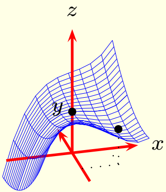
QMA Lecture 4

Value Through Time

Simon Angus



School of Economics

S2 2006

Agenda

1. Effective rate of interest;
2. Present value;
3. Equations of value;
    1. Under simple interest;
    2. Under compound interest;

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2. So the effective rate of interest I received over the year (at yearly compounding) is just,

$$\begin{aligned} r_e &= \frac{\text{amount I gained over year}}{\text{amount I invested}} - 1 \\ &= \frac{1.0824}{1} - 1 \\ &= 0.0824 \end{aligned}$$

Which leads to the following definition,

Definition: *Effective (equivalent) rates*

The effective rate r_e (at once-yearly compounding) that is equivalent to a nominal rate r compounded n times a year is,

$$r_e = \left(1 + \frac{r}{n}\right)^n - 1 \tag{1}$$

Example

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or using r_e,

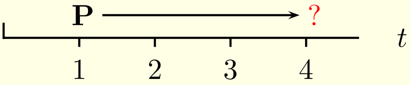
$$\begin{aligned} S &= P(1 + r_e)^t \\ &= (1)(1 + 0.0525)^{10} \\ &= \$1.67 \end{aligned}$$

Scenario

The tale of the generous parents

Your parents have been discussing your finances with you. They have (very generously) suggested that in 20 years' time, they are willing to give you $100,000 to help you out. Clearly this is very nice of them. However, you can't keep it out of your head to wonder, just **how** generous are they being? What would this kind of money mean **today** if they were just to give it to you?

- Previously, we have been interested to know what the **future value** (normally represented by S) of a certain sum (the principle, P) would be in a number of years time,



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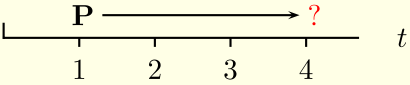

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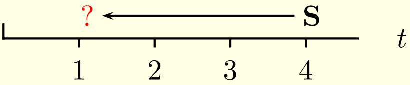

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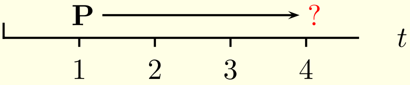

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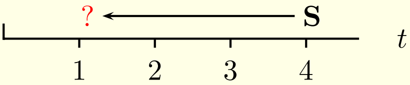

- Clearly, the way the money is being changed through time will matter. We'll consider a few.

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Suppose that (as with inflation) the money is not invested, and its value change is therefore the product of simple inflationary changes. Assume therefore, a compound interest scenario with yearly period and (average) inflation of 3% per annum.

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substituting,

$$\begin{aligned} P &= 100,000(1+0.03)^{-20} \\ &= \$55,368 \quad \text{(still a lot!)} \end{aligned}$$

Example (Approach 2: their super fund)

Now suppose they are going to use money from their superannuation fund (which they fortuitously can't touch for 20 years anyhow). Suppose that it receives no more outside money between now and 20 years into the future, just the interest it yields, which is calculated semi-annualy at around 11% per annum (it's a good fund). How much would need to be in there now?

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Definition: *Present Value (periodic)*

To obtain a compound amount of value S which has been maturing at the periodic rate of r for n periods, one needs to invest the starting amount, or principle,

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Definition: *Present Value (continuous)*

To obtain a compound amount of value S which has been maturing continuously at the nominal rate of r for t years, one needs to invest the starting amount, or principle,

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    1. We have an inherent preference for things now – we are impatient;
    2. Tomorrow is uncertain;

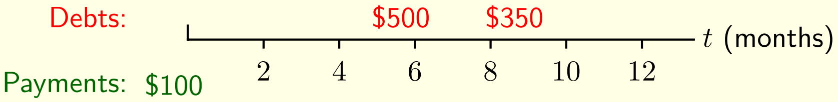
Equations of value

Scenario II

Actually, you owe your parents some money. At present, you owe them \$500 to be paid in 6 months and \$350 in 9 months. Unfortunately, you don't want to do installments, you'd prefer to pay \$100 **now** and the rest in **12 months time**. In negotiations, they agree to consider either simple or compounded (quarterly) interest. What will you do?

Solution technique:

1. Work out the timings;
2. Using the focal date bring all the payments and debts to it;
3. Set up the equation of value;
4. Solve.



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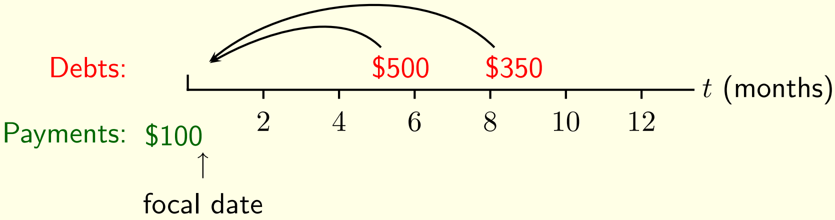

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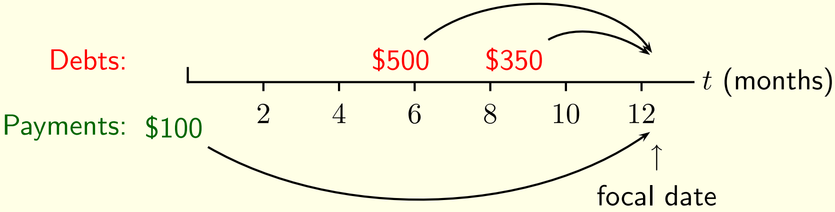

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Checking the two payments (now = \$715.63, 12 months = \$766.63), the value of the 12 month payment is,

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Caution!

When using **simple interest** in equations of value, the **focal date** must be agreed before hand, since it will affect the total value exchanged.

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3. Which repayment method would you pick?