Effects of phase transition on the hardness of shape memory alloys

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Based on the dimensional analysis and finite element calculations, the effects of phase transition on the hardness of shape memory alloys were unveiled. It is shown that the hardness of shape memory alloys increases with the increase in the phase transition stress, the increase in the plastic yield stress, and the decrease in the maximum transition strain. However, the ratio of the hardness to the phase transition stress or the ratio of the hardness to the plastic yield stress is not a constant and therefore, the hardness of shape memory alloys cannot be treated as a material property. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3160740]

Shape memory alloys (SMAs), such as near-equiatomic NiTi alloys have many engineering applications in microelectromechanical systems, biomedical devices, and implants due to their superelasticity, shape memory effect, and good biological compatibility.[1–4] In some applications, wear can become a major concern. High wear-resistant property of SMAs due to their superelasticity[5,6] plays an important role to ensure the uses of such systems and devices. Qian *et al.*[7] reported an anomalous relationship between the hardness and microwear property of a superelastic NiTi SMA and explained the increase of indentation hardness (Berkovich indenter) as a result of increased phase transition stress at elevated temperatures. However, the quantitative relationships between the indentation hardness of shape SMAs from a sharp indenter and the transition properties have not been reported yet.

In this letter, based on the dimensional analysis extended by Cheng and Cheng[8,9] into the indentation hardness of ordinary metals and by Yan *et al.*[10–12] into that of SMAs from a spherical indentation without plastic deformation, the effects of phase transition on the indentation hardness of NiTi SMAs from a sharp indentation containing the plastic deformation of stress-induced martensite phase are investigated. A sharp conical indenter with a half angle of 68°, which corresponds to a Berkovich indenter, is considered.[9] Finally, the quantitative relationships between the indentation hardness and the transition stress and the maximum transition strain are obtained from the finite element method.

In the finite element calculations, the deformation of the material due to phase transition from austenite to martensite and the deformation due to the plastic yielding of the transformed martensite are described by a "two-step shaped" idealized stress-strain curve under uniaxial loading, as shown in Fig. 1. Following this curve from the original point, phase transition will occur when the stress reaches the transition stress σ_{AM}. The maximum strain caused by the complete transition is denoted by ε^{tr}. Continuous loading will result in the elastic deformation of the transformed martensite until the stress reaches the yield stress of the martensite σ_{My}. Here, both of the deformations due to phase transition and plasticity are treated as perfect without hardening. Isotropic behavior is assumed under three-dimensional loading conditions. There is no need to deal with the deformation behavior under unloading as the indentation hardness of the materials is independent of the unloading process.

The dimensional analysis for the indentation hardness of SMAs with the plastic deformation of induced martensite is outlined as follows: For a given rigid conical indenter with a half angle θ and a given SMA, the force F on the indenter and the contact depth h_c during the loading part of indentation can be written as

$$F = E_A h^2 \Pi_1\left(\frac{\sigma_{AM}}{E_A}, \frac{\sigma_{My}}{E_A}, \varepsilon^{tr}, \frac{E_M}{E_A}, \nu_A, \nu_M, n, \theta, \mu\right), \quad (1)$$

$$h_c = h\Pi_2\left(\frac{\sigma_{AM}}{E_A}, \frac{\sigma_{My}}{E_A}, \varepsilon^{tr}, \frac{E_M}{E_A}, \nu_A, \nu_M, n, \theta, \mu\right), \quad (2)$$

where E_A, ν_A and E_M, ν_M are Young's moduli and Poisson ratios of austenite and martensite phases, respectively; μ is the coefficient of friction between the indenter and SMA, and n represents the hardening exponent of plastic deformation for the induced martensite. Recent study on ordinary metals found that the friction effect can be neglected if the

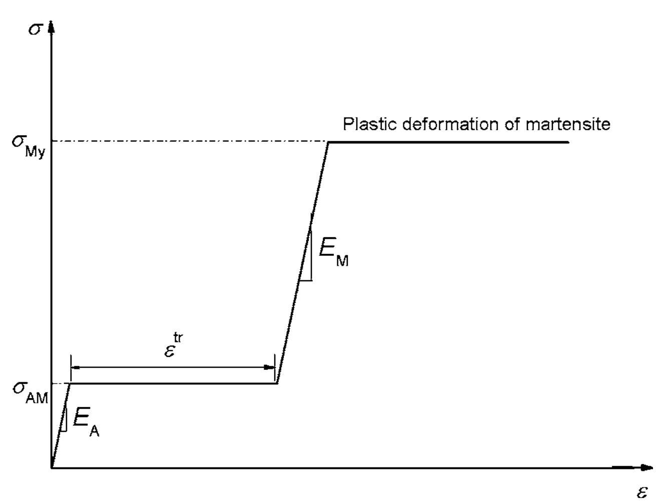

FIG. 1. Idealized loading stress-strain curve of an SMA with martensite plastic deformation.

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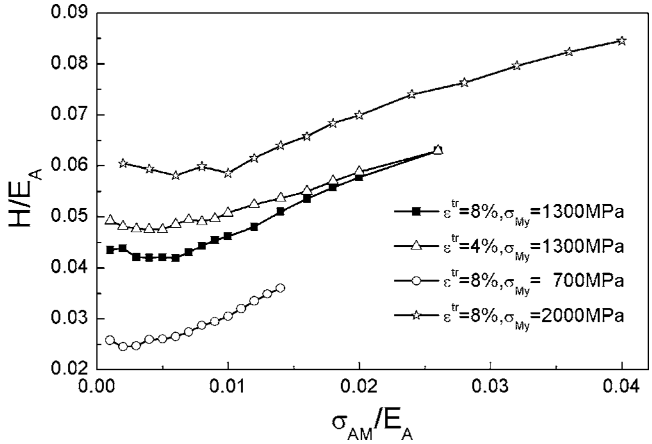

FIG. 2. Relationships between dimensionless hardness H/E_A and dimensionless phase transition stress σ_{AM}/E_A for different materials.

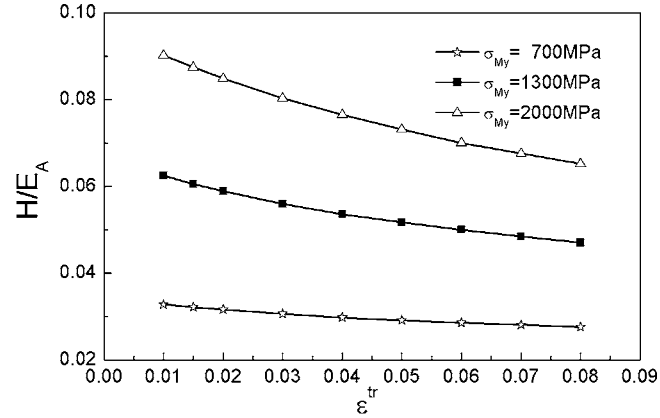

FIG. 3. Relationships between dimensionless hardness H/E_A and maximum phase transition strain ε^{tr} ($\sigma_{AM}=350$ MPa) for different yield stress values.

indenter angle is larger than 60°.[13] Therefore, here we consider a frictionless investigation with $\mu=0$. Furthermore, the plastic hardening behavior is neglected, i.e., $n=0$. If we assume that the Young's moduli and Poisson ratios of austenite and martensite phases are the same, and set the values of E_A (=50 GPa), ν_A (=0.3), and θ (=68°) to be constant, we can deduce the scaling relation of the indentation hardness H as,

$$H=\frac{F_m}{A_m}=\frac{F_m}{\pi h_c^2|_{h=h_m}\tan^2\theta}=E_A\Pi_3\left(\frac{\sigma_{AM}}{E_A},\frac{\sigma_{My}}{E_A},\varepsilon^{tr}\right), \quad (3)$$

where F_m and h_m are the maximum indenting force and depth, respectively, and A_m is the projected contact area under the load F_m. It implies that the frictionless indentation hardness of SMA with a given conical indenter depends on the elastic Young's modulus E_A, the transition stress σ_{AM}, the maximum transition strain ε^{tr}, and the martensite yield stress σ_{My}.

The scaling relationships [Eqs. (1)–(3)] indicate that the dimensionless force $F_m/E_A h_m^2$, contact depth h_c/h_m, and indentation hardness H of SMA under a sharp indenter are independent of the indenting depth h_m, which is in line with ordinary elastoplastic metals.[8,9] Such conclusions are confirmed by the numerical results from the finite element calculations. The quantitative relationships between the hardness and the phase transition stress σ_{AM} and maximum transition strain ε^{tr} from the numerical calculations are presented below.

Figure 2 shows that when the transition stress σ_{AM} is very small ($<0.005E_A$ for the cases with martensite yield stress $\sigma_{My}=1300$ MPa), the indentation hardness slightly decreases with the increase in σ_{AM}. After that, the hardness increases monotonically with σ_{AM}. The value of the turning point varies from $0.008E_A$ to $0.003E_A$ depending on the value of martensite yield stress σ_{My}. Experimental study by Qian *et al.*[14] found an anomalous increased hardness of superelastic NiTi alloy when the ambient temperature changes from 22 to 140 °C, which corresponds to the increase of the phase transition stress from near 400 to 1200 MPa while the yield stress of the martensite is almost unchanged within this range of temperature. The influence of the phase transition stress on the hardness shown in Fig. 2 quantitatively explains this experimental finding.

The effect of the maximum transition strain ε^{tr} on the hardness of NiTi SMA was not investigated in Ref. 14, while their experimental data shown a decreased maximum transition strain at elevated temperatures. Our calculated results in Fig. 3 conclude that if the transition stress and martensite yield stress are constant, a decreased maximum transition strain results in an increased indentation hardness, especially for the cases with a higher martensite yield stress. Figure 3 also illustrates that the effect of the maximum transition strain on the indentation hardness of SMA depends on the value of the transition stress. When the transition stress is relatively higher and closer to the martensite yield stress, the variation of the maximum transition strain hardly influences the hardness value. We can conclude that the anomalous increased hardness of superelastic NiTi alloy is caused by both the increased transition stress and the decreased maximum transition strain as the ambient temperature is elevated, and the transition stress becomes a dominated factor as the transition stress is close to the martensite yield stress.

Figure 4 further shows that the indentation hardness of SMA is not proportional to the transition stress, and the ratio of the indentation hardness to the transition stress decreases continuously with the increase of the transition stress, even if the transition stress is close to the prescribed martensite yield stress.

Figure 5 shows the relationships between the ratio of the indentation hardness to the martensite yield stress and the normalized yield stress of the SMA, compared with the results of an ordinary metal without any phase transition. For

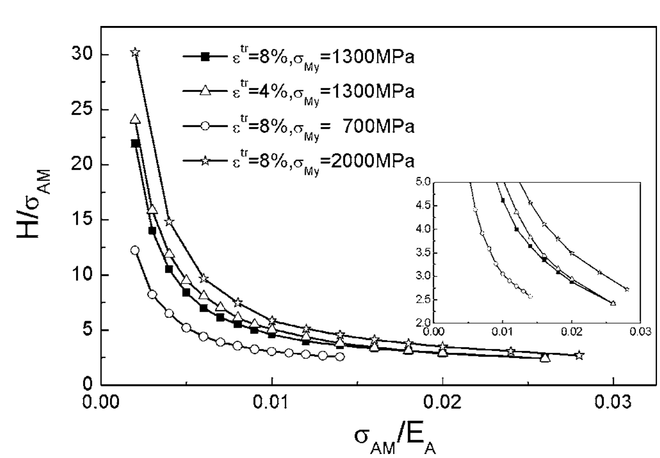

FIG. 4. Relationships between the ratio of the hardness to the transition stress H/σ_{AM} and dimensionless martensite yield stress σ_{AM}/E_A.

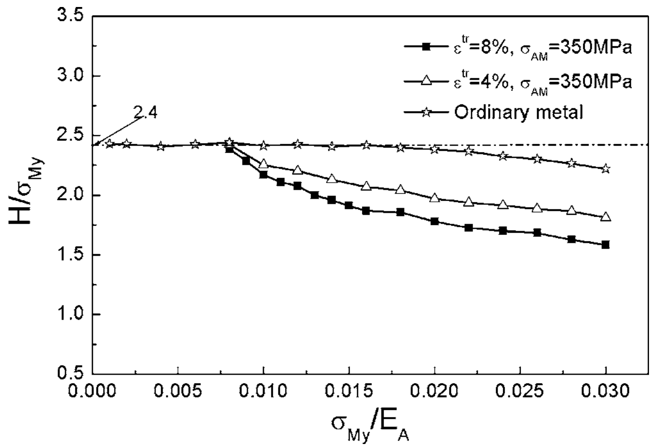

FIG. 5. Relationships between the ratio of the hardness to the martensite yield stress H/σ_{My} and dimensionless martensite yield stress σ_{My}/E_A, compared with an ordinary metal.

the ordinary metal ($E=E_A=50$ GPa), in the range of $0 \leq \sigma_{My}/E_A \leq 0.018$ (i.e., $0 \leq \sigma_{My} \leq 900$ MPa, which is a reasonable range of yield stress for many ordinary metals), the ratio of the indentation hardness to the yield stress has a constant value of 2.4, which is consistent with the results obtained by Cheng and Cheng.[8,9] However, for the SMA with phase transition, the ratio of hardness to yield stress decreases continuously with the increase of the martensite yield stress. We can conclude that the hardness of SMA is not proportional to the martensite yield stress, even within the range of small yield stress values. Bear in mind that the martensite yield stress should be larger than the prescribed transition stress, so the smallest value of the yield stress in Fig. 4 is 400 MPa for SMA, larger than the prescribed transition stress of 350 MPa.

In summary, we revealed the effects of phase transition and plastic deformation on the hardness of SMA by using dimensional analysis and finite element calculations. Our results indicate that the hardness depends on the phase transition stress, the maximum phase transition strain, and the martensite yield stress. A higher transition stress or a higher martensite yield stress or a lower maximum transition strain would lead to a higher indentation hardness, which is in an agreement with the experimental measurement,[14] and previous study on spherical indentation hardness.[12] For SMA alloy, the indentation hardness with a sharp conical indenter is independent of the maximum indenting depth. Either the ratio of the hardness to the phase transition stress or the ratio of the hardness to the plastic yield stress is not a constant. Both of them decrease with an increase in corresponding stress values. The hardness cannot be simply used as a measure of the phase transition stress or the plastic yield stress, which is in contrast to ordinary metals.

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[1] Y. Fu, H. Du, W. Huang, S. Zhang, and M. Hu, Sens. Actuators, A **112**, 395 (2004).
[2] W. Ni, Y. T. Cheng, and D. S. Grummon, Appl. Phys. Lett. **82**, 2811 (2003).
[3] X. G. Ma and K. Komvopoulos, Appl. Phys. Lett. **83**, 3773 (2003).
[4] J. A. Shaw and S. Kyriakides, J. Mech. Phys. Solids **43**, 1243 (1995).
[5] V. Imbeni, C. Martini, D. Prandstraller, A. Poli, C. Trepanier, and T. W. Duerig, Wear **254**, 1299 (2003).
[6] F. T. Cheng, P. Shi, and H. C. Man, Scr. Mater. **45**, 1083 (2001).
[7] L. M. Qian, X. D. Xiao, Q. P. Sun, and T. X. Yu, Appl. Phys. Lett. **84**, 1076 (2004).
[8] Y. T. Cheng and C. M. Cheng, Int. J. Solids Struct. **36**, 1231 (1999).
[9] Y. T. Cheng and C. M. Cheng, Appl. Phys. Lett. **73**, 614 (1998).
[10] W. Yan, Q. P. Sun, X.-Q. Feng, and L. M. Qian, Appl. Phys. Lett. **88**, 241912 (2006).
[11] W. Yan, Q. Sun, X.-Q. Feng, and L. Qian, Int. J. Solids Struct. **44**, 1 (2007).
[12] W. Yan, Q. Sun, and H.-Y. Liu, Mater. Sci. Eng., A **425**, 278 (2006).
[13] J. L. Bucaille, S. Stauss, E. Felder, and J. Michler, Acta Mater. **51**, 1663 (2003).
[14] L. Qian, Q. P. Sun, and X. D. Xiao, Wear **260**, 509 (2006).