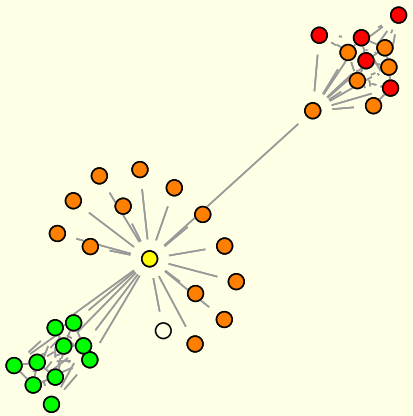
Cooperation Networks
Endogeneity & Complexity

Simon Angus

School of Economics, University of NSW, Sydney, Australia

Supported by the Commonwealth Government's APA scheme;
Research initiated at the SFI Computational Grad Workshop, 2004

ICCS, Boston, June 2006



Agenda

1. The problem of endogeniety;
2. A simple modification: properties and results;
3. Endogenous networks and complexity;
4. Areas for future interaction/work.

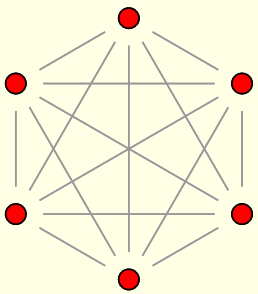
Cooperation: how does it work?

- Maynard Smith & Price: The logic of animal conflict (*Nature*, 1973);
- Axelrod & Hamilton: The evolution of cooperation (*Science*, 1981);
- Nowak & May: Evolutionary games and spatial chaos (*Nature*, 1992);
- Nowak & Sigmund: Evolution of indirect reciprocity by image scoring (*Nature*, 1998);
- Riolo, Cohen & Axelrod: Evolution of cooperation without reciprocity (*Nature*, 2001);
- Burtsev & Turchin: Evolution of cooperative strategies from first principles (*Nature*, 2006);

Analytical approaches

Uniform

- 'Trembling towards equilibrium' (best-response with mistake-making)
- Risk-dominant eq.
- e.g. KMR (1993)



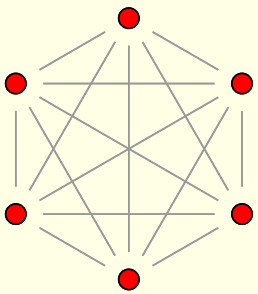
Analytical approaches

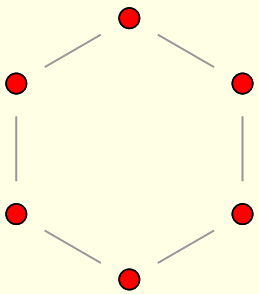
Uniform

- 'Trembling towards equilibrium' (best-response with mistake-making)
- Risk-dominant eq.
- e.g. KMR (1993)

Circle, Line, Grid

- Best-response, with **local** interactions
- Risk-dominant with acceleration
- e.g. Ellison et. al (1993–2000)





Analytical approaches

Uniform

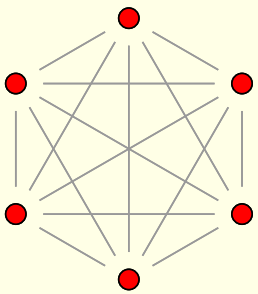
- 'Trembling towards equilibrium' (best-response with mistake-making)
- Risk-dominant eq.
- e.g. KMR (1993)

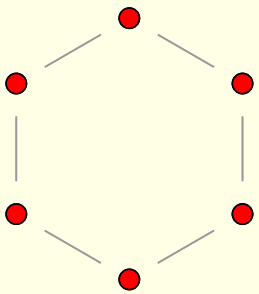
Circle, Line, Grid

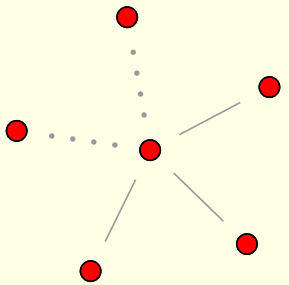
- Best-response, with **local** interactions
- Risk-dominant with acceleration
- e.g. Ellison et. al (1993–2000)

Dynamic

- Best-response graph-formation
- Inefficient and non-risk-dominant eq. possible
- e.g. Jackson & Watts (2002)







Limitations of Analytic Framework

- Strategies other than the Best-response (utility maximizing) hard to model analytically;
- Non-uniform (and non-regular) interaction spaces very challenging;
- Problem of *agency* very difficult – especially with strategic network formation;

Limitations of Analytic Framework

- Strategies other than the Best-response (utility maximizing) hard to model analytically;
- Non-uniform (and non-regular) interaction spaces very challenging;
- Problem of *agency* very difficult – especially with strategic network formation;
- boundedly rational behaviour + strategic network formation + dynamic interaction space = limits of analysis!

Limitations of Analytic Framework

- Strategies other than the Best-response (utility maximizing) hard to model analytically;
- Non-uniform (and non-regular) interaction spaces very challenging;
- Problem of *agency* very difficult – especially with strategic network formation;
- boundedly rational behaviour + strategic network formation + dynamic interaction space = limits of analysis!
- **But, computational, agent-based approaches well suited!**

Computational Approaches

Many models of boundedly rational play, but endogeneity of interaction?

- Ising models (incl. social-influence on small-world, random);
- Computation on a grid (e.g. 2D);
- Diffusion of technologies (again, structure-defined);
- IPD/CR (choice-refusal), endogenous, but network not strategic;

Computational Approaches

Many models of boundedly rational play, but endogeneity of interaction?

- Ising models (incl. social-influence on small-world, random);
- Computation on a grid (e.g. 2D);
- Diffusion of technologies (again, structure-defined);
- IPD/CR (choice-refusal), endogenous, but network not strategic;

Desirable Computational Model Qualities:

- 'Simple' set-up – relationship to previous literature;
- Truly *endogenous* (strategy-based, rather than observer based) interaction-space dynamics;
- Equilibria? Dynamics? Complexity?

Model Overview

Game

- Reward for cooperative, but risky play (modified IPD);
- Signal (#) play: fore-go payoff, establish link;
- Re-establishment each interaction.

	$\#_w$	C	D	$\#_s$
$\#_w$	(0,0)	$\cdots$		(0,0)
C	.	(3,3)	(0,5)	.
D	:	(5,0)	(1,1)	:
$\#_s$	(0,0)	$\cdots$		(0,0)

Model Overview

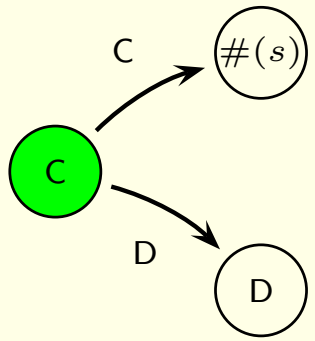
Game

- Reward for cooperative, but risky play (modified IPD);
- Signal (#) play: fore-go payoff, establish link;
- Re-establishment each interaction.

	$\#_w$	C	D	$\#_s$
$\#_w$	(0,0)	$\cdots$		(0,0)
C	.	(3,3)	(0,5)	.
D	:	(5,0)	(1,1)	:
$\#_s$	(0,0)	$\cdots$		(0,0)

Agents

Finite State Automata (FSA), GA updating



Control: length of interactions τ; number of ints/prd m

Model Overview

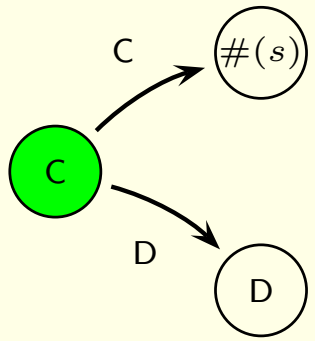
Game

- Reward for cooperative, but risky play (modified IPD);
- Signal (#) play: fore-go payoff, establish link;
- Re-establishment each interaction.

	$\#_w$	C	D	$\#_s$
$\#_w$	(0,0)	$\cdots$		(0,0)
C	.	(3,3)	(0,5)	.
D	:	(5,0)	(1,1)	:
$\#_s$	(0,0)	$\cdots$		(0,0)

Agents

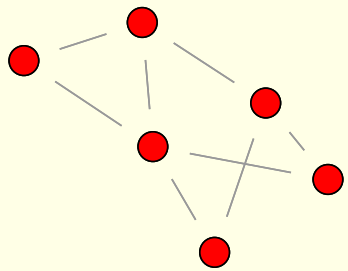
Finite State Automata (FSA), GA updating



Control: length of interactions τ; number of ints/prd m

Mixing

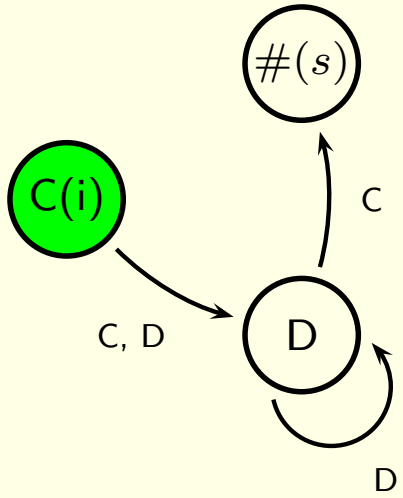
Uniform initially; then endogenous $\sim$ like, dislike, untried



Control: impact of 'like' η

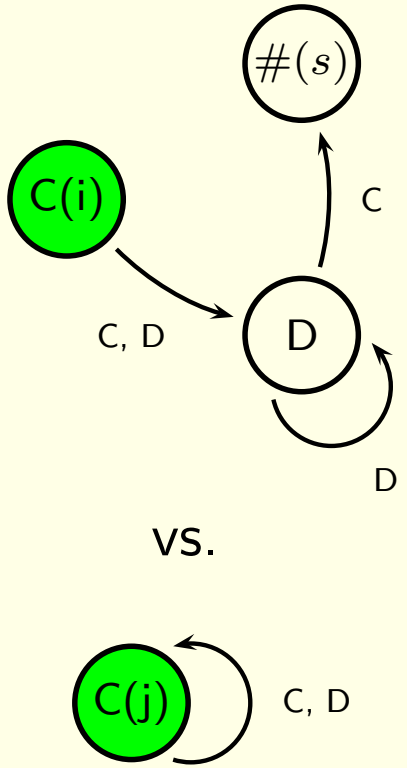
Example Interaction

1. Agent i addressed;



Example Interaction

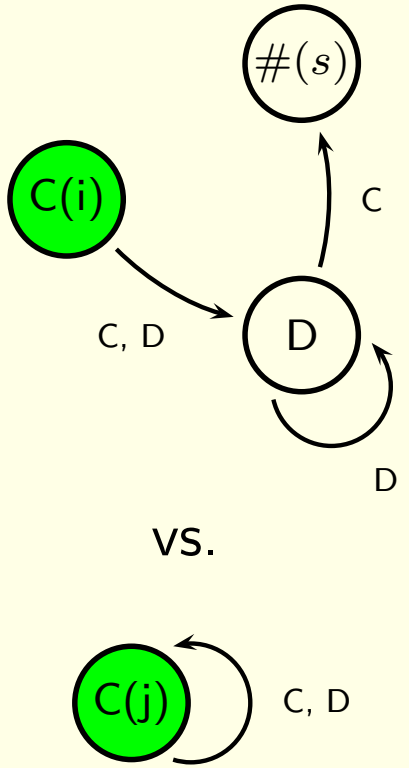
1. Agent i addressed;
2. Interaction probabilities determined from contact listing (like, dislike, untried) – meets j;



Example Interaction

1. Agent i addressed;
2. Interaction probabilities determined from contact listing (like, dislike, untried) – meets j;
3. IPD: interaction stops if # played, or τ iterations reached;

Iteration	s_i	s_j	π_i	π_j
1	C	C	3	3
2	D	C	5	0
3	$\#(s)$	C	0	0
$\sum \pi_x$			8	3

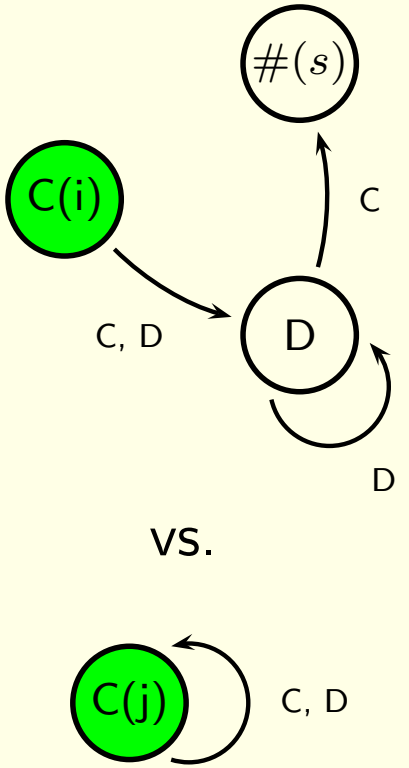


Example Interaction

1. Agent i addressed;
2. Interaction probabilities determined from contact listing (like, dislike, untried) – meets j;
3. IPD: interaction stops if $\#$ played, or τ iterations reached;

Iteration	s_i	s_j	π_i	π_j
1	C	C	3	3
2	D	C	5	0
3	$\#(s)$	C	0	0
$\sum \pi_x$			8	3

4. $\sum \pi_x$ added to period payoffs;
5. Update interaction structure (here, $i \nleftrightarrow j$).



Benefits of Approach

- Due to enhanced *agency*, network that arise are due to strategic play of individuals (not externally applied);

Benefits of Approach

- Due to enhanced *agency*, network that arise are due to strategic play of individuals (not externally applied);
- Networks can be 'good' *and* 'bad' for agents (not just arbitary decision of inquirer);
- Capacity to deal with multiple networks at same time (not single component or list);

Benefits of Approach

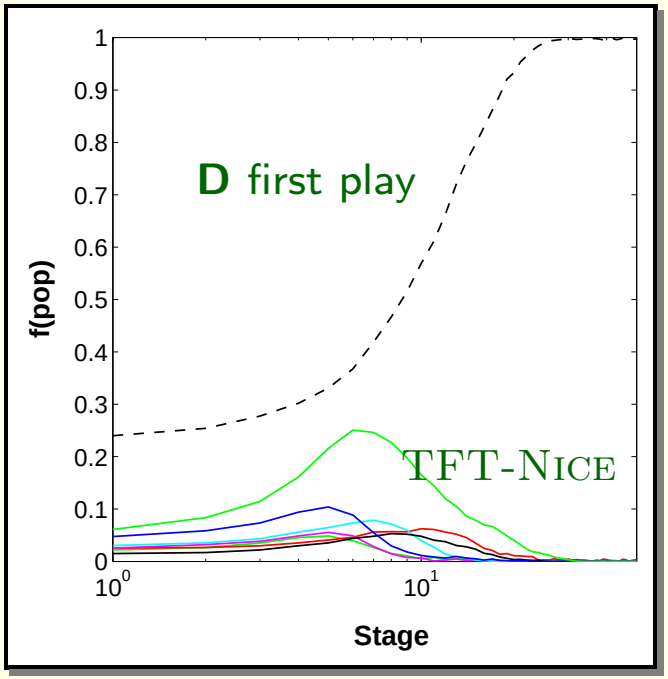
- Due to enhanced *agency*, network that arise are due to strategic play of individuals (not externally applied);
- Networks can be 'good' *and* 'bad' for agents (not just arbitary decision of inquirer);
- Capacity to deal with multiple networks at same time (not single component or list);
- FSA allows for large strategic space (e.g. for $\tau = 3 \longrightarrow 34$ *distinct* strategies;
- FSA encoding provides facile method of *learning* and *innovation/mistake-making* for agents;

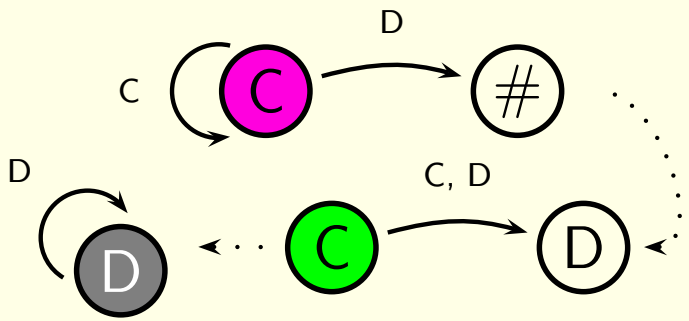
Without network formation, $\eta = 0$

- Is system still similar to standard IPD set-up?
- ... does the playing of # affect things?

Without network formation, $\eta = 0$

- Is system still similar to standard IPD set-up?
- ... does the playing of # affect things?
- Can show analytically that **D** play inevitable;
- Seen computationally (20 trials; 100 agents; $m = 20$).

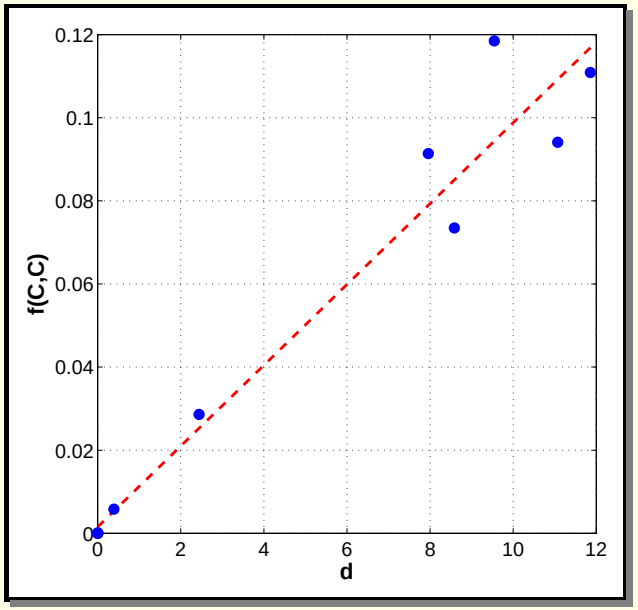




$\eta > 0$: Network formation & Cooperation

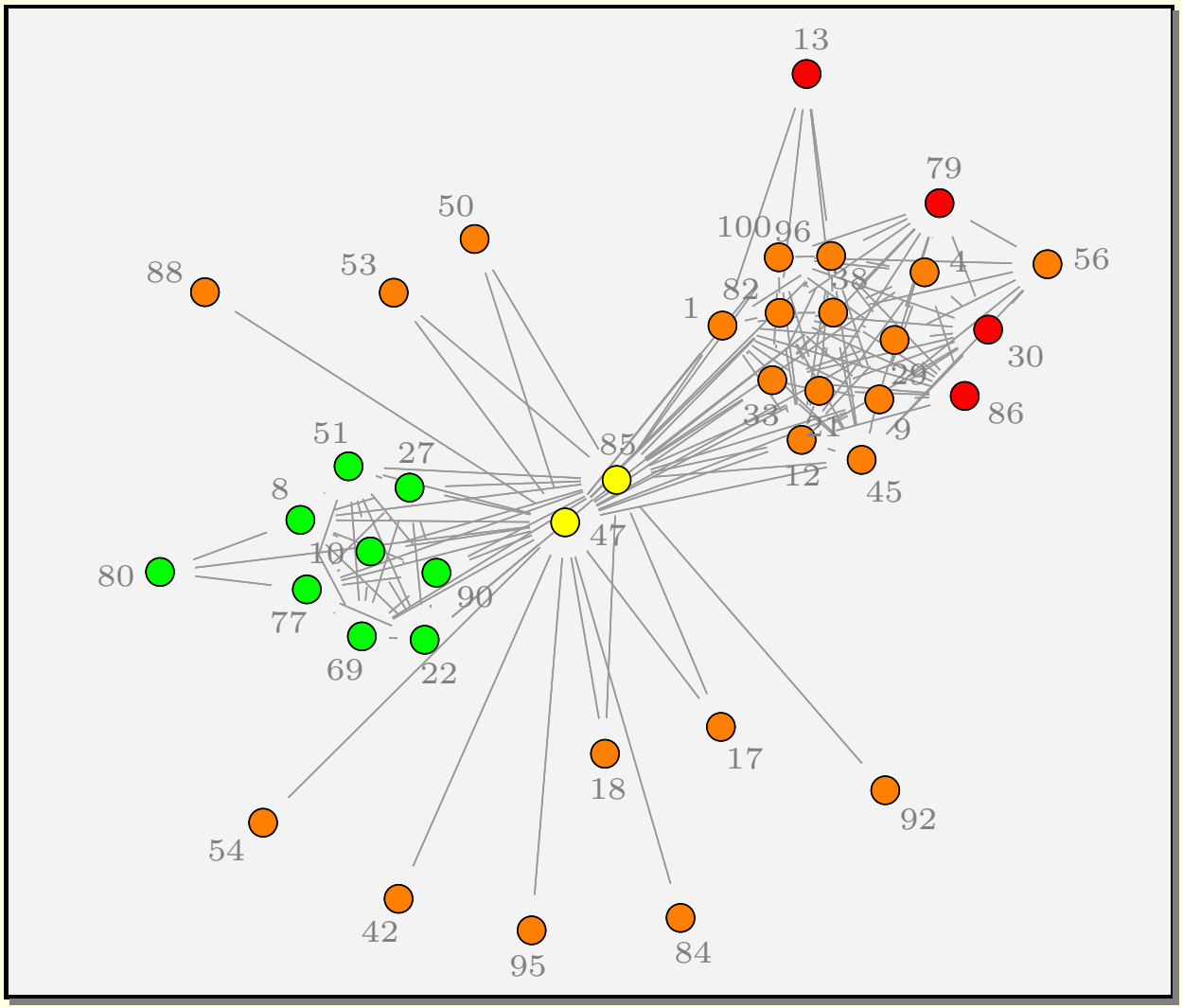
'Frequency' & 'Choice'

- Cooperation and average degree strongly related;
- Frequency of interaction AND 'impact' of edges necessary for sustainable cooperation-networks.

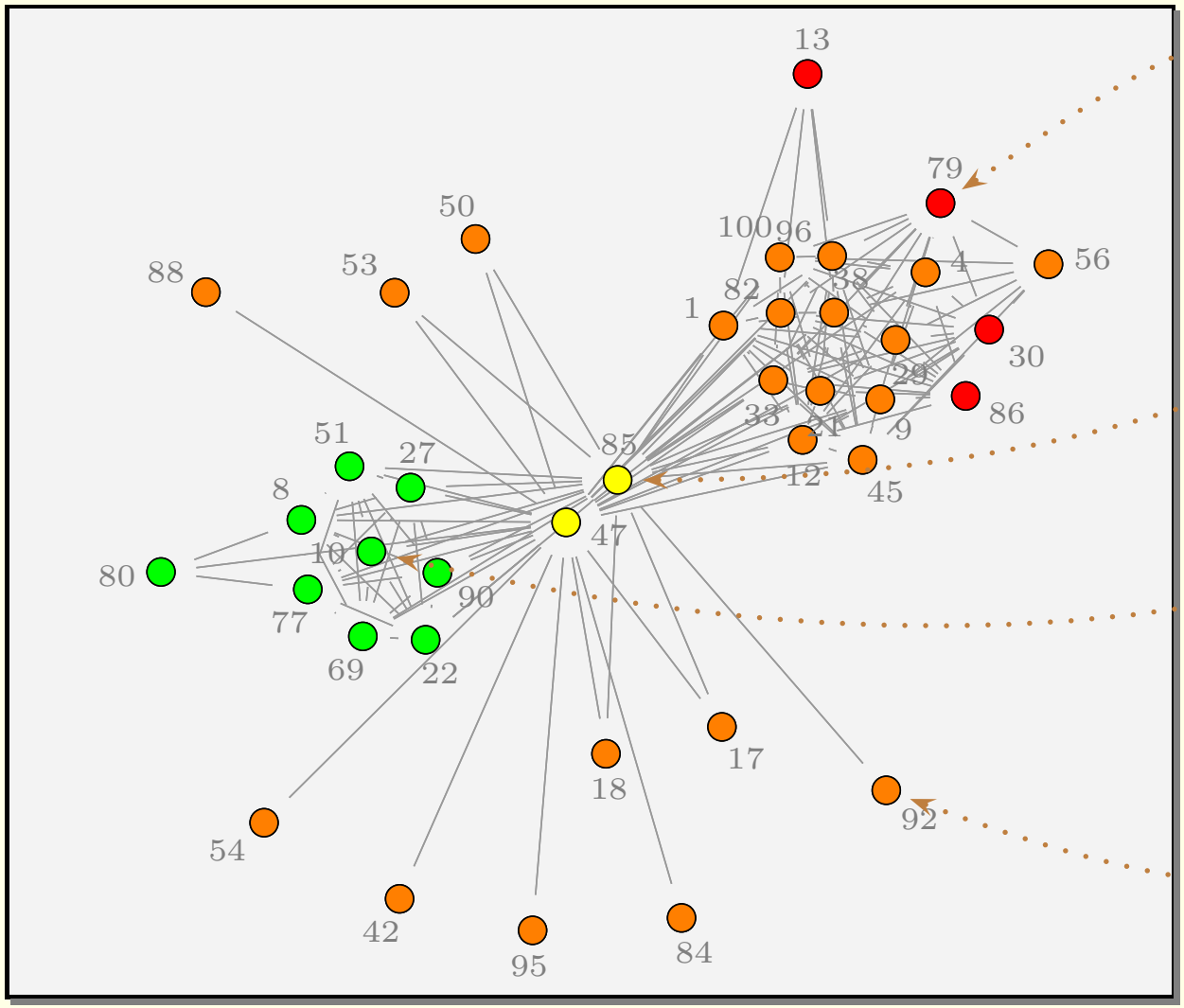


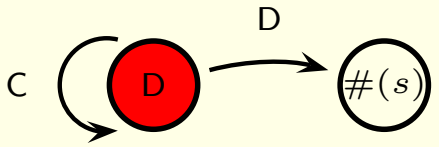
	$\bar{d}$			$f(C,C)$		
$m \setminus \eta$	0.80	0.90	0.95	0.80	0.90	0.95
10	0.000	0.000	0.000	0.000	0.000	0.000
14	0.004	0.001	0.391	0.000	0.000	0.006
18	2.441	11.859	8.587	0.029	0.111	0.074
20	7.959	11.073	9.548	0.091	0.094	0.119

Network formation I: usual suspects

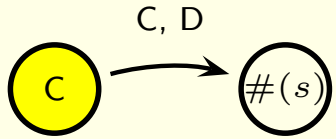


Network formation I: usual suspects

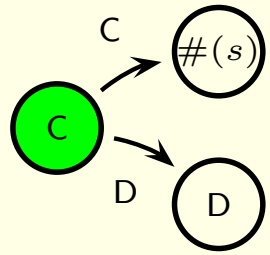




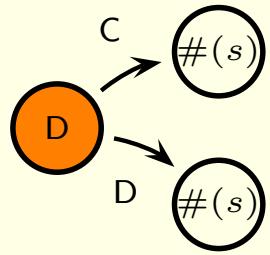
ID 79 ‘robust-D’



ID 85 ‘sucker’

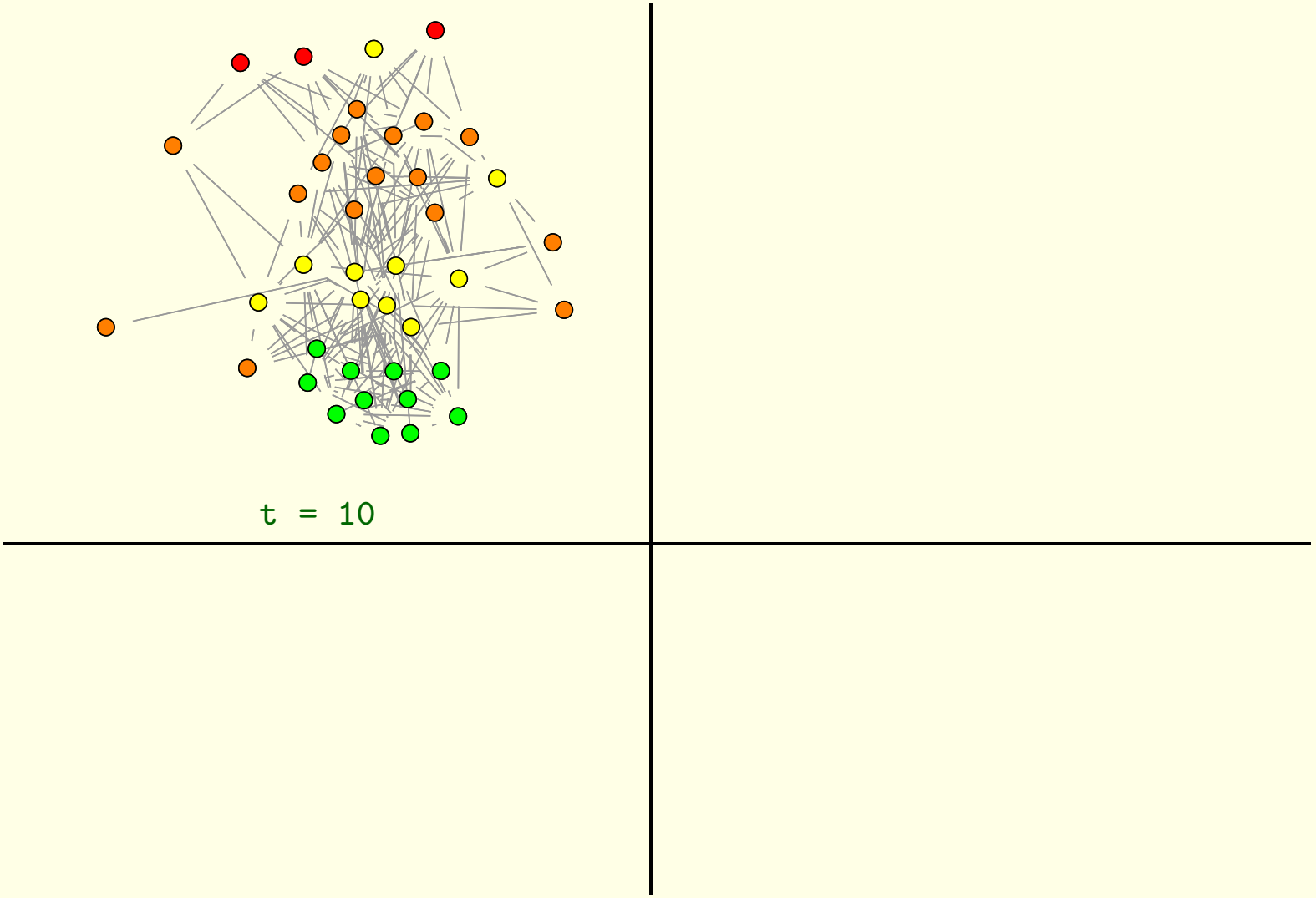


ID 10 ‘robust-C’

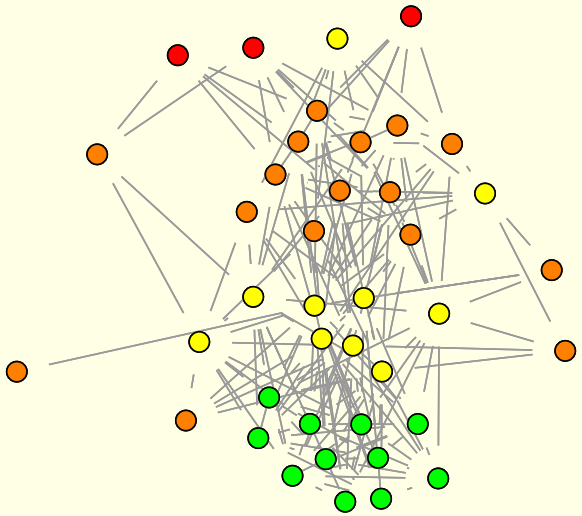


ID 92 ‘opportunist’

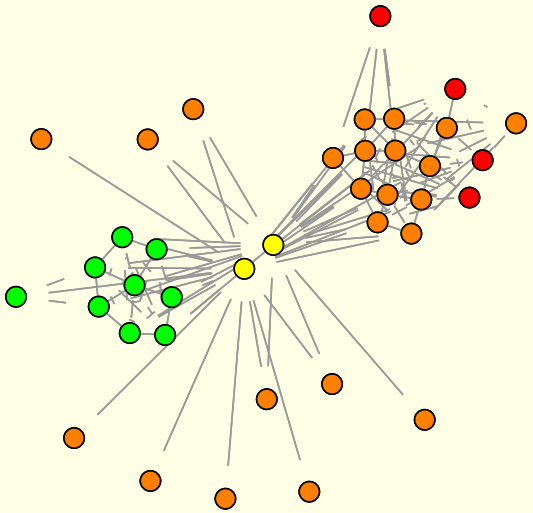
Network formation II: a character tour ...



Network formation II: a character tour ...

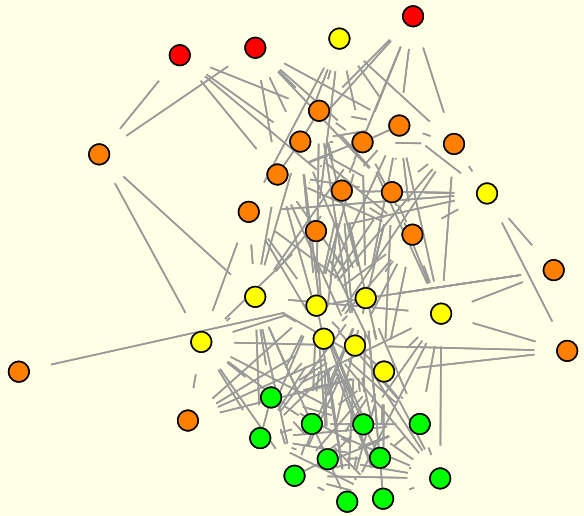


t = 10

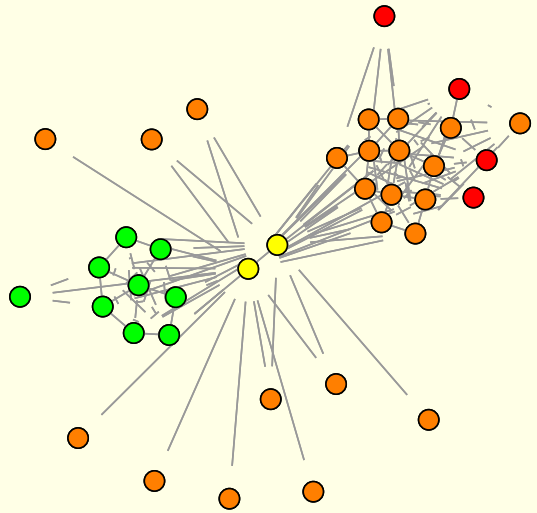


t = 13

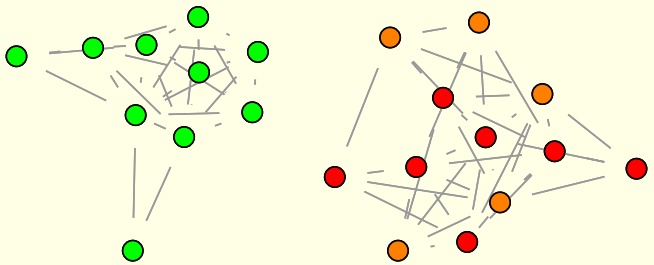
Network formation II: a character tour ...



t = 10

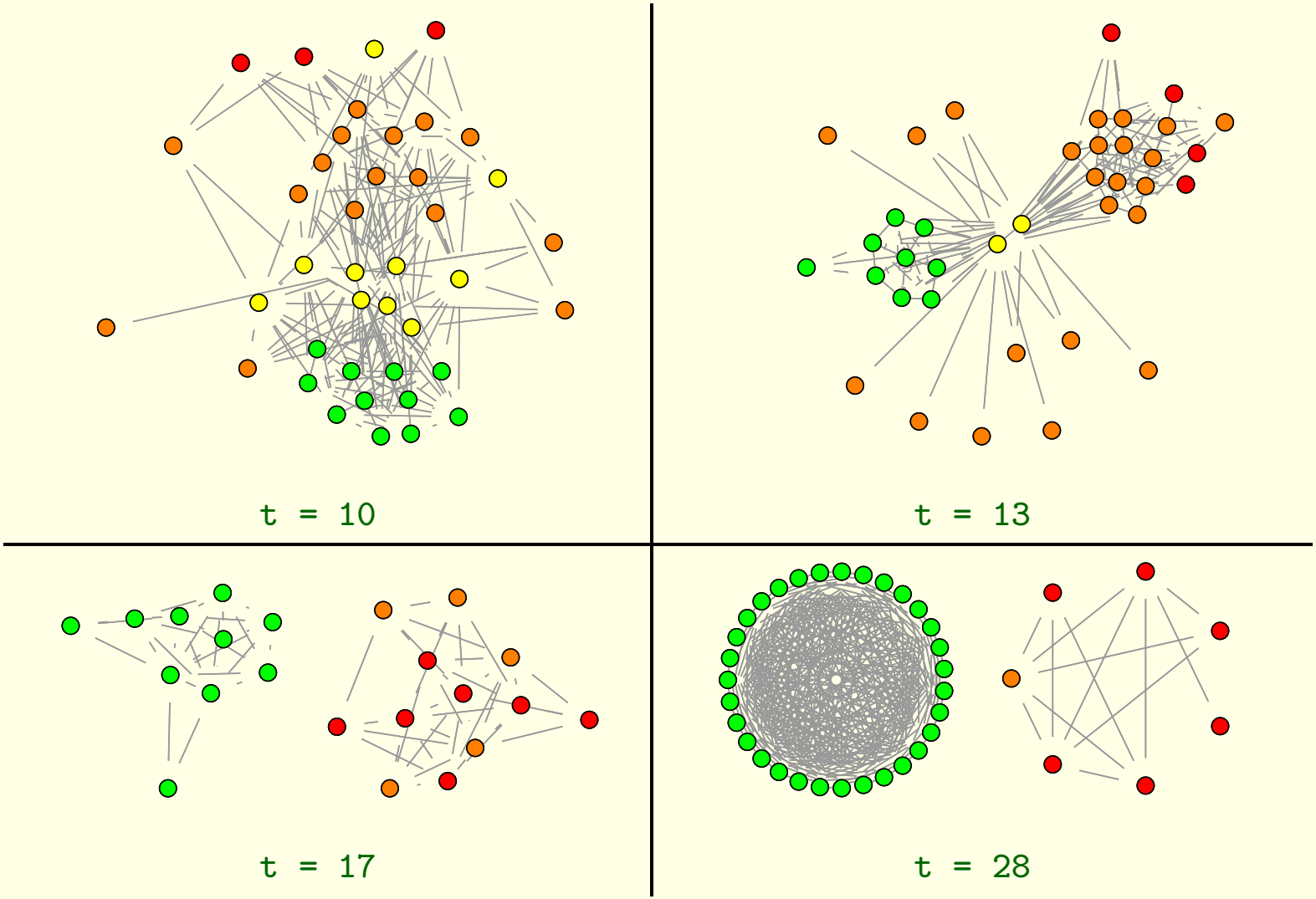


t = 13



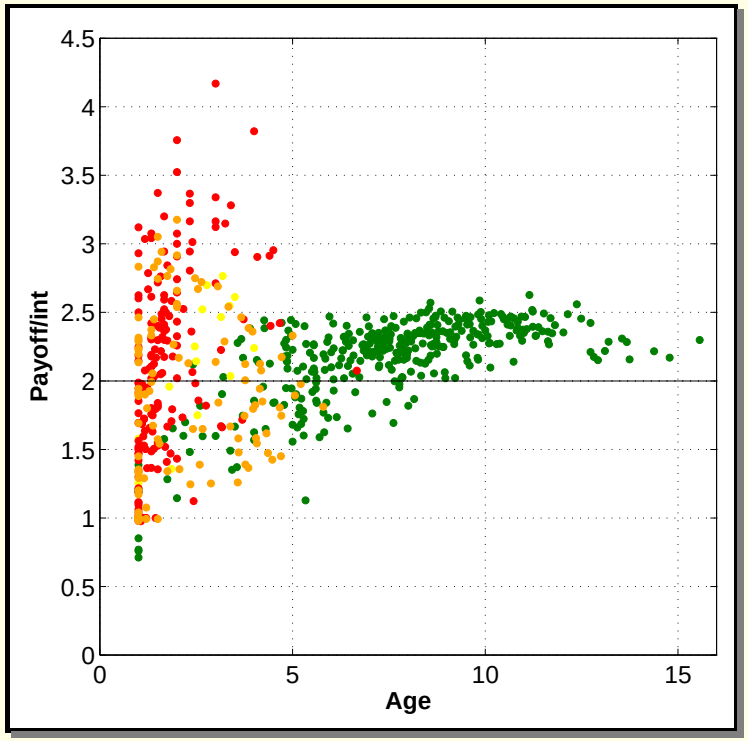
t = 17

Network formation II: a character tour ...



Network Purity & Stability

- High payoffs in mixed networks visible;
- Assortative (preferential) mixing leads to long-gevity;
- All-D payoff cut-point.



Network payoffs vs. mean age by dominant ($> 50\%$) type

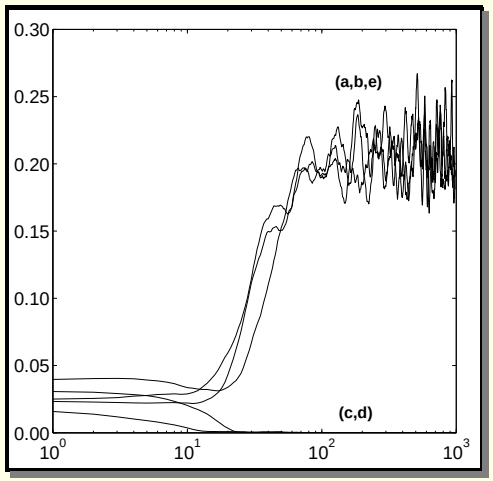
What about dynamics?

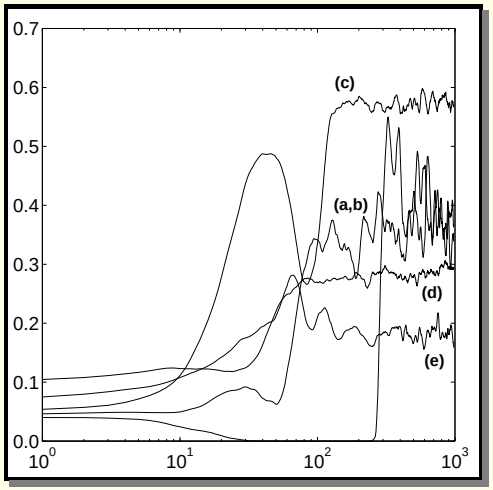
What is a state?

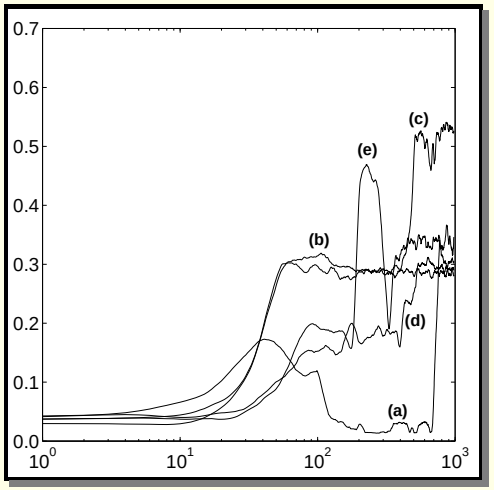
- State description is enormous (network + automata);
- Alternative, capture descriptive statistics that give aggregate description of state:
  - **Strategy measure** fraction of mutual cooperative plays, out of all plays ($f(C, C)$);
  - **Network measure** average agent link sponsorships ($\langle d \rangle$);

From simplicity to complexity...

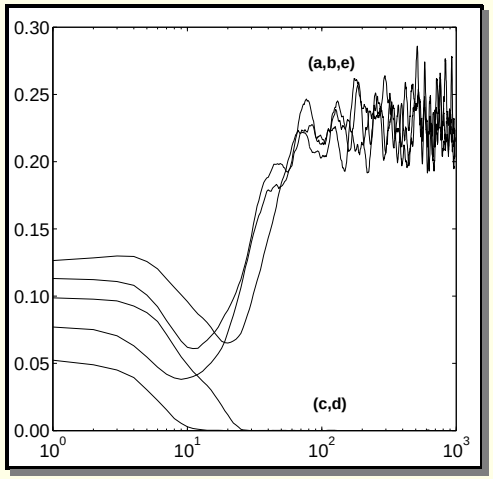
$\tau = 2$ $\tau = 3$ $\tau = 4$

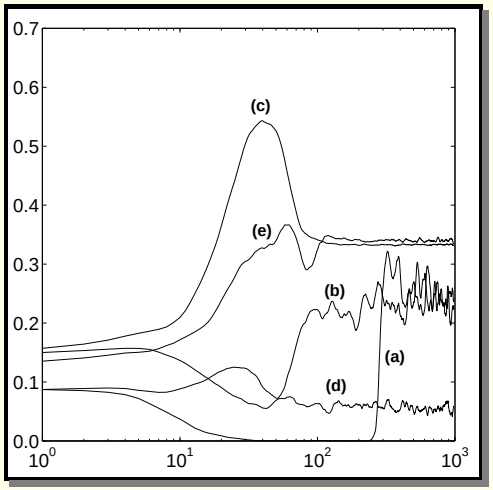


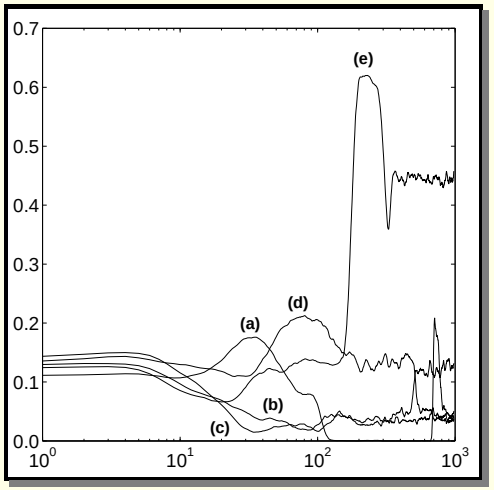




$\langle d \rangle$



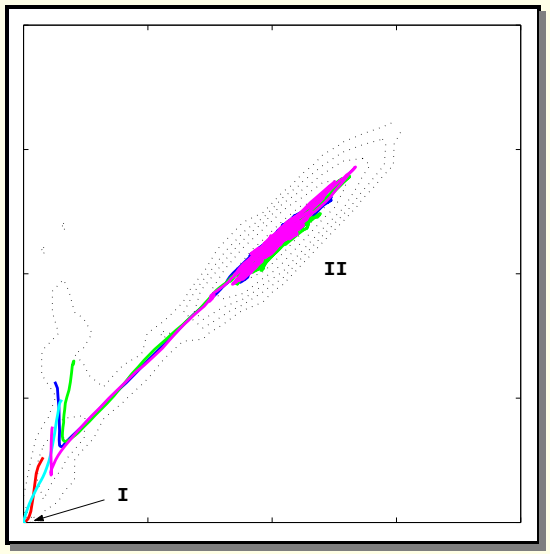




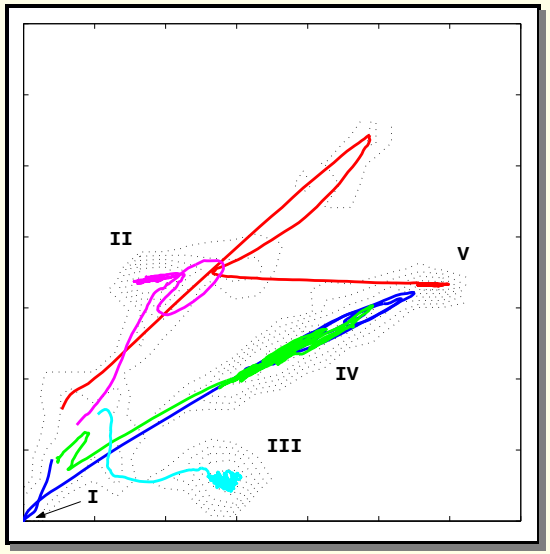
$f(C, C)$

In 'Network:Strategy' space

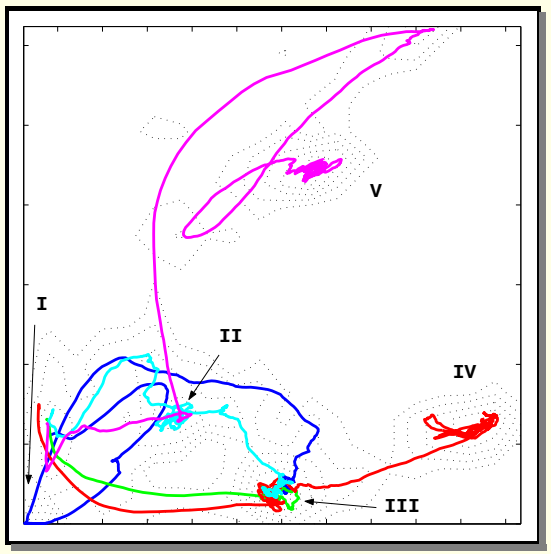
$\tau = 2$



$\tau = 3$



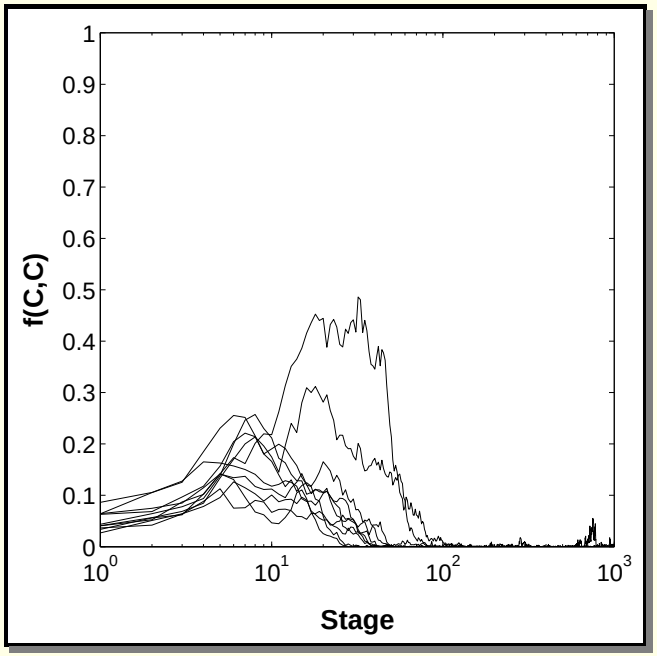
$\tau = 4$



Sources of complexity?

- Interactions?
- Strategies?
- Network dynamics?

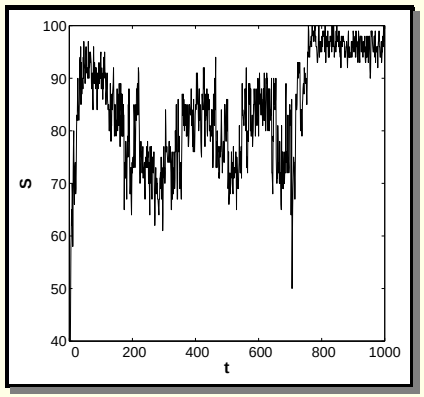
$\tau = 4$, network off



Endogeneity & self-organized criticality

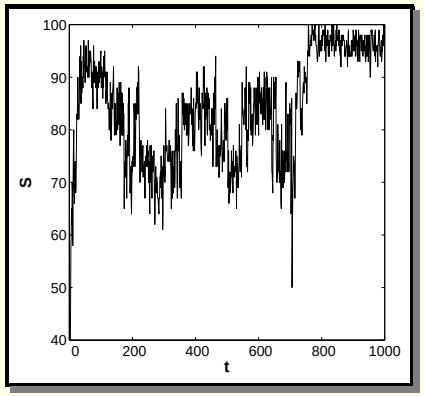
- Network changes could be source of complexity;
- Does the network show scaling over time and space?
- What is an **event**?
  - **Space** Frequency distribution of changes in network size (nodes, principle component);
  - **Time** Power spectra of size changes over time;
- Power-law scaling would indicate system criticality.

Analysing self-organized criticality on networks



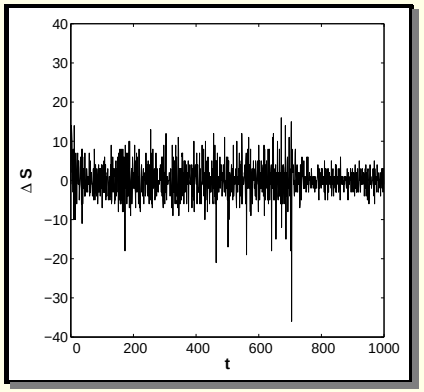
Network size series

Analysing self-organized criticality on networks



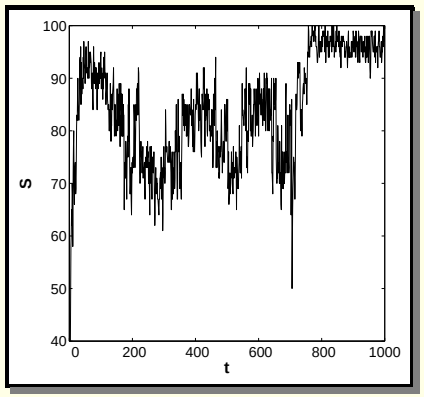
Network size series

$x_t - x_{t-1}$



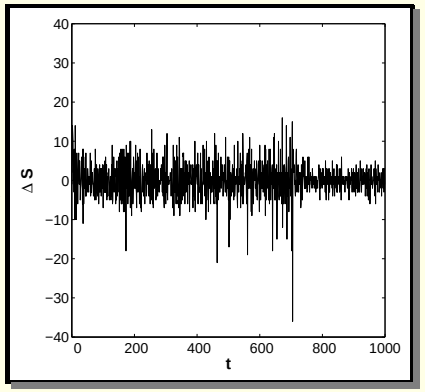
'Event' series: change in size

Analysing self-organized criticality on networks



Network size series

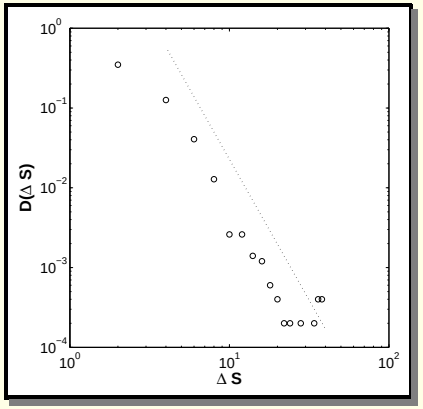
$x_t - x_{t-1}$



'Event' series: change in size

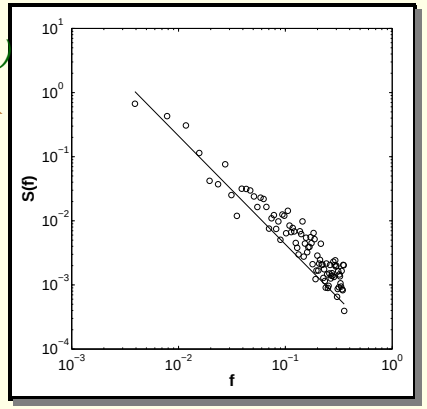
$F(x)$

Fractal scaling (space)



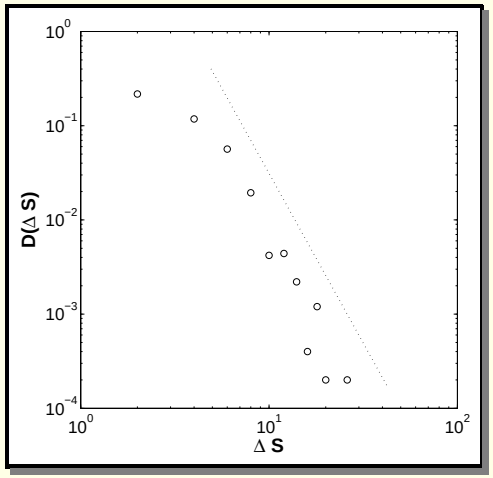
$DFFT(x)$

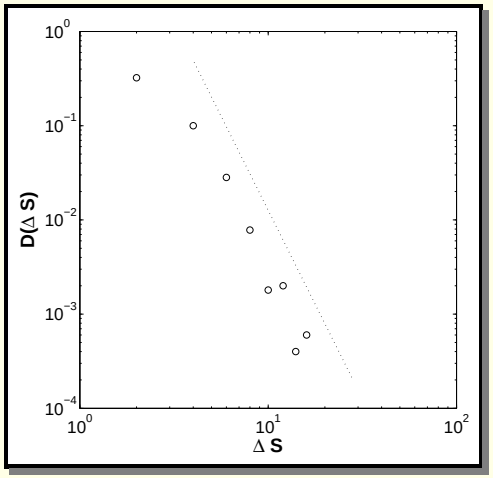
Power spectra (time)

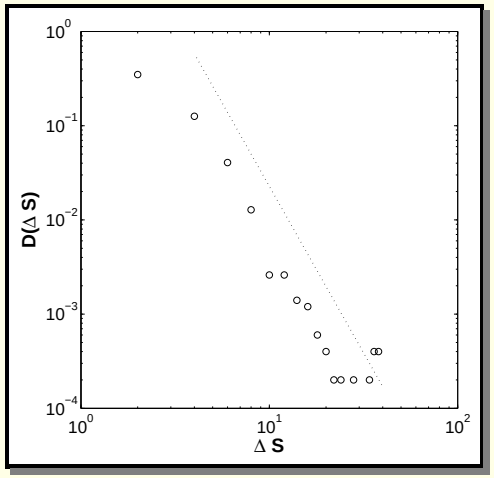


Power-law scaling (again...)

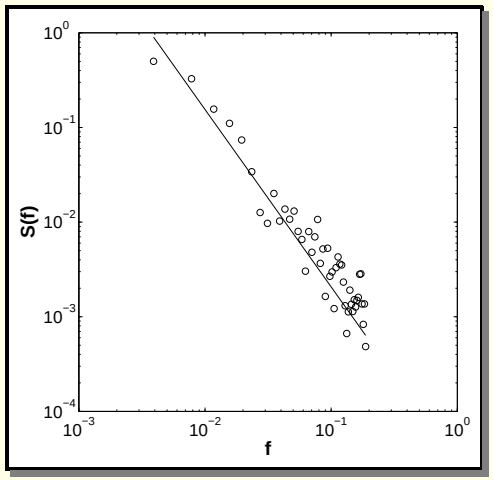
$\tau = 2$ $\tau = 3$ $\tau = 4$

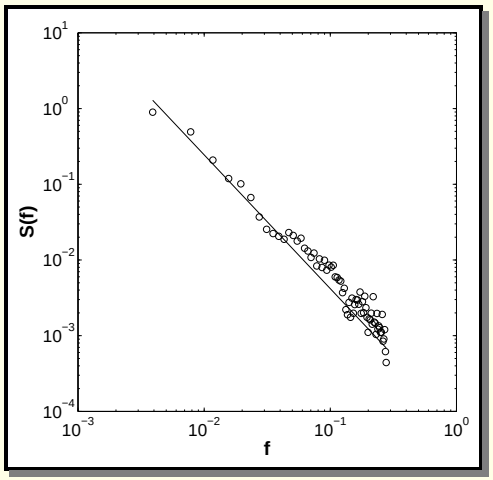


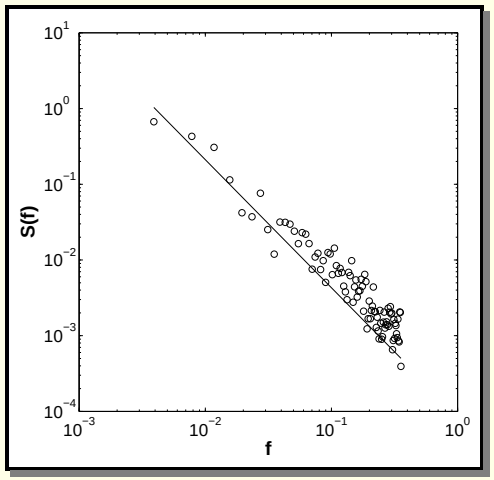




$D(\Delta S)$







$S(f)$

What does it mean?

- White noise would give slope of 0, here, slope: -1.8 ± 0.1;
- The system (analysed in these measures) displays critical behaviour (i.e. at/near a phase change);
- Impact of events propogate through spatial and temporal dimensions – connectivity;

What does it mean?

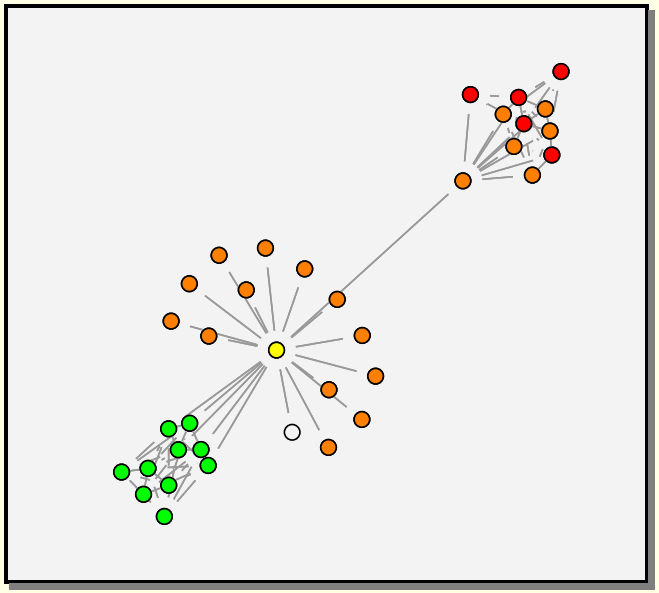
- White noise would give slope of 0, here, slope: -1.8 ± 0.1;
- The system (analysed in these measures) displays critical behaviour (i.e. at/near a phase change);
- Impact of events propogate through spatial and temporal dimensions – connectivity;
- Not surprising, although...
- Implies the SOC outcomes:
  - Tracking 'equilibrium' (resting) points becomes a *statistical* task (rather than by explicit prediction);
  - Seemingly small events can cause system-wide effects (although rarely) .. don't expect proportionality;
  - 'Simple' modification can upset canonical behaviour.

Future questions

- Coordination games on networks?
- Biological: self-replication with fitness?
- Economic: communication? reputation? signalling in network?
- Implications of SOC/complexity in these models – are we comfortable with disequilibrium? Long-run data on these effects?
- Social/government policy: dynamic control? What are the instruments? How costly are they? Where do they apply?

Thanks

Simon Angus, UNSW
s.angus@unsw.edu.au



$\eta > 0$: Mean Population Behaviours

Establishing the Network $(m = 20, \eta = 0.8)$

- Periodic behaviours observed: 'sucker' types; 'opportunists'; cooperation network builders; and defection network builders;
- 'Shake-out' period as before, but cooperation network resiliant;
- In network forming trials, cooperative network grows to encompass $\sim$ 60% of population