Puzzle corner 2

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Welcome to the Australian Mathematical Society *Gazette*'s Puzzle Corner. Each issue will include a handful of entertaining puzzles for adventurous readers to try. The puzzles cover a range of difficulties, come from a variety of topics, and require a minimum of mathematical prerequisites to be solved. And should you happen to be ingenious enough to solve one of them, then the first thing you should do is send your solution to us.

In each Puzzle Corner, the reader with the best submission will receive a book voucher to the value of $50, not to mention fame, glory and unlimited bragging rights! Entries are judged on the following criteria, in decreasing order of importance: accuracy, elegance, difficulty, and the number of correct solutions submitted. Please note that the judges decision — that is, my decision — is absolutely final. Please e-mail solutions to N.Do@ms.unimelb.edu.au or send paper entries to: Gazette of the AustMS, Birgit Loch, Department of Mathematics and Computing, University of Southern Queensland, Toowoomba, Qld 4350, Australia.

The deadline for submission of solutions for Puzzle Corner 2 is 1 July 2007. The solutions to Puzzle Corner 2 will appear in Puzzle Corner 4 in the September 2007 issue of the *Gazette*.

Coffee and doughnuts

In a certain mathematics class, if each boy purchases a coffee and each girl purchases a doughnut (all items costing an integer number of dollars), the class would spend a total of one dollar more than if each boy purchased a doughnut and each girl purchased a coffee. If there are more girls than boys in the class, what can one determine about the number of girls and the number of boys?

Solitaire

On an infinite chessboard, a game is played as follows. At the start, n^2 pieces are arranged on the chessboard in an $n \times n$ block of adjoining squares, one piece in each square. A move in the game is a jump in a horizontal or vertical direction over an adjacent occupied square to an unoccupied square immediately beyond. The piece which has been jumped over is then removed. Find those values of n for which the game can end with only one piece remaining on the board.

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Glass half full

You have a transparent glass which is in the shape of a right cylinder and it appears to be approximately half full of water. How can you accurately determine whether the glass is half full, less than half full, or more than half full, using as little equipment as possible?

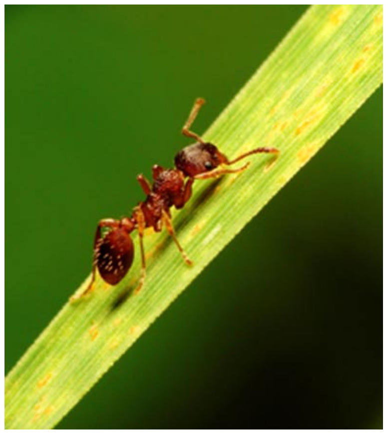
Secret salaries

Three mathematicians would like to know the average value of their salaries. How can they all determine the average without disclosing the value of their own salaries to each other?

Ambulatory ants

(1) A number of ants are distributed around a narrow circular path. At one particular instant, each of the ants chooses a direction and begins to walk along the path. The ants all walk at the same constant speed and, when two ants meet, they both instantaneously change directions and continue walking at the same speed.

Prove that at some later moment, every ant will be in its starting location.



(2) A number of ants are distributed along a thin stick one metre in length. At one particular instant, each of the ants chooses a direction and begins to walk along the stick. The ants all walk at the same constant speed and, when two ants meets, they both instantaneously change directions and continue walking at the same speed. Furthermore, when an ant meets an end of the stick, it instantaneously turns around and continues walking at the same speed.

(a) Prove that after two minutes, every ant will be in its starting location.

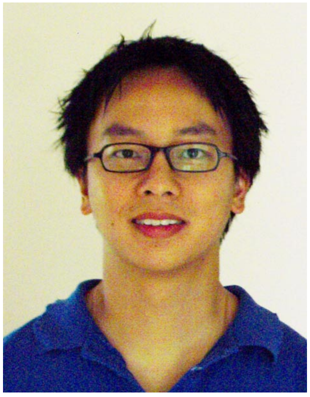
(b) Under what conditions is every ant in its starting location after one minute?

(3) Three ants find themselves on the hour, minute and second hands of an analog clock at noon. After that point in time, whenever one hand of the clock overtakes another, the two corresponding ants instantaneously swap positions. Twelve hours later, each of the ants has travelled a whole number of revolutions around the clock. Which ant has travelled the most number of times around?

An unusual identity

For a fixed value of n, choose any subset of n integers from the set $\{1, 2, 3, \ldots, 2n\}$. Now arrange them in increasing order to obtain the sequence $a_1 < a_2 < \cdots < a_n$ and arrange the remaining numbers in decreasing order to obtain the sequence $b_1 > b_2 > \cdots > b_n$. Determine all possible values of the expression

$$|a_1 - b_1| + |a_2 - b_2| + \cdots + |a_n - b_n|.$$



Norman is a PhD student in the Department of Mathematics and Statistics at The University of Melbourne. His research is in geometry and topology, with a particular emphasis on the study of moduli spaces of algebraic curves.