

By 14th of April:

(1) Implement Lax-Wendroff scheme. Use any programming language and the results of your previous homework to solve numerically one-dimensional continuity equation from the system of equations of hydrodynamics with an additional diffusive term:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) = k \frac{\partial^2}{\partial x^2} \rho,$$

where the diffusion coefficient k is defined as

$$k = \Delta t \cdot \frac{v_x^2}{2}.$$

For now, use a constant diffusion coefficient k and constant velocity v_x (however, declare an array for velocity vector, which will be used later).

Determine the time step Δt using Courant-Friedrichs-Lewy criterion

$$\Delta t = \frac{\Delta x}{v_{\max}}.$$

Use periodic boundary conditions and domain size $x=0..10$. As the initial condition, use Gaussian function centered around $x=2$:

$$\rho^{t=0} = \exp(-4 \cdot (x - 2)^2).$$

Plot the evolution of the solution in time. Note, to check how nicely your solution behaves in this very particular case, you can also plot the solution in the co-moving frame $x-ct$.