparison calculation revealed that the thermal stresses can be reproduced with sufficient accuracy by a plane strain model, if one considers a section through the rail and represents it by the halfplane $-\infty \le x \le \infty$, $z \ge 0$. Therefore, we consider an elastic halfplane under plane strain conditions, to which the temperature field Eqs. (7.1) and (7.2) is applied. The reference temperature $T_{ref}=0$. We introduce the modulus of elasticity E, Poisson's ratio ν and the integral thermal expansion coefficient α.

To find the stress field in the halfplane one may follow the well-known concepts of thermoelasticity; see e.g. Parkus [17].

The generalized Navier equations are

$$\Delta u_i + \frac{1}{1-2\nu}\frac{\partial}{\partial_i}\mathrm{div}(u_j) = \frac{2(1+\nu)}{1-2\nu}\alpha\frac{\partial T}{\partial_i}$$

$$i=x,y,z \quad \delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y}+\frac{\partial^2}{\partial z} \tag{9.1}$$

where u_j is the displacement vector in the x, y, z directions. Introducing the reduced coordinates

$$x=2a\xi \quad y=2a\eta \quad z=\delta\zeta$$

and the reduced temperature $\theta = T/T_{max}$, leads e.g. for $i=1$ (x-direction) to

$$\frac{\partial^2 u_1}{\partial \xi^2}+\frac{\partial^2 u_1}{\partial \eta^2}+\frac{4a^2}{\delta^2}\frac{\partial^2 u_1}{\partial \zeta^2}+\frac{1}{1-2\nu}\left(\frac{\partial^2 u_1}{\partial \xi^2}+\frac{\partial^2 u_2}{\partial \xi \partial \eta}+\frac{2a}{\delta}\frac{\partial^2 u_3}{\partial \xi \partial \zeta}\right) = \frac{2(1+\nu)}{1-2\nu}\alpha 2aT_{max}\frac{\partial \theta}{\partial \xi} \quad (9.2)$$

The transformation $\zeta = (2a/\delta)\tilde{\zeta}$ leads to a set of differential equations which is equivalent to Eq. (9.1), although with $i = \xi, \eta, \tilde{\xi}$.

A corresponding stress component is e.g.

$$\sigma_x = \frac{E}{1+\upsilon}\frac{1}{2a}\left[\frac{\partial u_1}{\partial \xi}+\frac{\nu}{1-2\nu}\left(\frac{\partial u_1}{\partial \xi}+\frac{\partial u_2}{\partial \eta}+\frac{2a}{\delta}\frac{\partial u_3}{\partial \zeta}\right) - \frac{1+\nu}{1-2\nu}\alpha 2aT_{max}\theta\right] \quad (10)$$

Since the displacements u_i are proportional to $\alpha 2aT_{max}$, the stress components are proportional to αT_{max}, too!

The stress problem is, therefore, solved in the $\xi, \eta, \tilde{\zeta}$ space for the dimensionless stress $\sigma_{ij}' = \sigma_{ij}/E\alpha T_{max}$. Two parameters appear in the solution, namely the Poisson's ratio ν and the dimensionless thermal penetration parameter $\tilde{\delta} = \delta/2a$. ν appears in the generalized Navier Eq. (9), $\delta/2a$ in the function $T = T_{max}\theta(\xi, \zeta)$ after substituting ζ by $(2a/\delta)\tilde{\zeta}$.

A proper analytical technique to find the stress field in the plane strain case is the establishment of a thermoelastic potential $\phi(\xi, \eta, \tilde{\zeta})$; see Ref. [17], chapters 2.4 and 3.1, leading to

$$\Delta\phi = \frac{1+\nu}{1-\upsilon}\theta(\xi,\tilde{\zeta}) \quad T = T_{max}\theta \quad (11.1)$$

and the corresponding Airy stress function

$$F = \frac{1}{1+\nu}(\psi - \phi) \quad (11.2)$$

ϕ is a particular solution of the differential Eq. (11.1). According to the Neuber–Papkovich concept, ψ is represented as

$$\psi = \xi\psi_\xi + \tilde{\zeta}\psi_{\tilde{\zeta}} \quad (11.3)$$

ψ_ξ, $\psi_{\tilde{\zeta}}$ are two harmonic functions which must be selected so that the stress boundary conditions are fulfilled which are

$$\zeta = 0: \quad \sigma'_{\tilde{\zeta}\tilde{\zeta}} = 0 \quad \tau'_{\xi\tilde{\zeta}} = 0. \quad (11.4)$$

The reduced stress components are

$$\sigma'_{\xi\xi} = \frac{1}{1+\nu}\frac{\partial^2}{\partial \tilde{\zeta}^2}(\psi - \phi)$$

$$\sigma'_{\tilde{\zeta}\tilde{\zeta}} = \frac{1}{1+\nu}\frac{\partial^2}{\partial \xi^2}(\psi - \phi) \quad (12)$$

$$\tau'_{\xi\tilde{\zeta}} = -\frac{1}{1+\nu}\frac{\partial^2}{\partial \xi \partial \tilde{\zeta}}(\psi - \phi)$$

Exactly this concept was followed already by Mow and Chen, [11], in the Fourier transform space and later by Ling in [5]. Tsuji et al. [2], followed a similar technique applying both a Laplace transformation and a generalized Fourier transformation. However, the inverse of the Laplace transformation can only be found by applying numerical techniques. Finally, the work by Goshima et al., e.g. Ref. [18], should be mentioned, which is based on the Muskhelishvili complex stress functions and the temperature field due to Mow and Chen [11].

Since the temperature field is defined in piecewise manner (see Eqs. (7.1) and (7.2)) the authors do not see an analytical way to solve the problem for stresses in closed form. Therefore, the authors prefer the finite element method as numerical concept since this method requires only first derivatives of the interpolation functions for the calculation of the stresses instead of second derivatives; see Eq. (12). Further, the finite element method has become a well established concept with effective error control algorithms.

The finite element method is now employed to find a full solution field of the thermoelastic problem, for which the relevant parameter is the dimensionless thermal penetration depth

$$\tilde{\delta} = \frac{\delta}{2a} = \sqrt{\frac{\kappa}{2a\upsilon}} \quad (13)$$

Considering steel with $\kappa = 9.1 \times 10^{-6}\ m^2\ s^{-1}$ and realistic data for a and υ ($a = 10^{-2}$ m, $\upsilon = 100\ km\ h^{-1} = 28\ m\ s^{-1}$) leads to $\tilde{\delta} \sim 0.004$. This means that, due to $\tilde{\zeta} = \delta\zeta/2a$, the coordinate $\tilde{\zeta}$ is only 0.004. Relative to this entity the heat input interval $0 \leq \xi \leq 1$ is very long. Therefore, an extremely fine mesh must be applied near the surface of the halfplane. Rectangular elements with a linear displacement field are used with length $\Delta\xi = 0.0025$ and a height of $\Delta\tilde{\zeta} = 0.0005$. In one element column 20 elements are applied in the $\tilde{\zeta}$ direction. In the area $-1 \leq \xi \leq 4$, $0 \leq \tilde{\zeta} \leq 0.01$ a total number of 40 000 elements is used. Outside of this area infinite elements are attached to represent the halfplane $-\infty \leq \xi \leq -1.0$, $4 \leq \xi \leq \infty$, $0.01 \leq \tilde{\zeta} \leq \infty$. The program ABAQUS Standard [19] is used. The relative temperature $\theta(\xi, \tilde{\zeta})$ in each nodal point is generated by a preprocessor. The calculations were performed for plane strain conditions as explained above. The total calculation time to gain one solution set (parameters $\tilde{\delta}$, ν) is about 7 min of CPU on an IBM-3BT RS/6000 workstation.

4. Results

For the stress state the following can be stated (see Figs. 3–5 which are drawn for $\tilde{\delta} = \delta/2a = 0.004$, $\nu = 0.3$ and the plane strain state):

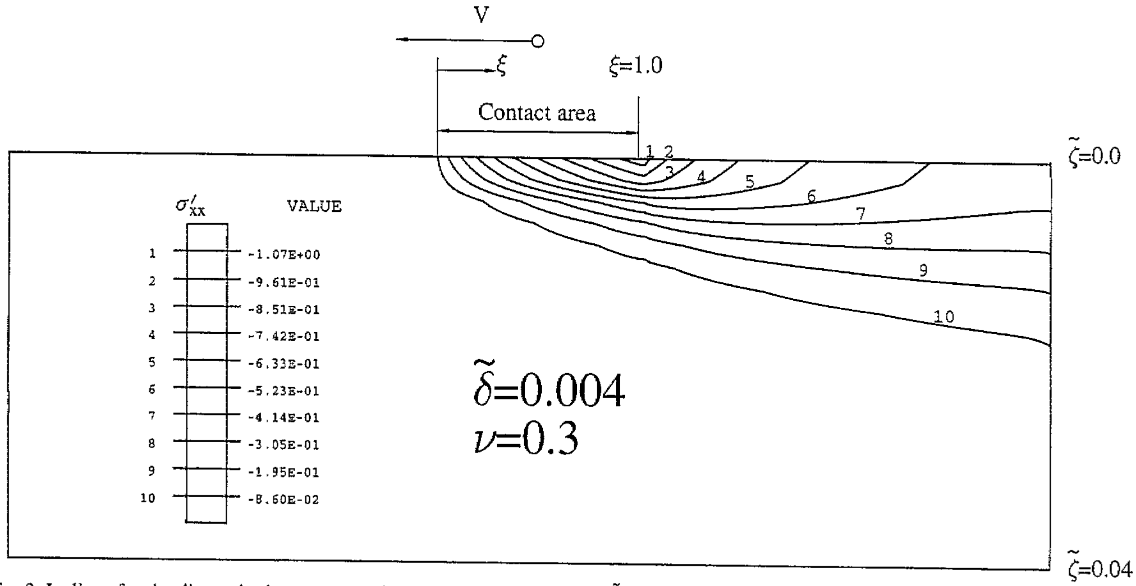

Fig. 3. Isolines for the dimensionless compressive stress component σ'_{xx}, $\tilde{\delta}=0.004$, $\nu=0.3$; plane strain; magnification in $\tilde{\xi}$ direction 50:1.

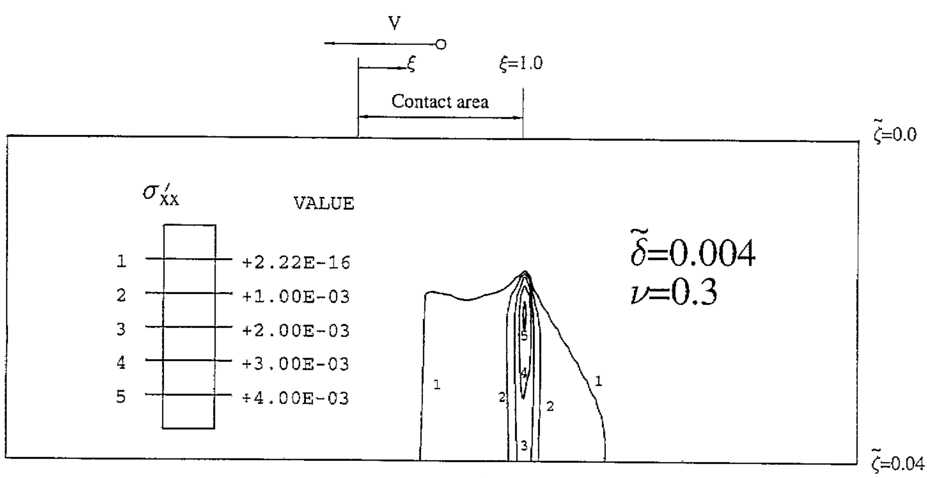

Fig. 4. Isolines for the dimensionless tensile stress component σ'_{yy}, $\tilde{\delta}=0.004$, $\nu=0.3$; plane strain; magnification in $\tilde{\xi}$ direction 50:1.

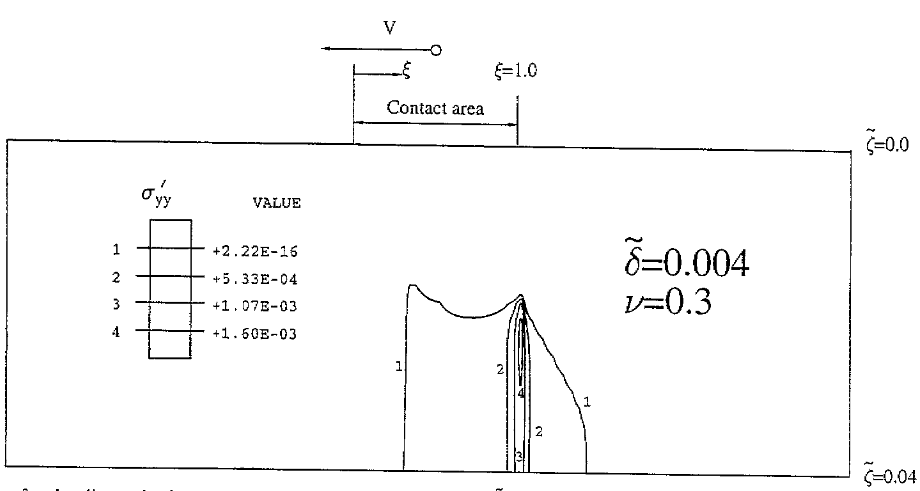

Fig. 5. Isolines for the dimensionless transverse stress component σ'_{yy}, $\tilde{\delta}=0.004$, $\nu=0.3$; plane strain; magnification in $\tilde{\xi}$ direction 50:1.

- A maximum compressive longitudinal stress component σ_{xx} ($\sigma'_{xx}=\sigma_{xx}/(E\alpha T_{\max})$) occurs near the trailing end of the contact area in the region $\xi=0$, Fig. 3, at the surface.
- A tensile longitudinal stress components σ'_{xx} occurs beneath the trailing end of the contact area; see Fig. 4. Its maximum appears in a depth of $\tilde{\xi}\sim 0.02$.

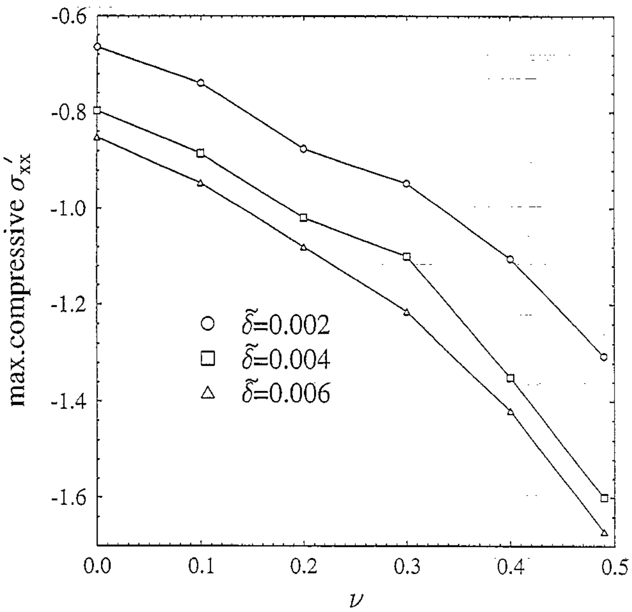

Fig. 6. Maximum compressive dimensionless stress component σ'_{xx} as function of Poisson's ratio ν depending on the dimensionless thermal penetration depth $\tilde{\delta}$. Actual stresses are found by multiplication with $E\alpha T_{max}$.

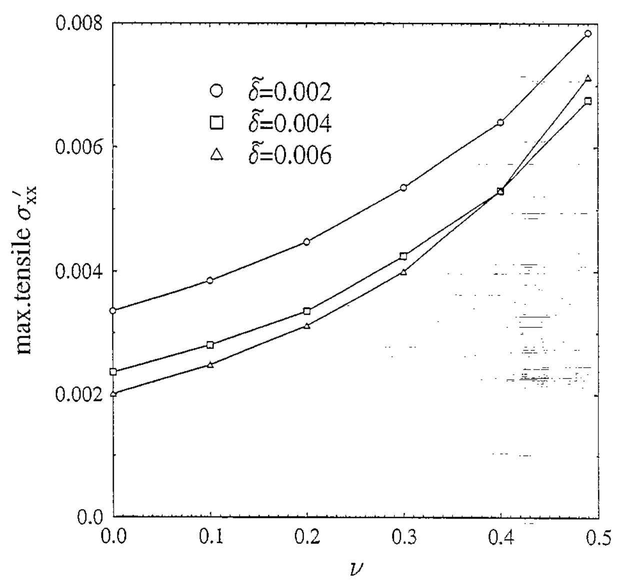

Fig. 7. Maximum tensile dimensionless stress component σ'_{xx} as function of Poisson's ratio ν depending on the dimensionless thermal penetration depth $\tilde{\delta}$. Actual stresses are found by multiplication with $E\alpha T_{max}$.

- Contours of the tensile transverse stress component σ_{yy} ($\sigma'_{yy}=\sigma_{yy}/(E\alpha T_{max})$) can be seen in Fig. 5. Their maximum appears in a depth of $\tilde{\xi}\sim 0.02$. σ'_{yy} is a principal normal stress.

It should be noted that the actual stress state is confined to in a very thin layer near the surface. Therefore, a magnification of 50:1 has been applied for the coordinate axis z, ζ in Figs. 3–5.

Fig. 6 presents a diagram for the maximum compressive surface stress σ'_{xx} as a function of ν for various $\tilde{\delta}$.

Fig. 7 demonstrates, in the same way, the maximum tensile surface stress σ'_{xx}.

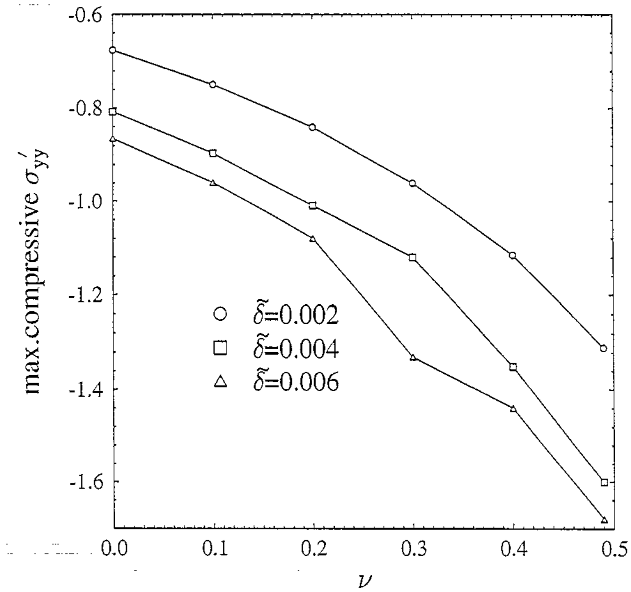

Fig. 8. Maximum compressive dimensionless stress component σ'_{yy} as function of Poisson's ratio ν depending on the dimensionless thermal penetration depth $\tilde{\delta}$. Actual stresses are found by multiplication with $E\alpha T_{max}$.

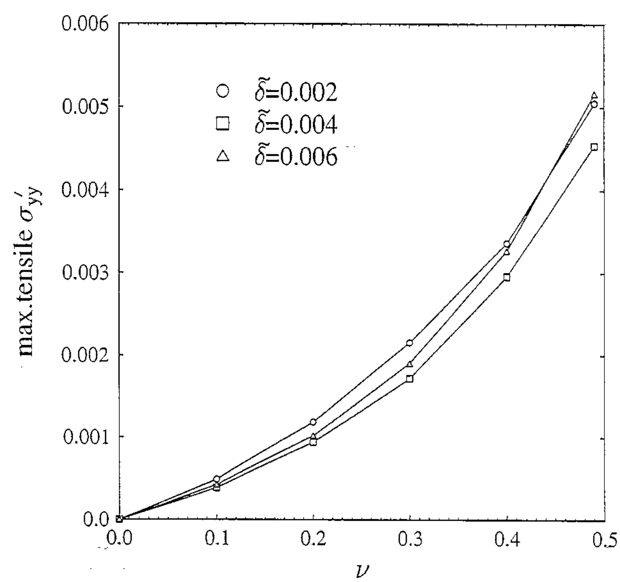

Fig. 9. Maximum tensile dimensionless stress component σ'_{yy} as function of Poisson's ratio ν depending on the dimensionless thermal penetration depth $\tilde{\delta}$. Actual stresses are found by multiplication with $E\alpha T_{max}$.

Figs. 8 and 9 give the corresponding values for σ'_{yy}.

It is also interesting to note that the stress components σ_{zz} are very small, even zero. This can be checked by the equilibrium conditions for a volume element keeping in mind that due to the plane strain state $\tau_{xy}=\tau_{yz}\equiv 0$. The following relations hold:

$$\frac{\partial\sigma_{yy}}{\partial y}=0 \qquad \frac{\partial\sigma_{xx}}{\partial x}=-\frac{\partial\tau_{xz}}{\partial z} \qquad \frac{\partial\sigma_{zz}}{\partial z}=-\frac{\partial\tau_{xz}}{\partial x} \tag{14.1}$$

Hence

$$\sigma_{yy}=\sigma_{yy}(x,z) \qquad \frac{\partial^2\sigma_{xx}}{\partial x^2}=\frac{\partial^2\sigma_{zz}}{\partial z^2} \tag{14.2}$$

An inspection of σ_{xx} along a line z = constant reveals that σ_{xx} can be well approximated by straight line segments. Hence, $\partial^2\sigma_{zz}/\partial z^2$ is zero nearly everywhere. Since σ_{zz} must be zero at the surface, it takes only very small values elsewhere.

For sake of comparison the authors shortly comment on the results by Yevtushenko and Chapovska [14]. They investigated the plane strain case for a circular and a parabolic punch. Their analysis only yields compressive stress components σ_{xx}. Furthermore, they reported small, however, not zero σ_{zz} components.

5. Conclusion

An analytical solution for the temperature distribution due to the sliding contact of a wheel–rail system in a coordinate system moving with the contact area can be given in a dimensionless form. The corresponding stress field in an elastic half plane as well as the extremes of the stress components are calculated by applying the finite element method for plane strain conditions. Diagrams are presented which allow the user to gain an immediate estimate of the stress state by multiplication of the reduced stress components by $E\alpha T_{\max}$.

Acknowledgements

F.D. Fischer express his thanks to Prof. K. Knothe, Berlin. The discussions with him motivated the authors to elaborate this paper.

Appendix A. Nomenclature

a	half contact width in forward direction (mm)
E	Modulus of elasticity (MPa)
F	Airy stress function (N)
p, p_o	contact pressure (MPa), p_o its maximum
$\dot{q}$	heat flow rate (W mm^{-2})
T	temperature (K)
$T_{\max}$	maximum contact temperature (K)
u_i	displacement component in the x, y, z direction
v	forward velocity (m s^{-1})
v_s	sliding velocity (m s^{-1})
x, ξ	coordinates along the surface of the x–z-halfplane
y, η	coordinates orthogonal to the x–z-halfplane
z, ζ, $\tilde{\zeta}$	coordinates in the x–z-halfplane orthogonal to the surface
α	thermal expansion coefficient (K^{-1})
$\tilde{\alpha}$	heat partition coefficient (1)
θ	dimensionless temperature (1)
δ	thermal penetration depth (mm)
$\tilde{\delta}$	dimensionless thermal penetration depth (1)
P_e	Peclet number (1)
κ	thermal diffusivity (m^2 s^{-1})
λ	thermal conductivity (W m^{-1} K^{-1})
ϕ, ψ	parts of the Airy stress function
σ'_{ij}	stress component (MPa)
σ_{ij}	dimensionless stress component (1), $\sigma'_{ij} = \sigma_{ij}E\alpha T_{\max}$
μ	coefficient of friction (1)
ν	Poisson's ratio (1)

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