By 12th of May:

The equation of motion for a damped pendulum is:

$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0.$$

The equation is second-order, linear, homogeneous ordinary differential equation with constant coefficients, and is easy to solve by substituting a trial function of the form of $e^{\alpha t}$ and solving the corresponding characteristic equation.

The analytic solution to this equation is given by:

$$x(t) = A_0 e^{-\frac{b}{2}t}\cos(\omega t - \phi),\ \omega = \sqrt{\omega_0^2 - \frac{b^2}{4}},\ \omega_0^2 = k,\ \text{if } \omega_0^2 \geq \frac{b^2}{4}.$$

Constants ϕ and A_0 can be found assuming the initial conditions $x(0) = x_0$, and $v(0) = \left.\frac{dx}{dt}\right|_{t=0} = 0$, and are given by:

$$\phi = \arctan\frac{b}{2\omega},\ A_0 = \frac{x_0}{\cos\phi}.$$

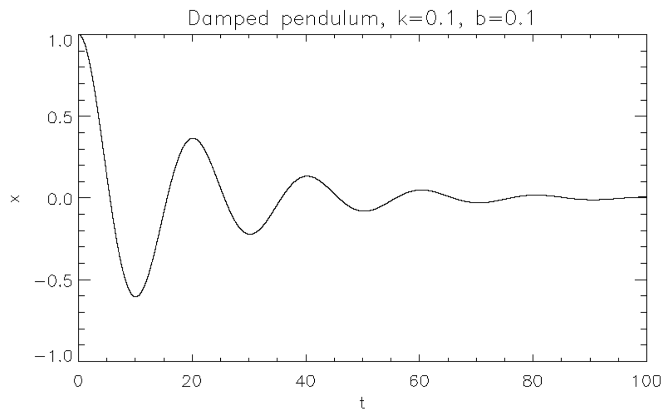
The period of damped oscillation described by these equations is given by $T = \frac{2\pi}{\omega}$.

As we have seen in one of the previous lectures, the (wave-like) equation can be converted into the following system of two first order differential equations:

$$\frac{dx}{dt} = v,$$
$$\frac{dv}{dt} = -kx - bv.$$

Solve this system numerically using the studied time advancement schemes to test their precision and convergence to the analytical solution given above. Compare Euler, Runge-Kutta second-order/Heun's/improved Euler, Runge-Kutta 4th order and Adams-Bashforth 4th order methods.

Use the following parameters: $k = 0.1$, $b = 0.1$, $t = 0..100$, which gives you the period $T \approx 20$ (in the figure to the right, the solution for these parameters is shown).



Define absolute global mean error of the numerical method as the mean of the modulus of the difference between the approximate numerical solution and the analytical solution. Calculate the absolute global mean error as a function of resolution per wave period.

To bootstrap the 4th order Adams-Bashforth scheme use the 4th order Runge-Kutta scheme.