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Determination of plastic yield stress from spherical indentation slope curve

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Abstract

The indentation slope curve from a spherical indentation on elastic-plastic materials is examined. By comparing it with that of an linear elastic material of the same elastic properties, we found that the start point of plastic yielding for an elastic-plastic material can be easily located from the indentation slope curve. Based on this analysis, a simple but effective method is proposed to measure the plastic yield stress of very small samples from a spherical nano-indentation slope curve.

Keywords: Characterization methods; Mechanical properties; Metals and alloys

1. Introduction

With the development of small scale engineering, such as micro-electro-mechanical system, and surface engineering, such as coatings and thin films, research has been carried out to use nano-and micro-indentation techniques to examine the mechanical properties of different materials in very small dimensions [1–7]. One of the focuses of such research is to search effective methods to extract material properties, e.g. Young's modulus and yield stress, from simple indentation tests. For ordinary elastic-plastic materials, many methods have been developed in the past years. For examples, Oliver and Pharr [1] developed a method to measure the Young's modulus from the initial slope of the unloading indentation curve while Cheng and Cheng proposed an energy based method to measure the Young's modulus [3]. Nayebi et al. [5] developed a method to measure the mechanical properties based on the minimization of error between the experimental and theoretical indentation curves. Tabor [8] estimated the yield stress from the hardness value whilst several fitting functions or numerical algorithms were developed to measure the elastic-plastic properties based on extensive numerical simulations [9–14]. To our knowledge, none of them have considered the information from the indentation slope curve. Stimulated by our recent work on the study of spherical indention of shape memory alloys [15], we describe in this paper an alternative simple method to estimate the yield stress of elastic-plastic materials from a spherical indention test. The basic idea is to locate the onset of plastic deformation on the indentation slope curve. Details of the method are described as follows.

2. Characterization method

We consider that a rigid spherical indenter tip with radius R is pressed into an elastic-plastic material. Here the material is assumed to follow the power law, i.e.,

$$\begin{aligned} &\sigma = E\varepsilon \text{ for } \varepsilon \le \sigma_y/E \\ &\sigma = K\varepsilon^n \text{ for } \varepsilon \ge \sigma_y/E \end{aligned} \tag{1}$$

where n is the work-hardening exponent and K is the work-hardening rate determined by $K=\sigma_y\,(E/\sigma_y)^n$. The uniaxial tensile stress–strain curve of a typical elastic-plastic material with $E=200$ GPa, $\sigma_y=600$ MPa and $n=0.1$ is shown in Fig. 1. At the beginning of loading, the material is in a linear elastic state. Once the tensile stress reaches the value of yield stress, the curve deviates from the purely elastic curve. The yield point corresponding to the onset of the plastic deformation can be identified from this tensile stress–strain curve.

If a frictionless spherical indentation is carried out on an elastic-plastic material, the indentation force F generally depends

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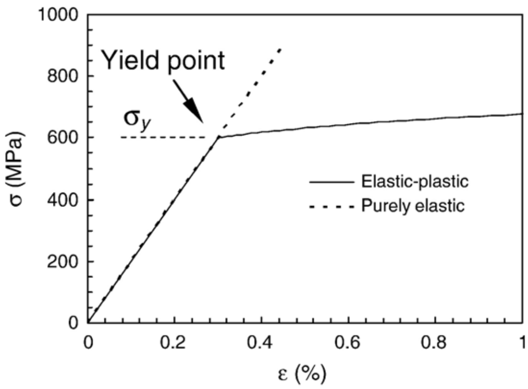

Fig. 1. The uniaxial tensile stress–strain curve of an elastic-plastic material and the yield point.

on the indentation depth h and the indenter radius R, as well as the material parameters σ_y, n, E and ν, i.e.,

$$F = Z(\sigma_y, n, E, \nu, h, R). \tag{2}$$

Similar to the tensile test, the material is in a linear elastic state at the beginning of the indentation due to the spherical shape of the indenter. Within the limits of small deformation, the Hertz contact theory can be applied to describe this elastic contact problem. The elastic indentation force before plastic deformation is therefore determined by [16]

$$F = \frac{4}{3}E^* R^{1/2} h^{3/2}, \tag{3}$$

where $E^* = E/(1-\nu^2)$. Once the maximum equivalent stress inside the material reaches the yield stress σ_y, plastic yielding will start and the indentation force-depth curve will start to deviate from the purely elastic indentation curve as demonstrated in Fig. 2(a). Here the material properties are the same as those shown in Fig. 1 and the radius of the indenter is 500 μm. The elastic-plastic indentation curve is obtained from the finite element simulation. The yield point on a spherical indentation curve is defined as the point at which the elastic-plastic loading curve starts to deviate from the purely elastic indentation curve, see Fig. 2(b). The force at the yield point is named as yield force F_y.

Because this yield point signals the start of the plastic deformation in the sample, it is an important characteristic point on the elastic-plastic indentation curve. It is noticed that the yield force F_y does not depend on the work-hardening of the material, i.e., it is not a function of n. In this sense, F_y is analogous to the yield stress σ_y shown in Fig. 1. However, the yield force is the response of the indented structure and must rely on the elastic properties of the material and the geometry of the indenter. Therefore, we have the following relationship:

$$F_y = Y(\sigma_y, E, \nu, R). \tag{4}$$

The corresponding dimensionless function is

$$\frac{F_y}{R^2 E} = \Pi\left(\frac{\sigma_y}{E}, \nu\right). \tag{5}$$

Eq. (5) can be determined either numerically or analytically by applying the Hertz contact theory for small deformation, the latter gives [16]

$$\frac{F_y}{R^2 E} = 17.92\left(\frac{\sigma_y}{E}\right)^3 \tag{6}$$

for $\nu=0.3$ as shown in Fig. 3. It is seen that a higher yield stress will result in a higher yield force, or vice versa.

We can use Eq. (6) to develop a method to determine the yield stress from the measured yield force by a spherical indentation test once we know the elastic constants, E and ν, of the material. Obviously, the accuracy of the extracted yield stress relies on the accuracy of the measured yield force. Practically, it would be difficult to obtain an accurate value from the indentation curve because it is hard to locate the yield point on the curve. For instance, in the case shown in Fig. 2(b) with $\sigma_y=600$ MPa, $E=200$ GPa, $\nu=0.3$ and $R=500$ μm, the estimated yield point from the indentation curve is at $h=49$ nm and $F_y=7.11\times10^4$ μN, which is very different from the theoretical point ($h=23.9$ nm, $F_y=2.42\times10^4$ μN) by Eqs. (3) and (6).

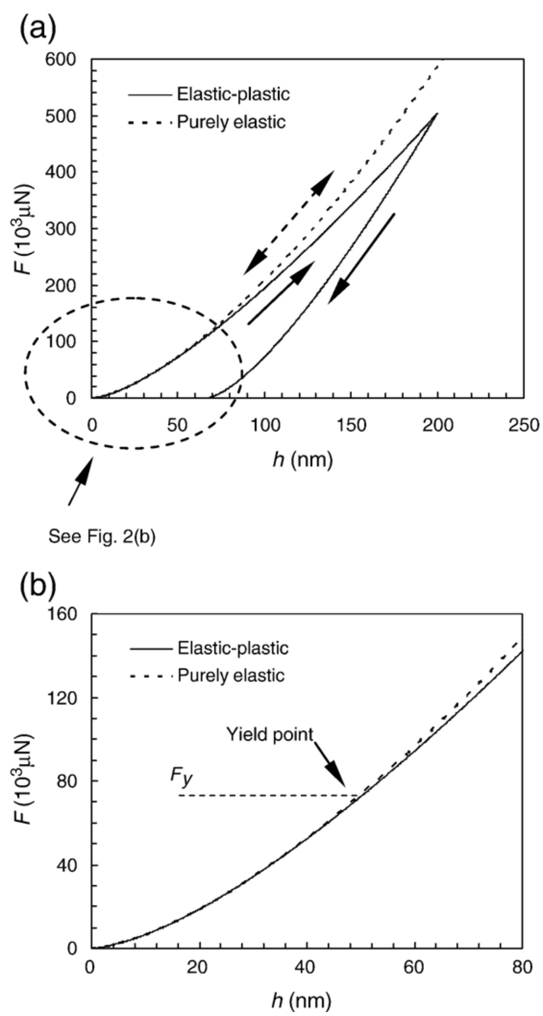

Fig. 2. (a) Comparison between a purely elastic indentation curve and an elastic-plastic indentation curve and (b) the yield point and the yield force F_y on the indentation curve.

To solve this problem, we propose using the elastic and elastic-plastic indentation slope curves, which is similar to that we proposed to measure the transformation stresses for shape memory alloys [15]. According to Eq. (3), the slope for the purely elastic indentation can be obtained as

$$\frac{dF}{dh} = 2E^{*}R^{1/2}h^{1/2}. \quad (7)$$

The slope for the elastic-plastic indentation can be obtained from a spherical indentation test in practice. To demonstrate the method, we give the elastic-plastic slope curve via a finite element simulation as shown in Fig. 4. The elastic slope curve from Eq. (7) is shown by the dashed line in Fig. 4. We can easily identify the yield point on the slope curve as the point where the elastic-plastic slope curve deviates from the purely elastic slope curve. After that, we can measure the corresponding indentation depth, h_y. In the example shown in Fig. 4, the value of h_y is about 31 nm. Combining Eq. (3) with Eq. (6), the yield stress σ_y is related to the yield indentation depth h_y by

$$\sigma_y = 0.434E\sqrt{\frac{h_y}{R}}. \quad (8)$$

Therefore, the yield stress can now be estimated from Eq. (8) as 684 MPa, which is about 14% above the real value of 600 MPa. This estimation is much better than the one from the indentation curve directly and the error, around 14%, is acceptable in most practical cases. Clearly, the accuracy of the slope curve method in determining the yield stress depends on how accurately we locate the yield point on the curve. Theoretically, it is related to the value of the work-hardening exponent n. The smaller n is, the more accurately we can locate the yield point. In the extreme case of $n=0$, which corresponds to an elastic-perfectly plastic material, the accuracy of this method can be over 90%. The major advantage of the slope curve method to measure the yield stress is that it is simple. In a real experiment, the extracting procedure is as follows:

(a). Draw the indentation loading slope curve from the measured spherical indentation curve.

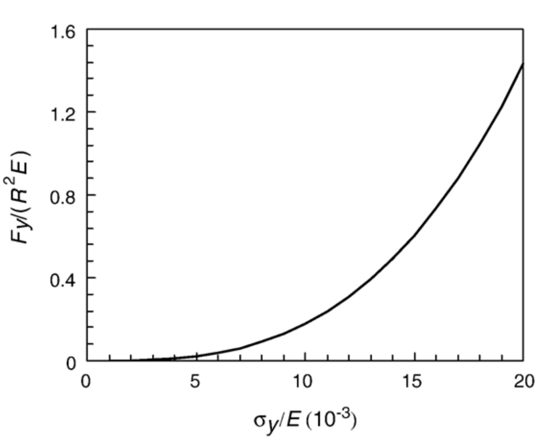

Fig. 3. Relationship between the normalized yield force $F_y/(R^2E)$ and the normalized yield stress σ_y/E for $v_a=0.3$ under small deformation condition.

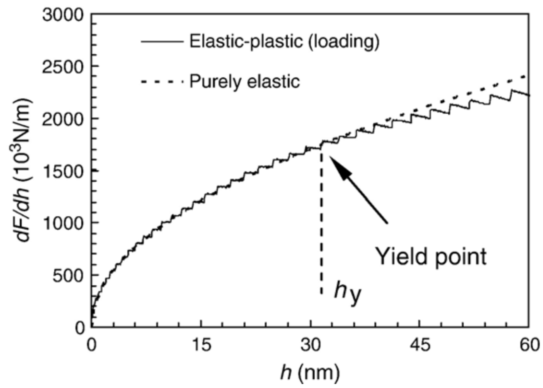

Fig. 4. Determination of the yield point and the corresponding indentation depth h_y from the purely elastic slope curve and elastic-plastic slope curve for the loading process.

(b). Draw the elastic indentation slope curve based on the Hertz elastic contact theory in the same diagram as shown in Fig. 4 for comparison.

(c). Determine the yield point by comparing the purely elastic indentation slope curve with the elastic-plastic slope curve. The corresponding indentation depth h_y can also be determined.

(d). Determine the yield stress σ_y according to Eq. (8).

3. Summary

We have identified the plastic yield point on a spherical indentation curve by comparing the purely elastic indentation curve with the elastic-plastic indentation curve. The corresponding yield force relies on the yield stress of the material, besides the elastic constants of the material and the indenter tip radius. This unique relationship provides the theoretical basis for the proposed method to determine the plastic yield stress of materials, especially in small specimens such as thin films. In order to improve the accuracy of the result, an indentation slope method is further proposed to locate the yield point and therefore to determine the yield stress.

References

[1] W.C. Oliver, G.M. Pharr, J. Mater. Res. 7 (1992) 1564.
[2] J.S. Field, M.V. Swain, J. Mater. Res. 8 (1993) 297.
[3] Y.-T. Cheng, C.-M. Cheng, Appl. Phys. Lett. 73 (1998) 614.
[4] K. Tunvisut, N.P. O'Dowd, E.P. Busso, Int. J. Solids Struct. 38 (2001) 335.
[5] A. Nayebi, R. El Abdi, O. Bartier, G. Mauvoisin, Mech. Mater. 34 (2002) 243.
[6] L. Qian, X. Xiao, Q. Sun, T. Yu, Appl. Phys. Lett. 84 (2004) 1076.
[7] W. Yan, Q. Sun, X.-Q. Feng, L. Qian, Int. J. Solids Struct. 44 (2007) 1.
[8] D. Tabor, The Hardness of Metals, Oxford University Press, London, 1951.
[9] A.E. Giannakopoulos, S. Suresh, Scr. Mater. 40 (1991) 1191.
[10] M. Dao, N. Chollacoop, K.J. VanVliet, T.A. Venkatesh, S. Suresh, Acta Mater. 49 (2001) 3899.
[11] M. Zhao, N. Ogasawara, N. Chiba, X. Chen, Acta Mater. 54 (2006) 23.
[12] B. Taljat, T. Zacharia, F. Kosel, Int. J. Solids Struct. 35 (1998) 4411.
[13] E.G. Herbert, G.M. Pharr, W.C. Oliver, B.N. Lucas, J.L. Hay, Thin Solid Films 398–399 (2001) 331.
[14] M. Beghini, L. Bertini, V. Fontanari, Int. J. Solids Struct. 43 (2006) 2441.
[15] W. Yan, Q. Sun, X.-Q. Feng, L. Qian, Appl. Phys. Lett. 88 (2006) 241912.
[16] K.L. Johnson, Contact Mechanics, Cambridge University Press, London, 1985.