Brunt-Väisälä frequency

$$\cancel{\rho_2' = \rho_1 + \frac{d\rho'}{dz} z}$$

$$\rho_2' = \rho_1 + \frac{d\rho'}{dr} r$$

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$$\rho_2' = \rho_1 + \frac{1}{\gamma} \frac{\rho_1}{P_1} \frac{dP'}{dr} r$$

Surrounding gas is not adiabatic.

So, we have:

$$\rho_2 = \rho_1 + \frac{d\rho}{dr} r \quad \text{(surround)}$$

$$\rho_2' = \rho_1 + \frac{1}{\gamma} \frac{\rho_1}{P_1} \frac{dP'}{dr} r$$

$$F = \rho \frac{\partial^2 r}{\partial t^2} = -g(\rho_2' - \rho_2) =$$

Adiabatic process

$$\frac{P_1}{P_2} = \frac{\rho_1^\gamma}{\rho_2^\gamma}$$

$$P_2 = \frac{P_1}{\rho_1^\gamma} \cdot \rho_2^\gamma$$

$$P = K_\gamma \cdot \rho^\gamma$$

$$\frac{dP}{dr} = K_\gamma \gamma \rho^{\gamma - 1} \frac{d\rho}{dr} =$$

$$= \gamma K_\gamma \rho^\gamma \frac{1}{\rho} \frac{d\rho}{dr}$$

$$\frac{dP}{dr} = \gamma \frac{P}{\rho} \frac{d\rho}{dr}$$

$$\frac{d\rho}{dr} = \frac{1}{\gamma} \frac{\rho}{P} \frac{dP}{dr}$$

$$= -g\left(\rho_1 + \frac{1}{\gamma}\frac{\rho_1}{P_1}\frac{dP'}{dr} r - \left(\rho_1 + \frac{d\rho}{dr} r\right)\right) =$$

$$= -g\left(\cancel{\rho_1} + \frac{1}{\gamma}\frac{\rho_1}{P_1}\frac{dP'}{dr} r - \cancel{\rho_1} - \frac{d\rho}{dr} r\right) =$$

$$= -g\left(\frac{1}{\gamma}\frac{\rho_1}{P_1}\frac{dP'}{dr} r - \frac{d\rho}{dr} r\right) = -gr\left(\frac{1}{\gamma}\frac{\rho_1}{P_1}\frac{dP'}{dr} - \frac{d\rho}{dr}\right) =$$

$$= -gr\rho_1\left(\frac{1}{\gamma}\frac{1}{P_1}\frac{dP'}{dr} - \frac{1}{\rho_1}\frac{d\rho}{dr}\right)$$

$$F = \frac{d^2r}{dt^2} = -gr\left(\frac{1}{\gamma}\frac{1}{P}\frac{dP}{dr} - \frac{1}{\rho}\frac{d\rho}{dr}\right)$$

$$\frac{d^2r}{dt^2} = \underbrace{-g\left(\frac{1}{\gamma}\frac{1}{P}\frac{dP}{dr} - \frac{1}{\rho}\frac{d\rho}{dr}\right)}_{N^2} r$$

$$\frac{d^2r}{dt^2} = -N^2 r \quad \text{(oscillator)}$$

Osc. sol: $r = r_0 e^{iNt}$

$N^2 > 0$ — oscillations, stable

$N^2 < 0$ — unstable.