Learning to Communicate: Communication Networks & Inductive Reasoning

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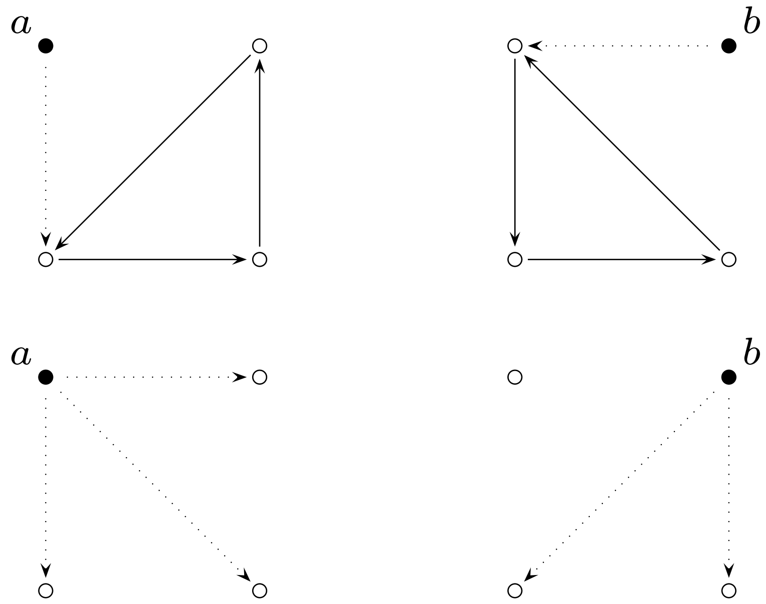

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Motivation

1. Increasing awareness of the role of *interactions* in economic behaviour

   Q: How do such networks form?
   Q: What are effecient networks?
   Q: What determines/controls human decision-making in these problems?

2. But analytical models are difficult ($|\mathcal{G}(n) \sim 2^{n(n-1)/2}$)

3. Examples of approaches:

   Network structure $\longrightarrow$ agent behaviour Anderlini and Ianni (1996, 1997): games on a torus Chwe (2000): network as coordination device

   Agents $\longrightarrow$ network structure Goyal and Joshi (2003): firm-firm committments,

   (Both) Agents $\longleftrightarrow$ Network structure Goyal and Vega-Redondo (1999): coordination games and network formation (complete, or stars), Ely (2002): choice of neighbourhood/strategy Jackson and Watts (2002) (e.g.): link costs non-trivial, network effects context dependant.

Motivation (cont.)

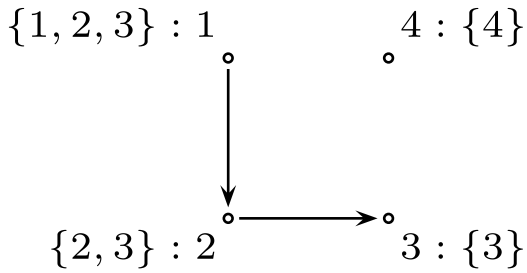
4. We focus on the *Non-cooperative Communication Network Formation* model of Bala and Goyal (2000)[1]
   - One of first 'pure' network formation papers (no strategic interaction thereafter);
   - Experimental evidence is available;
   - General setting, well known.

5. Rise of *artificial adaptive approaches* to 'difficult modelling' settings.

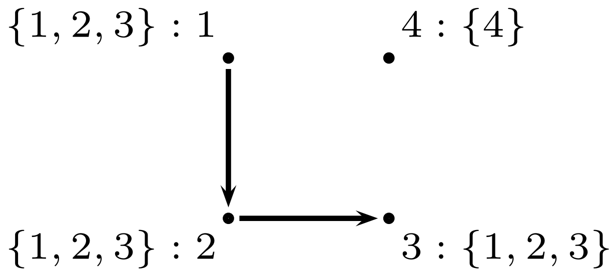
Refer to Bala and Goyal (2000) as **BG** *from here.*

[1] Bala, V. and Goyal, S. (2000), 'A Noncooperative Model of Network Formation', *Econometrica*, **68**(5), 1181–1229.

The BG model



(c) one-way



(d) two-way

1. One-, and two- way flows of information allowed (indirect observation);

2. Payoffs: total-information - total costs;

$$\pi_i(G) = \overset{\text{n(obs)}}{\mu_i(G)}V - \overset{\text{n(links)}}{\delta_i(G)}C \quad .$$

3. Agents update sponsorships according to (myopic) Best Response play at all times:

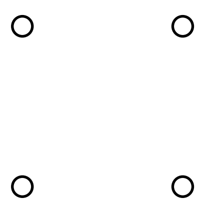
$$\max_{g_i \in \mathbf{g}} \left[\pi_i(\overbrace{g_i}^{\textit{my link vector this period}} \cap \underbrace{g_j^{t-1}}_{\textit{my opp.'s links last period}})\right] \quad \forall \quad j \in N/\{i\}$$

4. Convergence obtained in analytical model by applying *inertia* (don't update)

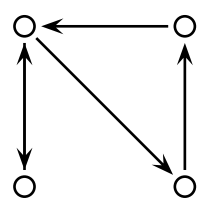
BG predictions

Flow	Edge Costs[a]	Structure				
		m1c	wheel	empty	m2c	cs-star
One-way	Low	△	▲*			
	High	△	▲*	▲		
Two-way	Low				△*	▲
	High			▲	△*	

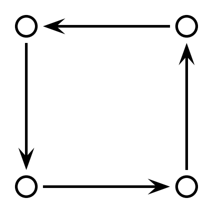
Notes: [a] Low $C \leq V$, High $C > V$; (△) non-empty nash, (▲) strict nash, (*) indicates that the structure is also *efficient* (following FK2003).



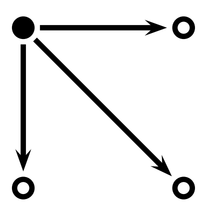
(e) empty



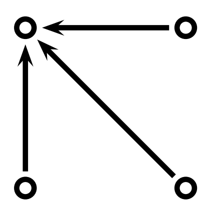
(f) min-con 1-way



(g) wheel



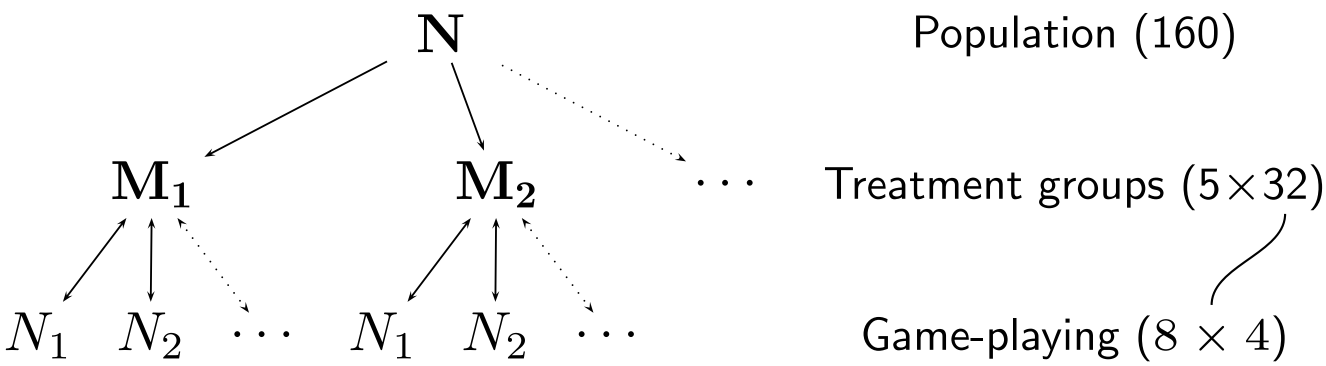
(h) cs-star



(i) min-con 2-way

In the Lab: Falk & Kosfeld (2003)

1. Exact replication of BG communication network formation set-up (4-player games, 160 subjects in total, five treatment groups, 5 round games, over 3 'stages');

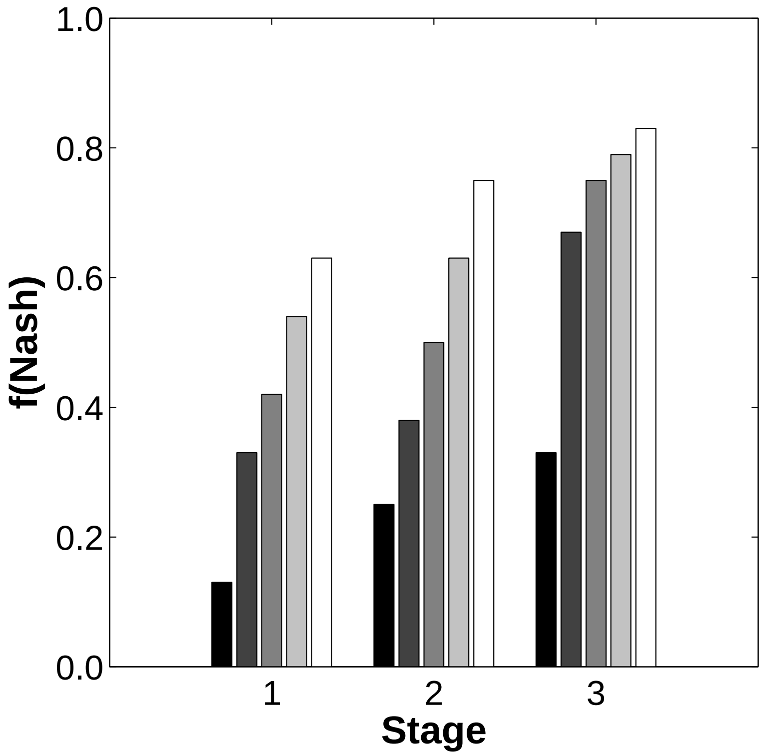


2. Using Swiss-Francs as incentives (avg. take-home $\sim AUS\$49.36$);

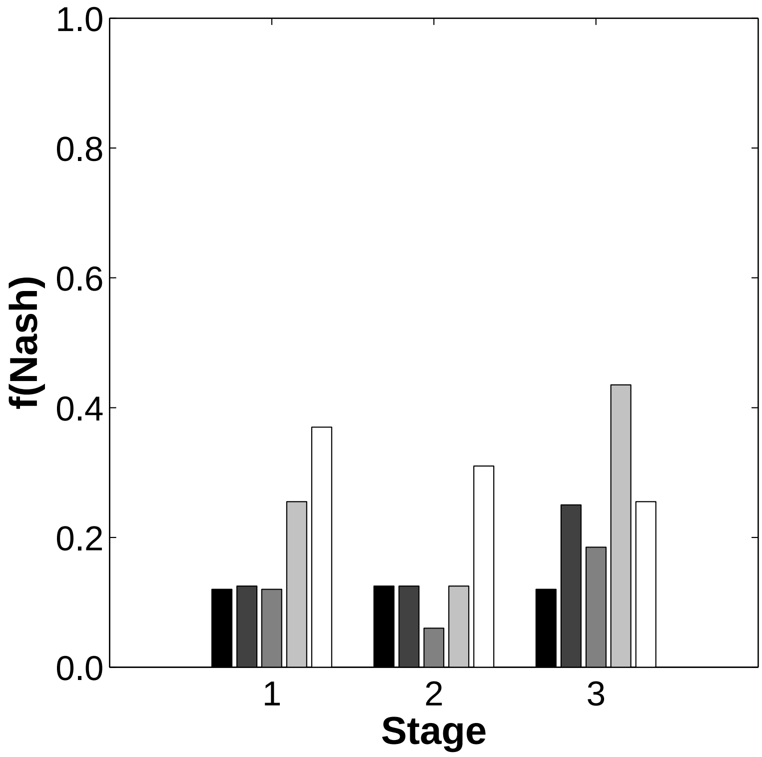
3. Findings:

   (a) One-way flow predictions *hold* (generally);
   (b) **But** Two-way predictions *not* realised (not a single *cs-star* formed during experiments);
   (c) Clear evidence of *intra-stage improvement* (learning?) observed both between rounds and stages;
   (d) Likelihood of Nash structures increased with link-cost (C) for one-way flows, but *decreased* with two-way flows;

FK2003 Subject Trials



(j) One-way



(k) Two-way

Theory & Reality: frequency of occurence

BG2000 Theory

Flow	Edge Costs[a]	Structure				
		m1c	wheel	empty	m2c	cs-star
One-way	Low	△	▲*			
	High	△	▲*	▲		
Two-way	Low				△*	▲
	High			▲	△*	

Notes: [a] Low $C \leq V$, High $C > V$; (△) non-empty nash, (▲) strict nash, (*) indicates that the structure is also *efficient* (following FK2003).

FK2003 Human Trials

Flow	Edge Costs	Structure				
		m1c	wheel	empty	m2c	cs-star
One-way	Low (5)	0.48	0.41			
	High (25)	0.59	0.49	0.10		
Two-way	Low (5)				0.31	0.00
	High (15)			(nr)	0.09	

One-way, Two-way: what's the difference?

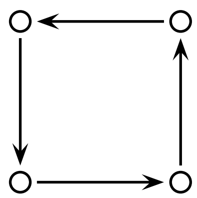
Main Differences

Stability of Nash networks in one-way case, around 82% likelihood to stay (if realised previous period); in two-way, only 11% (!);

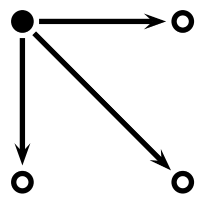
Distribution of links one-way cases, very narrow distribution around n links; in two-way case, much broader (indecision?)

Suggested Explanations

1. Symmetry:
   (a) wheel – symmetric in payoffs & strategies;
   (b) cs-star – *asymmetric* in payoffs & strategies;



(l) wheel



(m) cs-star

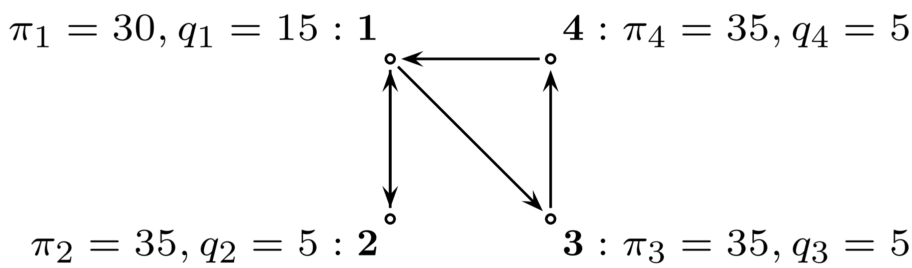
FK2003: Further Analysis

1. Ran regression models over the decision-making of each subject between rounds – did they revise their strategy? (did they exhibit *inertia*?);

2. (Probit) regression on *BRprevious*, and *PayoffInEquality*:

$$q_i(G) = \sum_{j \in N/i} \left| \pi_j - \pi_i \right|$$

3. Found, both strongly significant and positive – more likely *not* to revise if played BR in previous period, or experienced high relative payoff inequality;



A New Model(ling Approach)

Aim

To construct a richer non-cooperative communication model that explains as much of the observed behaviour as possible.

An Artificial 'Adaptive Agent' Model

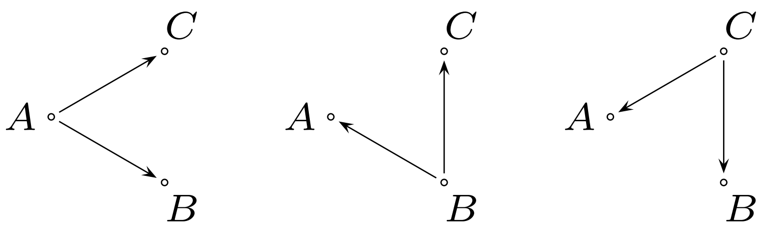
Action & Strategy Implement diverse agent decision-processes with a range of abilities;

Learning allow some agent plays to be rewarded, others to be punished and evolve the agent heuristics;

Testing Add various assumptions into behavior (such as BR-inertia, or inequality-inertia, or ...?);

A Complex Environment ...

1. Graph count: $\#[G(4)] = 4096$ (one-way flows)
   - Cognitively feasible??
2. *Simplification 1*: Retain 'response' nature of strategy decisions ⇒consider *absentee graph* $\mathbf{G}/\{i\}$;
   - now .. $\#[G(4-1)] = 64$?
3. *Simplification 2*: Not all graphs are actually distinct ...



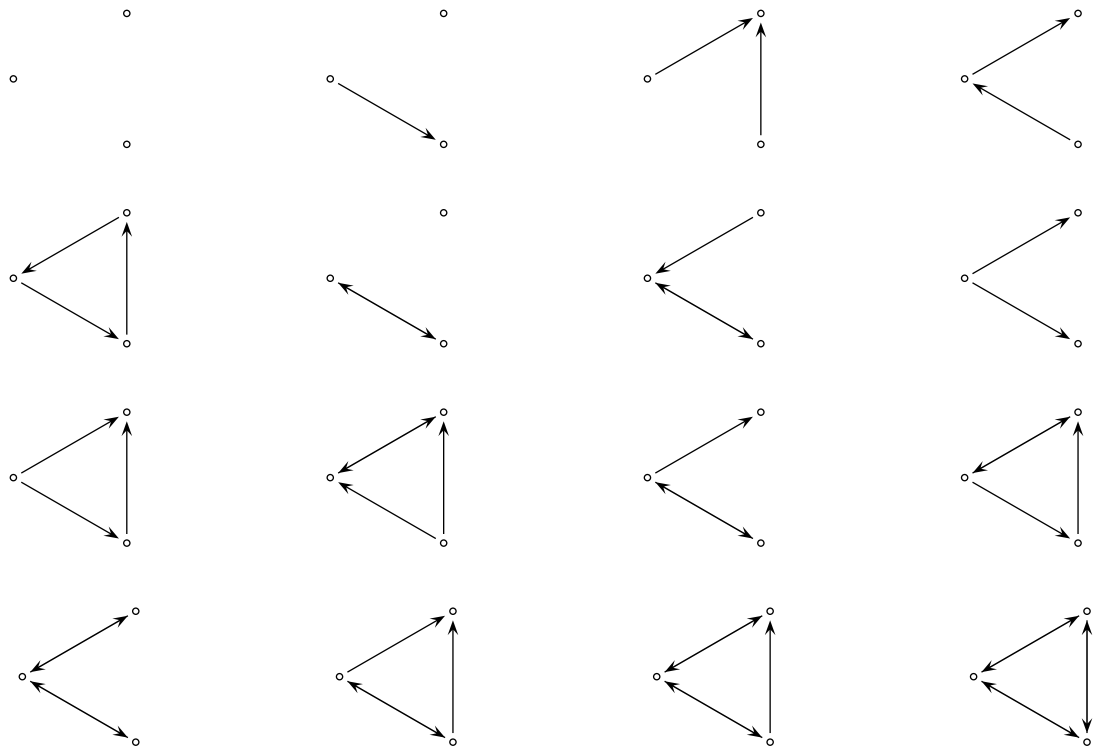
4. Therefore – consider *minimal absentee graphs*, call them the fundamental (or 'canonical') types,

$$\mathbf{T}(n) = \{\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_k\}$$

   - now .. $\#[T(4-1)] = 16$ .. OK!
5. And... define strategy decisions over $\mathbf{T}$, that is, define a *strategy* for player i, to be $\mathcal{S}_i \in \mathbf{S}$ such that

$$\mathcal{S} : \mathbf{T} \rightarrow \mathbf{g}$$

Full set of $T(3)$



Cognitive Assumptions

1. $\mathbf{A}$. **1. [Type Recognition]** *Given* k *un-identical graphs*
$$\left\{G_1(N_1^n, g), \ldots, G_k(N_k^n, g)\right\}$$
differing only in the ordering of elements in N^n *(e.g.* $N_1^4 = \{1, 2, 3, 4\}$ *and* $N_2^4 = \{2, 3, 1, 4\}$*), then any agent* $i \in N$ *will recognise* $\{G_1, \ldots, G_k\} \equiv \mathcal{T}_j$*, where* $\mathcal{T}_j \in \mathbf{T}(n)$*.*

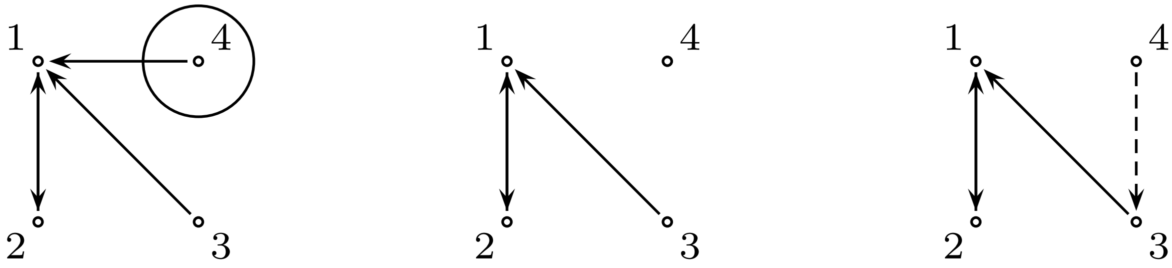
   - (Agents can tell which '$\mathcal{T}$' they are looking at)

2. $\mathbf{A}$. **2. [Context Invariance]** *Given any instance of an information network* G *which corresponds to a minimal graph* $\mathcal{T}$*, any agent* $i \in N$ *is able to apply the resultant edge sponsorship decision* $s(\mathcal{T})$ *to the context, and thus arrive at* g_i *that accords to the instance* G *before her.*

   - (Agents can apply their response to a given $\mathcal{T}$in the *actual* situation they have infront of them)

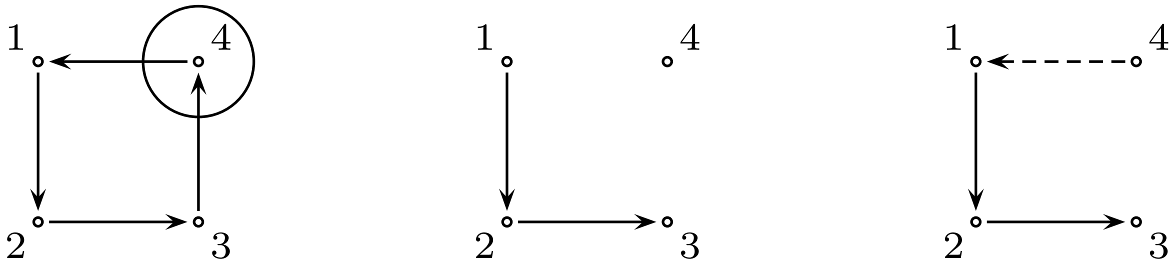
Decision-Making Process Examples

1. Example 1:



(r) G **(s)** $G/\{4\} \equiv \mathcal{T}^*$ **(t)** $\mathcal{S}_4(\mathcal{T}^*)$

2. Example 2:



(u) G **(v)** $G/\{4\} \equiv \mathcal{T}^+$ **(w)** $\mathcal{S}_4(\mathcal{T}^+)$

Learning

1. Record *public* plays of each agent;

2. Determine best performing agents(s) at the end of a stage, assign to *'teacher'* status, the rest, to *'students'*;

3. Students *learn* from teachers via *imitation* and *innovation* (mistakes):

   - NB: a *one-way* form of transfer (cultural transmission)

$$\begin{aligned} \mathcal{S}_t &= \Big(s(\mathcal{T}_1), \ldots, \overbrace{000, 110, 001}^{\text{section to be imitated}}, 101, \ldots, s(\mathcal{T}_k)\Big) \\ \mathcal{S}_s &= \Big(s(\mathcal{T}_1), \ldots, 011, 010, 011, 001, \ldots, s(\mathcal{T}_k)\Big) \\ &\Downarrow \\ \mathcal{S}_s^* &= \Big(s(\mathcal{T}_1), \ldots, 000, 11\underline{1}, 001, 101, \ldots, s(\mathcal{T}_k)\Big) \end{aligned}$$

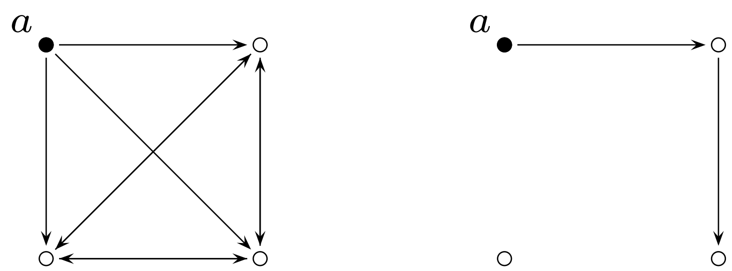
4. Assumptions 1 & 2 guarantee successful application;

Who should be the teacher(s)? Objective function trials

1. **Payoffs**:

$$\bar{\pi}_i = \frac{1}{R}\sum_{r=1}^{R}$$

   - Simple, orthodox, but relatively low information



2. **'Value'**:

$$f_i(\mu_i, \delta_i) = \frac{\mu_i V + C}{C(\delta_i + 1)} \; ,$$

   re-written,

$$f_i(\mu_i, \delta_i) = \left(\frac{1}{\delta + 1}\right)\left[\left(\frac{V}{C}\right)\mu + 1\right] \; ,$$

   - Value of information and cost of links weights measure;

3. **'Nieve'**: Same as 'Value' (frequency etc.) but *choose teacher at random*. (just immitation only?)

First cut: Objective functions

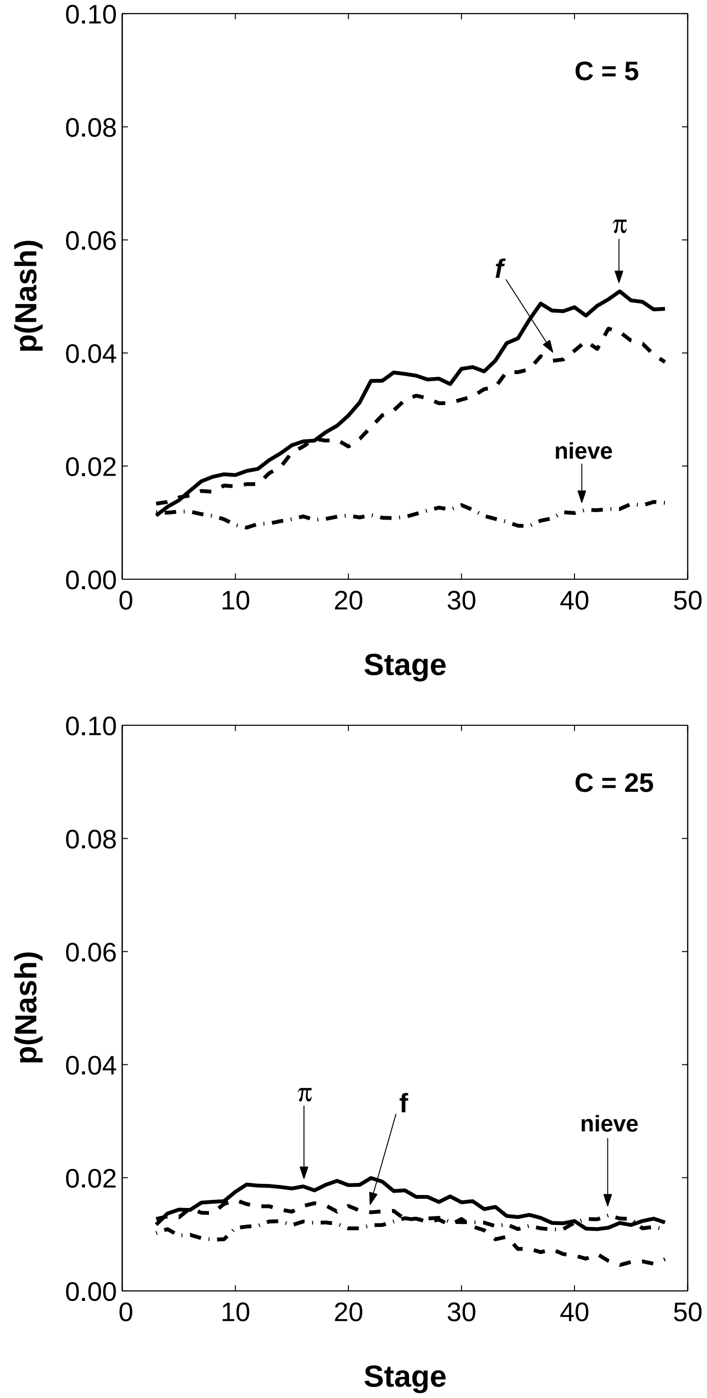


Figure 1. Nash (non-empty) structures under one-way information flows: (left) $C = 5$; and (right) $C = 25$, under different objective measures: payoffs (π), benefit/cost ratio (f) and naive (random) learning.

First cut: Link sponsoring

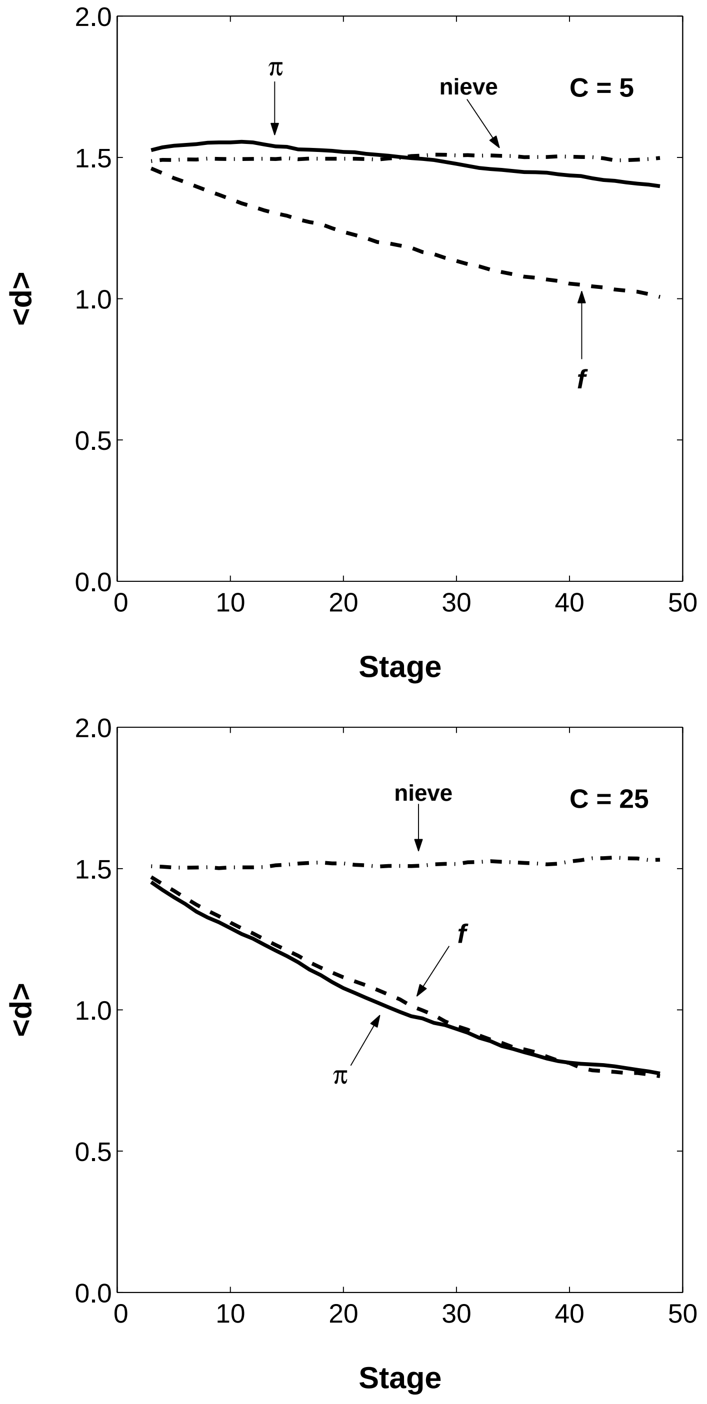


Figure 2. Average agent degree under one-way information flows: (left) $C = 5$; and (right) $C = 25$, objective measures as for Fig. 1.

- .. Under-sponsoring compared to humans.

Increase Link Sponsoring by Reciprocity Measure

1. **Simple Reciprocity Measure**:

In-d	Out-d	R Measure
0	0	0
$\geq$1	0	0
$\geq$1	$\geq$1	1
0	$\geq$1	2

2. Combine objective measure and reciprocity:

$$\Omega_i = \alpha \langle r_i \rangle + (1 - \alpha)\{\langle \pi_i \rangle, \langle f_i \rangle\}$$

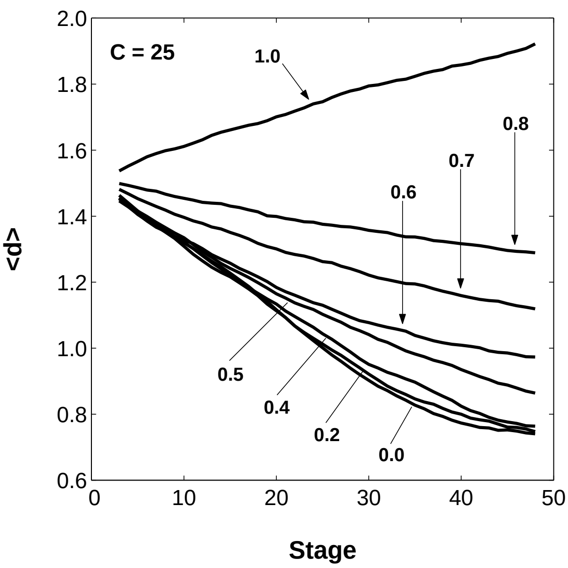


Figure 3. Combined (altruism,benefit/cost ratio) objective measure calibration results at different α values.

Long(er)-run study with Reciprocity

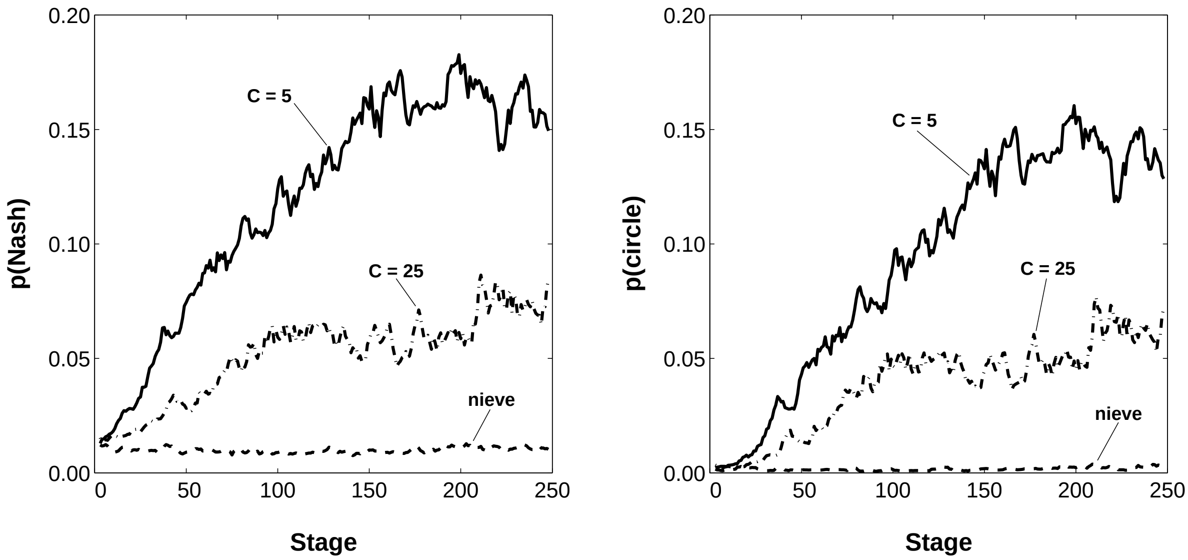


Figure 4. Results of long-run study under combined altruism – benefit/cost ratio measure at different costs. Naive learning included as a control. Nash structures (non-empty) are predominantly comprised of the Strict Nash (one-way) circle structure.

More is better?

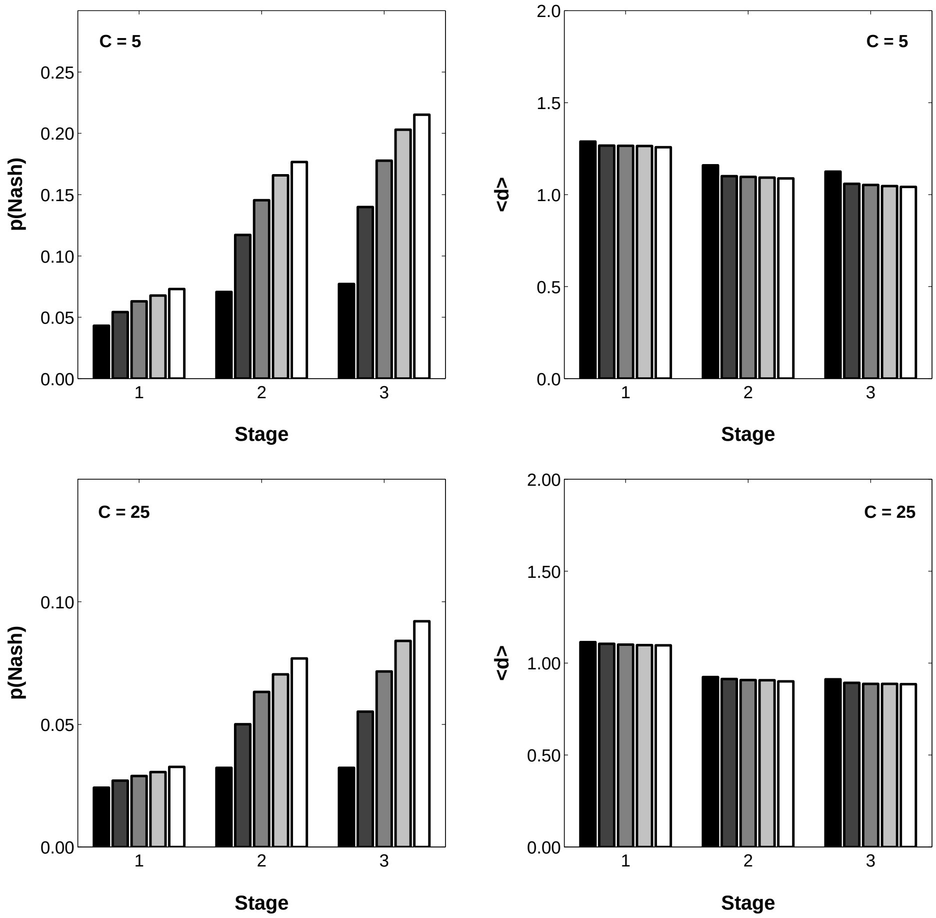
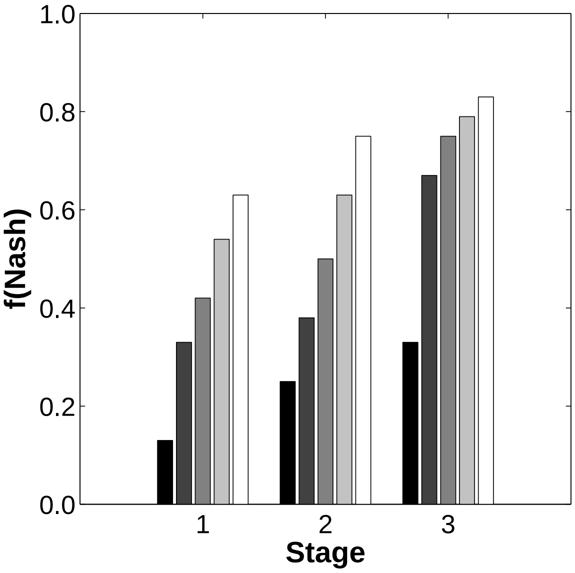


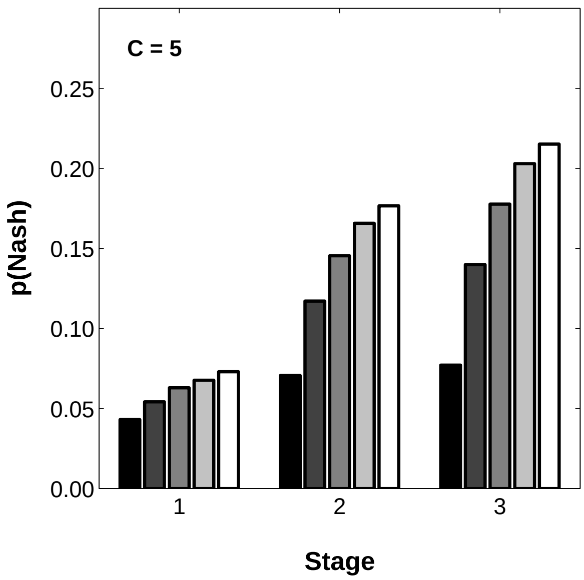
Figure 5. Within- and between- stage learning as evidenced by improving (non-empty) Nash structure probability. Average agent degree also shown (right), showing little within-stage variation, despite large equivalent performance variance (left). Data shown is average over all mixing groups and repeats.

- Strong improvements within ‘stages’;
- Improvements between ‘stages’;
- More is better? .. no.. strategic learning!

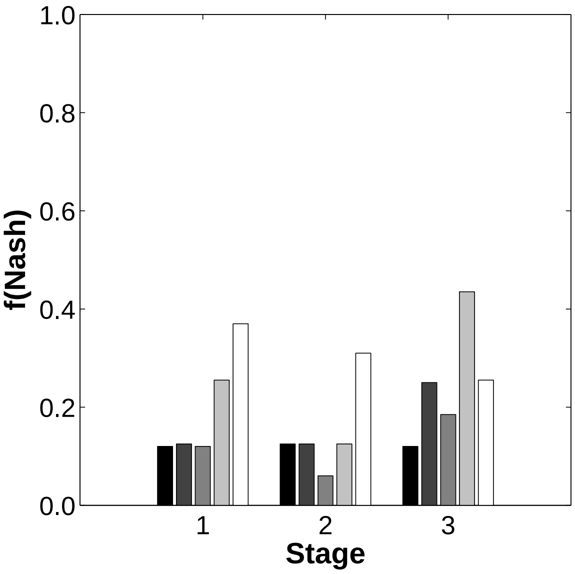
Humans vs. Artificial Agents

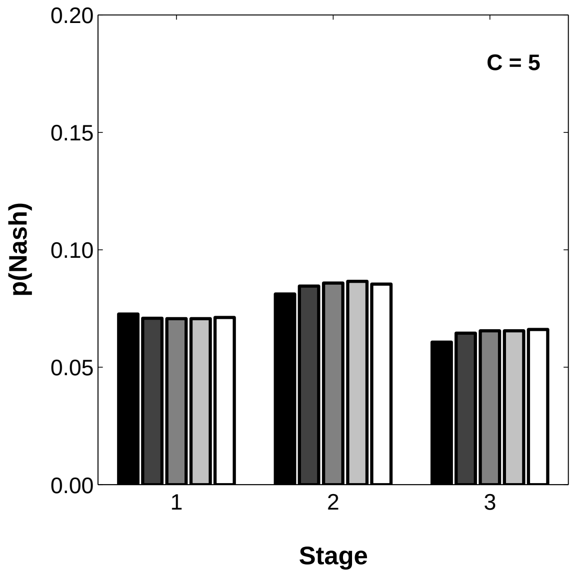
One-way (Nash)





Two-way (Nash)





The Rise of Inductive Reasoning

Question: *Are agents able to predict the next round of play?*

- Simple measure of 'prediction'
- Strategy *this period* versus:
  1. Realised graph *last period*
  2. Realised graph *this period*

$$M_i^r = \mathsf{sign}\Big[f(g_i^r \cap g_{-i}^r) - f(g_i^r \cap g_{-i}^{r-1})\Big]$$

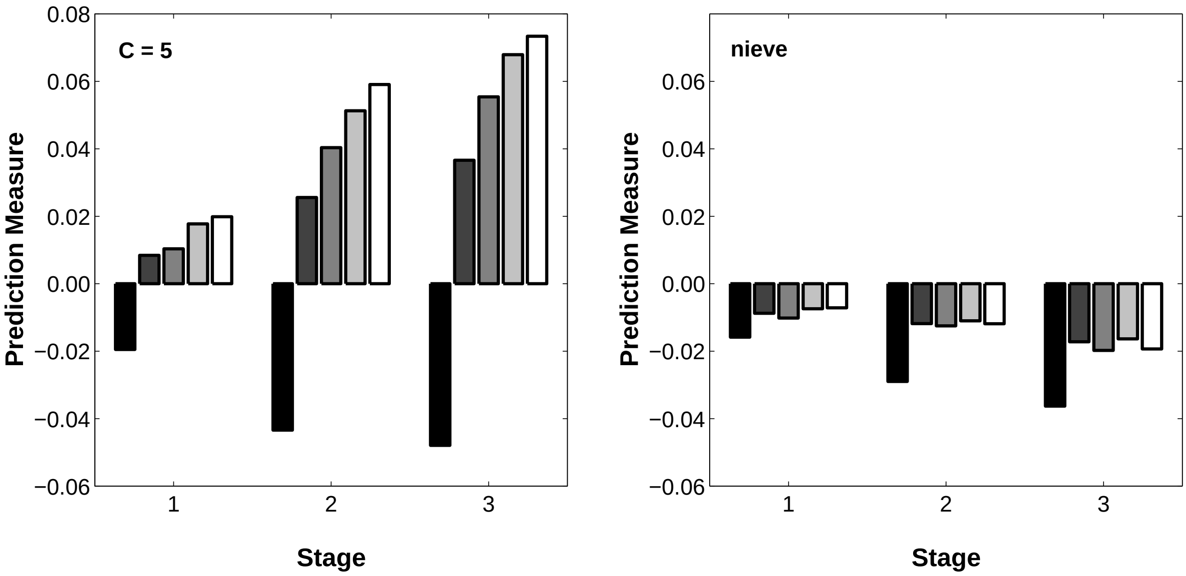


Figure 6. Prediction measure results for within- and between-stages for combined and naive learning rules for comparison. A strong correlation with performance is clear.

Concluding Comments

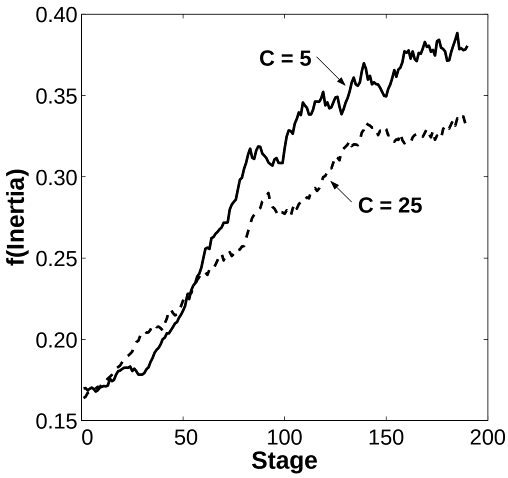
1. AAs replicate many stylized facts of experimental work
   (a) Nash structures (predominantly circles) in one-way case;
   (b) Very few cs-stars, Nash outcomes in two-way case;
   (c) Within stage, and between stage improvement (learning?) in one-way, but not two way;
   (d) *Stategic* improvement rather than just link-based;
   (e) Emergence of inductive/predictive reasoning despite single-period backward-looking play.

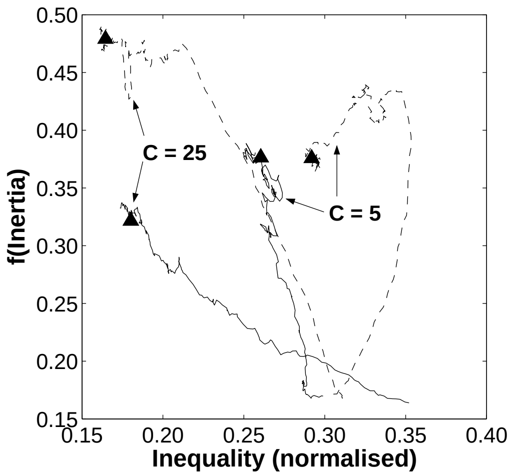
2. Why don't the AAs achieve same *magnitude* of performance?
   (a) No focal structure – model completely agnostic with respect to each (of 4096) possible structure;
   (b) Only 1 period of memory (role of signalling etc.)
   (c) Relatively limited cognition – 'value' measure, with reciprocity only.

3. What else would one want to know?
   (a) The misses: if they aren't playing Nash, what are they playing? (measure for 'off-play')
   (b) What coordination mechanisms could be used to induce cs-star play? (predictions for the lab?)
   (c) How complex are the strategies of individuals? Does *diversity* have something to say, especially in the initial group (predictions for the lab?)

Strategic Inertia & Emergence

- Strategic inertia: $s_t = s_{t-1}$ not part of model process;
- Emergent phenomenon – correlated with 'good' play (one-way) or 'sponsor-none' (two-way).





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