

Games People Play¹

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I. Rock, paper, scissors & that last piece of pie

Economics, some would say, is the study and practice of resource allocation under scarcity – ergo, ‘No scarcity. No economics.’ You may disagree with the definition, but few would disagree with the reality of its content. One need only remember the last time that you and a group of friends set to work on a cake or pie, presumably finding blatant imperfections in the distributive mechanism. What to do? A coin flip often smacks of pure chance, whereas arm wrestling (and its variations) occupy the alternative pole – all ability, and no lady luck to smooth tempers. In Australian culture, the simple but effective game of ‘rock-paper-scissors’ (RPS) usually fills the void; a quick and simple tournament approach (after necessary clarification of the rules of engagement) yielding a solution to the pie division in seconds: to the victor, the pie; and to the vanquished, the solace of battle ‘fairly’ lost. But why?

Perhaps you’ve never thought about RPS, but I doubt it. If you’re much like me, and have ever found yourself the vanquished party, you will spend at least a few moments analysing the tournament, possibly with such anguished questions as: ‘where did I go wrong? why didn’t I play ‘rock’ to his ‘scissors’? how could I have missed the signals?! I’m sure his left eye-brow was raised, that always means scissors!!’ Unfortunately, such a process rarely leads to any constructive insight. Rather, after a moment of jubilation, a very public pie-consumptive humiliation ensues and the nuances of the game-play pass quietly into history. All that arises from one’s reflection is a steely resolve for next time (and a groaning stomach).

However, help is at hand, for it is to just such a situation that the Theory of Games (or Game Theory) speaks. Game theory is the (analytical) study of strategic behaviour – that is, where individual behaviour depends upon the actions of others. Where most other micro-economic theory limits its perspective to the individual, game theory attempts to incorporate the decisions of all players in a strategic situation. And as promised, we can say quite a lot about RPS through the lens of game theory.

II. ‘One-two-three...’: Simultaneity and the Normal Form game

RPS, as described above, fits the rudimentary type of game that game theory studies: the ‘normal form game’. In a normal form game, we require several elements: *players*, their *strategies*, the *payoffs* accruing to each player under the different possible outcomes, and finally (and importantly), simultaneous game-play.

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The players in RPS are obviously the two with their fists in the air (let's call them A, and B), and their strategies are naturally one of 'rock', 'paper', or 'scissors'. All that remains is to describe the possible payoffs, which are simply taken from the set {win,lose,replay} and follow from the physical nature of each strategy combination. Hence, if A plays 'rock' and B plays 'scissors', then the rock beats the scissors, and A receives: 'win', B: 'lose'. However, and here is the rub, if B played 'paper' instead, then A would lose, and B win. Infact, any strategy that that a player plays can yield 'win' or 'lose' based on what the other player plays. This is the very kernel of the theory of games, my outcome doesn't just depend on me, it depends on each player's decisions.

Important to this game is that the strategies of each player are revealed *simultaneously*. Ever wondered why you shake your fist three times in the air, chanting aloud, 'one-two-three' before revealing your play? For precisely the reason of simultaneity. The shaking and chanting serve to 'lock-in' the timing so that each player's strategy is known at *exactly* the same time.

Timing is very important in game theory, and we shall conclude on this point below, however, for the moment, we notice the clincher of RPS: there is no strategy that any player can play that ensures a 'win' for them. They simply don't know what the other player will play – they might win, or they might lose, there are no guarantees. We say that there is no *dominant* strategy for a player in RPS, and this, coupled with the timing of the game ensures that there is no way that any player can get the 'upper hand'. They must simply 'wait and see'. Moreover, it is worse than this, not only is there no dominant strategy to play, but there is no combination of strategies that the two players can play which will result in them both being 'happy' with the outcome at the same time. If you lose a game of RPS, then you always know (in hind-sight) what you should have played. The catch is, that if you'd played that way, your opponent would have played differently too...and so on. By its nature, RPS has no equilibrium.

III. When everyone does their best: the Nash equilibrium

Not all games turn out like this, however, as opposed to RPS, it is possible that *all* players can be playing their best response to their opponents' strategies *at the same time*. Such a scenario has a special (and now popular) name: the Nash Equilibrium. Consider a game known (a little unhelpfully) as the 'battle of the sexes'. It goes like this: suppose A (the guy) and B (the girl) have an outing booked for the evening, to be enjoyed at either the football (A's preference) or the ballet (B's preference). However, suppose further that A and B work separate jobs, do not own mobile phones (it is possible!) and in the rush of life haven't confirmed the evening's location — that they are going out is sure, but where to, of the two options, they do not know. We naturally assume that both A and B would prefer to be at the same place for the evening (although A would like that to be at the football, and B at the ballet), but since they can't know before-hand where the other will go, we recognise that they have a coordination problem.

Now in this situation, if A was to choose the football, B's best response would be to go to the football too; and likewise if B chose the ballet, then A should choose the ballet.

This game is interesting since it has not one, but two Nash equilibria. Both the event where A and B arrive at the football together, and the reverse, where A and B arrive at the ballet together are situations in which each player is playing their best response to the other player's decision. Given that A chooses ballet, B's best response would be the ballet too; and likewise, given that B chooses the ballet, A's best response is similarly the ballet. Neither player has an incentive to deviate. It is in this sense that we have *equilibrium*.

Now around 1950 John Nash laid down a very powerful theorem. He asserted that any normal form game (just as we have described above) has at least one Nash equilibrium. But hang on, what about RPS? you ask. Didn't we conclude that there is no combination of strategies for the two players that give their best response to each other? That's true, but what we haven't mentioned so far is that up to this point, our play has only considered what are known as *pure strategies*. The alternative is for players to play *mixed strategies*. The former means that players play some strategy with probability 1, whereas the latter implies that players don't confine themselves to one strategy, rather, in any single game, they play one of a number of strategies *some of the time*. And that, if you are an old-hand at RPS, is exactly what you most likely do. You see, the Nash equilibria in the RPS game is when both players play each of rock, paper and scissors exactly 1/3 of the time each. That is, if you were to play a nine round tournament of RPS with a friend, then for you to be playing the Nash equilibrium, you should both aim to play each of rock, paper and scissors three of the nine times (in some random order, of course). And that, my friends, is the power of game theory. It provides us with a systematic analysis of hitherto intractable strategic environments.

IV. Concluding remarks

We have covered some considerable ground here, but I would not want you to think that we have exhausted the learnings of game theory. Indeed, we have but scratched the surface! For instance, what happens if the 'moves' of the game don't occur at the same time, such as chess or bargaining (for example)? This is another class of games known as *sequential games* and they have their own fascinating outcomes: the game of chess is completely solvable from move one; 'having the honour' – going first in a tournament – can actually be a curse; and perhaps closer to home, knowing that a sequential game between you and a friend has an end to it will cause you both to be as nasty as possible to each other!

Perhaps at this point, you will be rolling your eyes, commenting that such predictions are the stuff of 'common sense'. In most cases, I would happily agree with you, however, it is by applying the Theory of Games that we come to an appreciation of what is going on 'behind the scenes'. And frankly, when it comes to analysing the 'common sense' of the cold war, nuclear weapons proliferation, and other apocalyptic scenarios (not to mention matters of pie division amongst friends), a more rigorous understanding of strategic behaviour seems more than sensible, infact, it's the only worthwhile game in town.