By 30th of March:

(1) Use any programming language to solve two-dimensional diffusion equation with a constant diffusion coefficient *k*:

$$\frac{\partial \phi}{\partial t} - k\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) = 0,$$

applying the central difference approximation of second derivative given by

$$\frac{d^2 f}{dx^2} \approx \frac{1}{\Delta x^2}(f_{i+1} - 2f_i + f_{i-1}),$$

and Euler-type time advancement scheme

$$\frac{df}{dt} \approx \frac{1}{\Delta t}(f^{t+1} - f^t).$$

Define an equidistant square grid $i,j=0..n-1$, which covers $x,y=0..1$. Use $k=0.1$, $\Delta t=0.00001$. Use fixed boundary conditions for the spatial derivative calculation, i.e. the values of partial derivatives at $i=0$, $j=0$, $i=n-1$, $j=n-1$ are zeroes; therefore the solution at these points does not change in time. Use two-dimensional Gaussian function as the initial condition:

$$\phi^{t=0} = \exp(-100 \cdot ((x-0.5)^2 + (y-0.5)^2)).$$

Plot the solution and demonstrate evolution of the solution on time.