QMA Lecture 4
Value Through Time

Simon Angus[*]

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1 Introduction

We spend a little more time looking at how the value of money is actually dependent on the time at which it is in our hand. That is, as before, we note that there exists a **time value of money**; apparently having \$100 in your hand today, will be more acceptable than even \$105 tomorrow. (What would you take?).

There are various explanations for this fact, but we will consider some of the common ones later. As you might have reflected above – 'I'd prefer the money today, since I'm not sure what will happen tomorrow,' – that is the **uncertainty** of our lives kicking in. Just one of the factors that contribute to the apparent value of money changing through time.

Today we'll deal with a couple of scenarios mostly, both involving parents – one where they are going to give you a lot of money, but in the future; and the other, where you are trying to work out a more preferably repayment scheme for money that you owe them. The first case speaks directly to the nature of present and future values, whereas the second will cause us to consider what are known as **equations of value**.

Agenda

1. Effective rate of interest;

2. Present value;

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[*]School of Economics, Room 113, John Goodsell Building, University of New South Wales; E-mail: s.angus@unsw.edu.au.

3. Equations of value;

   (a) Under simple interest;
   (b) Under compound interest;

2 Effective rates

Suppose we want to compare interest rates quoted in non-yearly periods versus compound rates on a standard yearly schedule. It would be useful to bring them to **the same playing field**.

1. This is just the same as saying, suppose I start with \$1, with nominal rate 0.08 and compounding at 4 times per year. Then the value at the end of the year is,

$$S = (1)\left(1 + \frac{0.08}{4}\right)^{1\times 4} = \$1.0824$$

2. So the effective rate of interest I received over the year (at yearly compounding) is just,

$$
\begin{aligned}
r_e &= \frac{\text{amount I gained over year}}{\text{amount I invested}} - 1 \\
&= \frac{1.0824}{1} - 1 \\
&= 0.0824
\end{aligned}
$$

Which leads to the following definition,

Definition 1 (Effective (equivalent) rates). *The effective rate r_e (at once-yearly compounding) that is equivalent to a nominal rate r compounded n times a year is,*

$$r_e = \left(1 + \frac{r}{n}\right)^n - 1 \tag{1}$$

What do we gain by calculating the **effective rate**? It doesn't give us a rate that would work for simple interest applied for the number of years in question, rather it translates between **non-yearly** compounded rates and **yearly compounded rates**. The following example should help to show this.

Example:

A bank is offering two different types of savings accounts – the first has a nominal rate of 5.15% compounded quarterly, whilst the other just has a single yearly compounding at a rate of 5.35%. Which would you choose?
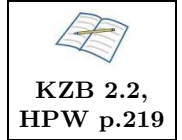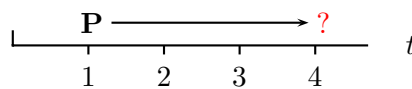
3   Present value

Scenario

The tale of the generous parents
Your parents have been discussing your finances with you. They have (very generously) suggested that in 20 years' time, they are willing to give you $100,000 to help you out. Clearly this is very nice of them. However, you can't keep it out of your head to wonder, just **how** generous are they being? What would this kind of money mean **today** if they were just to give it to you?

- Previously, we have been interested to know what the **future value** (normally represented by S) of a certain sum (the principle, P) would be in a number of years time,

$$\mathbf{P} \longrightarrow \textcolor{red}{?}$$
$$\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \quad t$$

- However, in our current scenario, we want to know what the present value of a future amount is, given that it represents the end-points of some value-adding proceess, that is,

$$\textcolor{red}{?} \longleftarrow \mathbf{S}$$
$$\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \quad t$$
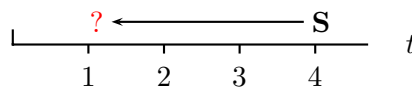
- Clearly, the way the money is being changed through time will matter. We'll consider a few.

✎ *Example: Approach 1: just inflation*

Suppose that (as with inflation) the money is not invested, and its value change is therefore the product of simple inflationary changes. Assume therefore, a compound interest scenario with yearly period and (average) inflation of 3% per annum.
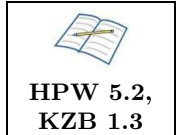
Example: Approach 2: their super fund

Now suppose they are going to use money from their superannuation fund (which they fortuitously can't touch for 20 years anyhow). Suppose that it receives no more outside money between now and 20 years into the future, just the interest it yields, which is calculated semi-annualy at around 11% per annum (it's a good fund). How much would need to be in there now?

Example: Approach 3: the 'Endowment Trust' fund

Suppose instead, that the money will come from a dedicated fund for this purpose offered by their bank. This account uses **continuous compounding** at a nominal rate of 11%. How much would need to be invested now?

Example: Dummy check case study: A can of Coke

If a can of (regular) Coca-cola costs $1.80 today, what would it have cost me when I was a kid (20 years ago)? (Assume 3% inflation through these years).

Definition 2 (Present Value (periodic)). *To obtain a compound amount of value S which has been maturing at the periodic rate of r for n periods, one needs to invest the starting amount, or principle,*

$$P = S(1+r)^{-n} \tag{2}$$

otherwise called the present value of S.

> **Definition 3 (Present Value (continuous)).** *To obtain a compound amount of value S which has been maturing continuously at the nominal rate of r for t years, one needs to invest the starting amount, or principle,*

$$P = Se^{-rt} \tag{3}$$

> *otherwise called the present value of S.*

- Why? Why is money **today** apparently 'more valuable' than the same amount tomorrow?:

- A topic of lots of economic discussion, but at this stage, we consider a relatively simple explanation:

    1. We have an inherent preference for things *now* – we are impatient;

    2. Tomorrow is uncertain;

4   Equations of value

Equations of value

Scenario II
Actually, you owe your parents some money. At present, you owe them $500 to be paid in 6 months and $350 in 9 months. Unfortunately, you don't want to do installments, you'd prefer to pay $100 **now** and the rest in **12 months time**. In negotiations, they agree to consider either simple or compounded (quarterly) interest. What will you do?

KZB 1.4, 2.6

4.1   Simple Interest

Solution technique:

1. Work out the timings;

2. Using the focal date bring all the payments and debts to it;

3. Set up the equation of value;

4. Solve.
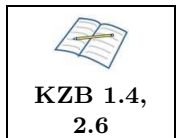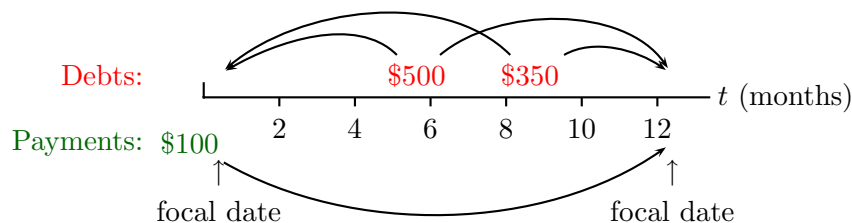
Example:

Using a focal date of **now** or in **12 months**, and simple interest at the nominal value of 7%, what would be the single sum you owe?

Checking the two payments (now = \$715.63, 12 months = \$766.63), the value of the 12 month payment is,

$$
\begin{aligned}
P &= 766.63(1 + 0.07)^{-1} \\
&= \$716.48 \quad \textit{!!!}
\end{aligned}
$$

⚠️

*When using **simple interest** in equations of value, the **focal date** must be agreed before hand, since it will affect the total value exchanged.*

4.2   Compound Interest

1. Try the compound interest version yourself;

2. Check if the value of the future (12 month) payment is the same as the current payment.

3. Which repayment method would you pick?

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