Study of localized damage in composite laminates using micro–macro approach

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Abstract
A robust multiscale scheme referred to as micro–macro method has been developed for the prediction of localized damage in fiber reinforced composites and implemented in a finite element framework. The micro–macro method is based on the idea of partial homogenization of a structure. In this method, the microstructural details are included in a small region of interest in the structure and the rest is modeled as a homogeneous continuum. The solution to the microstructural fields is then obtained on solving the two different domains, simultaneously. This method accurately predicts local stress fields in stress concentration regions and is computationally efficient as compared with the solution of a full scale microstructural model. This scheme has been applied to obtain localized damage at high and low stress zones of a V-notched rail shear specimen. The prominent damage mechanisms under shear loading, namely, matrix cracking and interfacial debonding, have been modeled using Mohr–Coulomb plasticity and traction separation law, respectively. The average stress at the notch has been found to be 44% higher than the average stresses away from the notch for a 90 N shear load. This stress rise is a direct outcome of the geometry of the notch.

1. Introduction
Failure in composites often originates from stress concentration regions in the structure. The onset of such a failure is not only determined by the material properties of the composite but also depends on the lay-up and geometry of the stress raisers, for example, at the edge of circular cut-outs, notches, etc. This makes the task of developing a general theory for predicting the damage initiation in composites even more challenging. It is important to note that the present computational strategies for predicting damage initiation and evolution, in the proximity of stress raisers, are either inapplicable in the regions of stress singularities or very expensive computationally [1–4].

A number of strength-based macro-mechanical models have been proposed in the literature over the years for the prediction of failure in composites. The macro-mechanical failure models typically used are Tsai–Hill [5], Hashin’s [6] and Puck’s damage criteria [7]. However, the macro-mechanical failure models do not capture the failure mechanisms at the fiber and matrix level. The most discernible approach of studying damage at the microscopic level is to carry out a full-scale microscopic analysis of the structure by explicitly modeling all the heterogeneities. A direct consequence of inclusion of the microstructural details in the whole structure is the inevitable complexity and massive computational cost. In order to reduce computational cost and to study the microscale behavior of composites, various multiscale/multi-level methods are used, as illustrated in Fig. 1.

Multiscale methods offer the efficiency of macroscopic models and the accuracy of microscopic models. The scope of multiscale modeling is the designing of more efficient combined computational methods than the solving of the full microscopic model providing microscopic information to the desired accuracy, at the same time. To study microscale behavior of composites, researchers like González and Llorca [8], Totry et al. [1,9] have demonstrated the use of a Representative Volume Element (RVE) for the prediction of damage mechanisms at the microscale of fiber-reinforced composites. An RVE is a volume element that represents the composite statistically and includes a sample of all the heterogeneities that are present in the composite. These studies
were focused on deformation and failure mechanisms of fiber-reinforced composites under multi-axial stress states. Appropriate damage mechanisms were incorporated in the constitutive behavior of the microstructural elements such as fiber, matrix and interface. The properties of the RVE should be independent of its position in the structure. Therefore, models based on RVEs often employ periodic boundary conditions [1,8,9]. Ghosh et al. [2] proposed a coupled multiscale two-dimensional model for composites. Homogenized material coefficients were derived by performing the Voronoi Cell Finite Element Method (VCFEM) analysis using periodic boundary conditions on the base cell. A base cell is the microstructure cell containing all the inclusions and voids modeled exclusively. A finite element based multiscale model was introduced by Feyel [3] called the FE2 method. In this method, macroscopic analysis was performed using the homogenization of periodic media. An RVE was assigned to each integration point at the macroscopic scale and a separate finite element computation was performed at the microscale, simultaneously. However, the assumption of periodicity is lost at the points close to free edges or points of high stress concentration, and thus, the approaches presented in the literature [1–3,8,9], cannot be used in these areas. To address the issues related to the unsuitability of periodic boundary conditions at stress raisers, different approaches have been developed which are discussed later in the paper.

Markovic and Ibrahimbegovic [4] proposed a two-scale computational strategy for modeling the inelastic behavior of composites. The finite element method was applied to the structural scale as well as on the microscale. The localized Lagrange multipliers were used to derive the interface conditions between the micro and macroscales. Gonzalez and Llorca [10] simulated the fracture behavior of a notched fiber-reinforced composite sample under tension, where the fibers were aligned in the direction of loading and perpendicular to the notch. Their method was based on an embedded cell approach. In front of the notch tip, where the damage is concentrated, actual fiber/matrix topology was modeled while the rest of the beam was represented by a transversally isotropic homogeneous solid.

Sun and Wang [11] proposed a multi-level approach in which microscale results were obtained in a small area of interest. In the initial macro-analysis, the effective properties were derived using a suitable homogenization method, e.g., laminate theories were utilized to predict the effective (macroscopic) stress and effective (macroscopic) displacement fields. Subsequently, in the microanalysis, microscale results were obtained in the area of interest by employing macro-stresses and/or macro-displacements as boundary conditions on the microscale domain. All the microstructural details like fibers, matrix, interface, etc., were explicitly modeled in the area of interest (for example, stress concentration area). The size of the local domain was selected on the basis of a "local domain test". The sub modeling methodology available in

ABAQUS Standard® [12] was used for analyzing a part of the structure with more accuracy using a finer mesh density. Therefore, this method can potentially be used to implement the idea of homogenization and de-homogenization [11].

However, at present, the sub modeling method uses constant time step increments which limit its application to highly non-linear problems. Wang and Yan [13] proposed an inter-scale theory to study matrix failures in composite laminates. This theory utilized the macro-field of the laminate obtained after homogenization to recover the microfield at the fiber and matrix scale (de-homogenization). Borokovand Sabadash [14] have proposed a partial homogenization approach for obtaining microstresses in the regions of interest. They proposed an approach in which the microscale and macroscale are coupled as a single domain and analyzed simultaneously. Both the models proposed by Sun and Wang [11] and Wang and Yan [13] give accurate results at the microstructural scale only when the region of interest lies away from the boundaries of the microdomain. This limitation is addressed by Borokov and Sabadash [14] and the present work follows upon the idea of partial homogenization to develop and implement the proposed micro–macro approach.

In this paper, an attempt has been made to formulate a multiscale modeling approach which can address the limitations of the existing modeling approaches mentioned earlier and can be utilized to predict damage mechanisms in the regions of high stress concentrations. The ideas of homogenization and de-homogenization by Wang and Yan [13] have been extended and combined with the idea of partial homogenization provided by Borokov and Sabadash [14] for the formulation of the micro–macro approach. The micro–macro method however extends this idea to inelastic deformation while it was restricted to only linear deformations in the previous studies [14]. Following Borokov and Sabadash [14], microscale and macroscale are coupled as a single domain and analyzed simultaneously in the micro–macro approach. The microstructure is enriched with all the details like fibers, interface, etc. The use of partial homogenization leads to less computations as compared to multiscale methods described in the previous studies [2–4,10]. The drawback of the works which present local analysis via multiscale methods [11,13] are also addressed, since the micro-structural solution remains accurate even at the boundaries of the microdomain. The details of the formulation of the scheme and its application for investigating damage mechanisms in regions near a notch root are outlined in the following sections.

The next section presents the locality principle on which the micro–macro method has been formulated. The details of the finite element geometry, material models and the boundary conditions used in the model are mentioned in this section. The micro–macro model is validated by comparing the predicted shear response of a cross-ply laminate against the experimental shear response obtained from V-notched rail shear test [15]. After model validation,
the effects of phase and interface properties in the microdomain have been characterized. Finally, two different micro–macro models have been developed to distinguish the damage initiation and propagation near the stress raisers and away from it. In the first case, damage evolution near a notch root is studied whereas the second case deals with damage evolution away from the notch root.

2. Formulation of the micro–macro approach based on the locality principle

The locality principle states that the effect of homogenization of the structure does not influence the homogenized part farther than the characteristic length of the structure. Typically, the characteristic length is of the order of the unit cell containing a single fiber [14]. This principle can be used to model the microstructure in the region of interest and the remaining area could use homogenized or effective properties.

In order to confirm that the proposed micro–macro methodology based locality principle is valid, a prediction of localized damage in the composite via a micro–macro analysis is compared with a full-scale microstructural analysis. The damage in the epoxy matrix is modeled using Mohr–Coulomb plasticity and the accumulated plastic strain is taken as a measure of the damage. Fig. 2(a) shows a randomly distributed fiber array consisting of 25 fibers which are surrounded by the matrix material. It also shows the center fibers around which the accumulated damage in the matrix is captured. Fig. 2(b) shows the corresponding partially homogenized lamina as per the proposed micro–macro scheme. Fig. 2(c) shows the mesh geometry of the micro–macro analysis. A shear load in $X$–$Y$ plane is applied on the lamina as shown in Fig. 2(b). Fig. 3 shows the damage in the epoxy matrix versus the volume averaged shear strain in the localized region of interest for three different approaches, viz., complete microstructural modeling, sub-modeling and micro–macro. The two-step sub modeling approach has also been used predict the localized damage and compared with the proposed micro–macro method. The solution time for the sub-modeling approach is double that of the micro–macro method since it requires two separate analyses (one at macroscale and another at microscale). It can be seen that the micro–macro scheme prediction is in good agreement with the full-scale microstructural analysis and the two-step sub-modeling.

Note that the micro–macro method evaluates the distribution of stresses in the macrostructural and the microstructural domains via a coupled concurrent analysis of both the domains. This method uses partial homogenization in which the region other than the area of interest (where the microstructure is explicitly modeled) is homogenized. The basic difference between conventional multi-level methods, i.e., inter-scale theory and the partial homogenization method is that the microstructural and macrostructural analyses are not performed simultaneously in the multi-level methods. On the other hand, in the partial homogenization method, the two length scales are analyzed simultaneously. This leads to the required coupling of the two scales, eliminating the need for two separate analyses. In the proposed scheme, initially, a region of interest (or a local domain) is identified in the structure, which is typically the region of a stress concentration, free edges, crack tips, etc. This region is modeled with the microstructure of the composite, that is, randomly distributed fibers in the matrix. The rest of the structure is modeled as a homogeneous continuum with effective properties. Strong kinematic coupling is incorporated between the local domain and the homogeneous continuum. This modified structure, that is, the original structure along with the modified local domain is then solved and the required micro-stresses in the region of interest are obtained. Fig. 4 illustrates the micro–macro method. Fig. 4(a) shows the combined micro–macro model, Fig. 4(b and c) show details of the microdomain in the combined model.

Note here that the most of the multi-level models reported in the literature assume elastic homogenized properties for the
macro-domain [11,13]. The global response for non-homogenous strain rate cannot be studied via this approach; ply-level failure can be incorporated by using quadratic failure criteria, such as Hashin’s and Tsai–Hill criteria. Consequently, some form of macro-level failure can be incorporated.

The micro–macro approach described here will be used in the following sections for developing a model to analyze the damage behavior of the specimen used in V-notched rail shear test [15]. In-plane loading was chosen for developing the model, since the damage mechanisms are fairly complex under shear. The model utilizes the Mohr–Coulomb criteria for simulating damage in the matrix, and a traction separation law for simulating fiber–matrix interface failure. Damage mechanisms in the stress concentration area near the notch root can be investigated and compared against the damage mechanisms away from the notch root.

3. Finite element modeling

3.1. Model geometry

This scheme requires the modeling of both the microstructure and the macrostructure. The geometry of the structure is same as the geometry of the specimen used in the ASTM-D7078 standard [15], as shown in Fig. 5(a). The [0/90] laminate specimen contains a double V-notch and a significant portion to either sides of the V-shaped notches is clamped tightly during the experiment and does not undergo deformation. Consequently, these sections are not included in the finite element model geometry to reduce the computational load. This can be seen in Fig. 5(b) which provides the details of the geometry of the structure used in the FE model. The microstructure consists of two cubical cells with fibers in one cell oriented orthogonally with respect to fibers in the other cell to represent a [0/90] laminate. Each cell contains randomly distributed glass fibers in the matrix with a fiber volume fraction of 28%. The fiber volume was measured as 28% post fabrication. Hence, the model used a volume fraction of 28% to corroborate the experimental response. However, this is not a limitation of the model. The fiber diameter is 24 μm and 24 fibers are modeled within a cube of 0.2 mm edge length. The remainder of the cube is composed of matrix material. The cohesive layer between the fiber and the matrix is modeled as a layer of cohesive elements with zero initial thickness. 8-Noded 3D cohesive elements were used for meshing this layer in ABAQUS Standard®. The mesh is created by offsetting a layer of zero thickness from the mesh containing fiber and the matrix. The size of the cube can be determined by the volume required to capture all the relevant microstructural details. The detailed geometry of the microstructure is as shown in Fig. 6.
Fig. 7 shows a magnified view of the section of the structure which is cut by the plane of symmetry PQRS and contains the line AB. The position of the slot with reference to the structure is clearly illustrated by the cross sectional view of the slot in the plane PQRS. The dimensions of the slot are 0.2 mm × 0.2 mm × 0.4 mm. The microstructure is placed in that region as shown in Fig. 7.

3.2. Material and damage models

Each layer of the macrostructure is considered to be a homogeneous unidirectional elastic lamina under plane stress conditions and the stiffness values are provided in Table 1, obtained by using laminate theory. E-glass (ER-459L) fibers were modeled as linear elastic isotropic solids and their constants are given in Table 2. The epoxy matrix (EPOFINE-556) with FINEHARD-951 hardener was assumed to behave as an isotropic, elasto-plastic material and its elastic constants are also provided in Table 2.

The plastic deformation is governed by the multi-axial Mohr–Coulomb criterion. The Mohr–Coulomb criterion described in Jiang and Xie [16] considers that yielding takes place when the shear stress, $\tau$, acting on a specific plane reaches a critical value, which is a function of the normal stress, $\sigma_n$, acting on that plane, as indicated in Eq. (1). The corresponding yield surface, written in terms of the maximum and minimum principal stresses ($\sigma_1$ and $\sigma_3$) is given by Eq. (2):

$$\tau = c + \sigma_n \tan \phi \quad (1)$$

$$F(\sigma_1, \sigma_3) = (\sigma_1 - \sigma_3) + (\sigma_1 + \sigma_3) \sin \phi - 2c \cos \phi \quad (2)$$

where $c$ and $\phi$ are matrix cohesion and the matrix friction angle, respectively. The matrix tensile strength, $\sigma_{mt} = 75$ MPa and $\sigma_{mc} = 105$ MPa have been measured experimentally and used for the calculation of $c$ and $\phi$ using Eq. (3).
The fiber–matrix interfacial decohesion was simulated using standard cohesive surface elements in the ABAQUS Standard [12]. The mechanical behavior of the interface was simulated using a traction–separation law which relates the displacement across the interface to the force vector acting on it. In the absence of any damage, the interface behavior was assumed to be linear with a high value of initial stiffness, \( K \), to ensure displacement continuity at the interface. The linear behavior ends at the onset of damage, using a maximum stress criteria expressed as:

\[
\max \left\{ \frac{t_n}{N}, \frac{t_s}{S} \right\} = 1
\]

where \( t_n \) and \( t_s \) are normal and tangential tractions transferred by the interface, respectively. \( N \) and \( S \) are the normal and tangential interfacial strengths, assumed to be equal, for simplicity. After the onset of damage, the damage evolution of the interface was displacement controlled and linear, as shown in Fig. 8.

Fracture energy, \( \Gamma' \), is another parameter which controls the interface behavior other than cohesive strength \( (N, S) \). It is defined as the total energy needed for complete decohesion of the interface. The interface failure model assumes that the energy consumed during the fracture of the interface is independent of the loading path. The fracture energy, \( \Gamma' \) is described as

\[
\Gamma' = \frac{1}{2} t \Delta \delta
\]

where \( t \) (\( t_n \) or \( t_s \)) is the cohesive strength of the interface and \( \Delta \delta \) is the total change in displacement across the interface. The interface properties for the material system have not been explicitly reported in the literature. The energy necessary for complete decohesion has been chosen as 100 J/m\(^2\) for all the simulations in the present work, which is a reasonable range for glass fiber/epoxy matrix composite laminate, as reported by Zhou et al. [17].

### 3.3. Boundary and contact conditions

The boundary conditions applied on the structure are illustrated in Fig. 9. Since the loading applied on the specimen is purely in-plane shear, the face ABPQ is fixed and \( U_x \) is set to zero on the opposite face DCRS. The structure is loaded by giving some arbitrary positive displacement \( \delta_0 \) to face DCRS and setting \( U_y \) to \( \delta_0 \). Displacement controlled loading has been used such that the load carried by the structure decreases as the structure fails which allows for a slower rate of failure.

The connection between the microstructure and macrostructure has been established through kinematic constraints. The constraint type used in this work is a surface-based tie constraint [12].

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**Fig. 8.** Standard traction–separation law.

**Fig. 9.** Boundary conditions on the finite element model for in-plane shear.

**Fig. 10.** Mesh seeds in the microstructure.
work were carried out by keeping the number of mesh seeds fixed at 30, as no appreciable improvements in the results have been observed by increasing mesh density any further. SC8R continuum shell elements were used to discretize the macrostructure. C3D8R 3D solid elements were used for discretizing the fiber and the matrix. COH3D8 cohesive elements were used for the modeling of the fiber–matrix interface. A total number of 142,526 elements and 157,265 nodes have been used in the microscale model.

4. Model validation

Throughout this section, \( \tau_{12} \) and \( \varepsilon_{12} \) refer to the in-plane shear stress and in-plane shear strain averaged over the volume of the microstructure, unless mentioned otherwise. Volume averaged stresses and strains are determined by Eqs. (6) and (7).

\[
S_{ij} = \frac{1}{V} \int_S s_{ij} dV
\]

\[
E_{ij} = \frac{1}{V} \int_S e_{ij} dV
\]

where \( S_{ij}, E_{ij} \) are the volume averaged stresses and strains respectively and \( s_{ij}, e_{ij} \) are the microscopic strains in a microdomain whose volume is \( V \).

Since the strain gauges measure average strains over a relatively large area as compared with the micro-domain of micro–macro method, the experimental response depicts the average value obtained from the strain gauges. Fig. 12 shows that the shear response predicted by the micro–macro scheme plotted as \( \tau_{12} \) versus \( \varepsilon_{12} \) is in good agreement with the experimental response [18]. The curve showing experimental response illustrates the in-plane shear response of the rail shear specimen tested according to ASTM Standard D7078 [15]. For doing micro–macro simulations, the matrix friction angle, \( \varphi \), and matrix cohesive strength, \( c \), are determined as \( 10^5 \) and 44.7 MPa, respectively, from Eq. (3). The interface stiffness is chosen to be 10 GPa/mm, the interface strength as 30 MPa and the interface energy as 100 J/m². As shown in Fig. 12, the linear portion of both the predicted as well as experimental response extends up to 1.5% strain. The maximum prediction error is limited to ~8%. The difference in the experimental response can be attributed to the assumptions in the model and errors in measurement and the variation in the properties of the constituents due to local defects and in-homogeneities. Once the matrix starts yielding, the response becomes non-linear in nature. The transition between the linear and the non-linear regimes is relatively smooth in the experimental data. However, the transition is more pronounced in the model and the non-linear response commences at a strain of 1.5%. This sudden transition could be explained by the initiation of matrix yielding. After model validation, a parametric study was performed to study the effect of various phases and interface properties on the stress–strain response of the microstructural domain.

5. Effect of the phase and interface properties of the microdomain

Interface and phase properties can affect the damage initiation and progression. This section is focused on quantifying the effects of interface and phase properties which can affect the damage response of the microstructural domain. It has been observed in authors’ previous work [18] that for the current material system (volume fraction and/or phase and interface properties) the effect of delamination is negligible on both local and global response. As a result, the effect of delamination was not explicitly considered in the present model. However, it can be easily implemented in the model for other loading conditions where delamination may be significant.

5.1. Effect of interface fracture energy

The effect of the variation of fracture energy is discussed in this section. The interface energy is a difficult quantity to measure experimentally and the values can vary within an order of magnitude depending upon the test method. The values for the fracture energy reported in the literature lie anywhere between 1 J/m² and 100 J/m² [17]. The proposed approach can potentially capture the effects of variations in the material properties which can be very useful for design purposes. One can estimate the variation in the local response for the lower and upper bounds of the interfacial energy and other parameters that can affect the interface damage and the matrix damage occurring in a local region. The fracture energy is determined by the relationship given in Eq. (5), which indicates the amount of energy released during the failure of the interface. The strength and the stiffness of the interface
are kept constant with values equal to 30 MPa and 10 GPa/mm, respectively. The matrix friction angle \( \phi \) is kept as 10°. It can be seen from Fig. 13 that increasing the fracture energy reduces the amount of interfacial damage at a particular strain value. At constant strength and stiffness, increasing the fracture energy increases the separation, \( d \), for the final fracture of the interface. Consequently, the completion of the debonding process (final separation) is delayed, which ensures high stress values at the same strain for higher fracture energy.

Fig. 14 shows the contour plots of interface damage at 8% shear strain. The legend indicates the value of the variable, which detects the final fracture of the interface. The variable can take any value between zero and one. A value of less than one indicates that the damage has initiated, while a value of one indicates the completion of the debonding or decohesion process. It can be seen in Fig. 14 that the contour plot of the laminate with a fracture energy equal to 10 J/m\(^2\) shows a significant amount of completely damaged interfaces; whereas, the laminate with fracture energy equal to 100 J/m\(^2\) does not show complete fracture anywhere. This is an expected result (as discussed in this section) because the amount of interface damage decreases with increasing fracture energy.

5.2. Effect of matrix friction angle

Other than the interface fracture energy, it is expected that the matrix properties may also affect the response of the microdomain. The shear stress–strain responses of the microdomain predicted by this micro–macro scheme are shown in Fig. 15. The interface properties are kept fixed interface stiffness as 10 GPa/mm, interface strength as 30 MPa and interfacial energy as 100 J/m\(^2\). The matrix friction angles have been changed from 5° to 15°, which are commonly reported values of friction angle for the epoxy matrix [8]. Since the matrix tensile strength was assumed to be constant and equal to 75 MPa, changes in the friction angles modifies the cohesive strength ‘\( c \)’ of the matrix as given in Eq. (3). The cohesive strength of the matrix increases from 37.5 MPa (\( \phi = 0^\circ \)) up to 48.8 MPa (\( \phi = 15^\circ \)). It can be seen from Fig. 15 that the onset of non-linear behavior is affected by changing the friction angle in composites. A lower value of the friction angle leads to an earlier onset of matrix yielding because the cohesive strength reduces when the friction angle is decreased and, therefore, the resistance to shear deformation decreases.

6. Application of micro–macro analysis for characterization of stress raisers

The first step in performing the micro–macro analysis of any structure is to identify the regions of interest based on the stress distribution in a continuum model of the structure. Fig. 16 shows the von-Mises stress distribution in a double V-notch cross-ply laminate under shear loading. The stress concentration region can be seen near the notches. Fig. 16 also shows the location of
the two microscale domains selected on the basis of the stress distribution. One microscale domain is placed below the notch at the top and the second one is placed away from the notch so that there is no stress concentration near the second microscale domain. The idea behind placing two microscale domains is that the results from both cases will clearly elucidate the differences in the damage initiation and propagation due to a stress raiser.

The effect of the stress-raiser in the microdomain is fairly pronounced if the shear stresses are plotted as a function of the applied shear load as shown in Fig. 17. It can be seen that if the microdomain lies in the stress raiser (near the notch root) the volume averaged shear stress developed is 37.5% higher than the average stresses developed in the region away from the stress raiser under a 40 N shear load. The volume averaged stresses in the stress raiser increases multifold (~1.45 times the stresses developed in the region away from the stress raiser) if the applied shear load is increased to 90 N. Hence, it can be interpreted that the micro–macro approach is useful in characterizing damage initiation and propagation in the hot zones, such as notches, holes, free edges and geometric discontinuities.

It is important to note here that the difference between stresses at two hotspots, i.e., one close to the notch and another away from the notch, can be quantified through macro model analysis. However, to quantify the damage initiation and evolution in terms of accumulated matrix damage accumulated and fiber–matrix debonding the analysis of micro-domain is essential. These microscopic damage mechanisms distinctly elucidate the differences in the behavior of regions close to stress singularities with those away from such singularities. It may be noted that this knowledge can be used for enhanced design/reinforcement at the hot spots.

### 6.1. Evolution of stresses and strains in microdomain

It may be noted that the accumulated plastic strains and stresses in the microdomains under the notch and away from it can potentially provide information about damage initiation and propagation as shown in Totry et al. [19]. As seen previously in Fig. 17, at a given shear load, the microdomain near the notch has higher volume averaged shear stresses as compared to the microdomain away from it. Consequently, it is expected that the stress evolution in the microdomains in the stress raiser and away from it will be different. To characterize the stresses in the two microdomains, the contour plots of von-Mises stress at a 90 N shear load are shown in Fig. 18. The maximum von-Mises stress in the region near the notch root is 82% higher than the maximum von-Mises stress induced in the region away from the notch root. This clearly shows that the effect of stress raisers on the micro-stresses can be characterized effectively via the micro–macro approach. Note that Fig. 19 shows the accumulated plastic strain in the matrix. It can be seen that the plastic strain accumulates in the form of shear bands.
in the matrix and is not uniformly distributed in the matrix. These shear bands run parallel to the fibers and interact with each other, which leads to deformation in the matrix. It is seen that significantly high plastic strains are induced at the same load in the stress raiser and the maximum accumulated plastic strain in the stress raiser is 160% higher than the maximum accumulated plastic strain induced in the microdomain of the region away from the notch. Note that stresses and strains at the boundaries of the microdomain may be problematic, however as mentioned in Section 2, that homogenization of the structure does not influence the homogenized part farther than the characteristic length of the structure. The stresses and strains predictions away from the boundary are relatively accurate.

6.2. Damage evolution at the microdomain

Besides evaluating the stresses and strains, the micro–macro method can also be used for characterizing the damage evolution in the matrix. Fig. 20 shows the evolution of the damage in the matrix as a function of the shear load for both the microdomains (in the vicinity of the notch root and away from it). It is observed that the damage in the matrix is higher at the notch as compared to away from the notch at every instance. This is expected because when the von-Mises contour plots of Fig. 18 show that at any location, the magnitude of stresses at the notch are higher; this typically gives rise to larger plastic deformation of the matrix. Damage in matrix material at notch is ~10% higher as compared to damage in matrix for away from the notch at a 90 N load.

The damage at the fiber–matrix interface is plotted in Fig. 21. It can be observed that the interface damage is more pronounced in

![Fig. 19. Contour plot of accumulated plastic strain in the matrix of cross-ply laminate at (a) notch (b) away from the notch at a 90 N load.](image_url)

![Fig. 20. Comparison of evolution of the percentage of matrix elements damaged in the microstructure between the notch and away from it.](image_url)

![Fig. 21. Contour plot of damage of the fiber–matrix interface (a) notch (b) away from the notch at a 90 N load (the value 1 indicates total failure of the interface).](image_url)
the microdomain at the notch root as compared to the microdomain away from the notch. The maximum damage value increases by 75% due to the presence of a stress raiser. It can also be observed that the fiber–matrix decohesion progresses from the center to the edges of the fibers in both the cases.

7. Conclusions

A robust micro–macro multiscale scheme has been proposed in this work which can be applied to predict local response and microscale damage in laminates under multi-axial loading. This paper is focused on using this scheme for predicting microscopic stress fields for shear loading of cross-ply laminate. It has been found that the response to in-plane shear loading is greatly influenced by the properties of the phase and interface. Furthermore, this work investigated the effect of the stress-raiser and the local response in the microdomain near notch root of a rail shear specimen used in ASTM Standard D7078. The response has been compared with the local response of the microdomain away from the notch root and significant differences in stress evolution and damage behavior have been found. The key findings can be summarized as follows:

- The predicted response with proposed micro–macro scheme is in good agreement with the experimental response and other established multiscale techniques, such as sub-modeling.
- Interfacial fracture energy has significant effect on the material response. Increase in fracture energy enhances the plastic response of the composite laminate. The experimental results indicate that interfacial fracture energy of 100 J/m² is appropriate for capturing the constitutive response in the plastic zone.
- A lower value of the matrix friction angle leads to an earlier onset of matrix yielding causing significant reduction in stiffness of the material.
- It has been observed that 37.5% higher volume averaged shear stress is developed in the micro-domain near the notch root as compared average stress developed in the micro-domain away from the notch at 40 N shear load. The volume averaged stress value in the stress raiser is ~1.45 times the average stress developed in the region away from the stress raiser if the applied shear load is increased to 90 N.
- The maximum von-Mises stress and accumulated plastic strains in the region near the notch root are 82% and 160% higher than the maximum von-Mises stress induced in the region away from the notch root, respectively.
- Damage in matrix material at notch is ~10% higher as compared to damage in matrix far away from the notch at a 90 N load.
- It can also be observed that the fiber–matrix decohesion progresses from the center to the edges of the fibers in both the cases. Maximum interfacial damage is 75% higher in the region near the notch as compared to the region away from the notch.

References