

Some suggestions

For those of you taking this thing for credit, I require something in writing. What I want is a batch of solved problems organized around some theme. I encourage you to come up with something on your own, but I have some ideas. I don't just want a batch of unrelated problems, but what constitutes a "theme" is quite loose. The problems you choose don't have to be Putnam problems — in fact, I welcome others. You may even include a few standard results — if the topic is unusual, it's likely a good idea to give a small introductory example or even a basic tutorial. (Ex gratia, I can't really see making more than 2 pages out of a thing on Pick's Theorem, but if you did, a proof would be appreciated. I can see a thing made out of uses of the mean value theorem(s) of calculus, but you shouldn't bother proving it (them); state it (them), though.)

Anyhoo, here are some ideas about "themes", with a sampling of possible problems that deal with them. They are, in this instance, all from old Putnams.

1. Games/probability. Lots of problems involve these topics. Incidentally, very few of the "probability" problems have anything to do with heavy-duty probability theory — they are either counting problems, or calculus. (In the latter case, they often involve finding areas or volumes.) (B2,'93), (B3,'93), (B3,'82), (A2,'97), (A2,'01), (A4,'02), (B5,'95), (B4,'85)
2. Leibniz' rule/funky integrals. Occasionally, you find a problem where differentiating under the integral sign comes in handy. (A5,'05), (A3,'82) Other times, a little ingenuity does the trick. (A2,'95), (B1,'87), (B5,'85) [Marc-André: This is yours if you want it, but please show up for the practice sessions.]
3. Problems with the problems. Very occasionally, a problem winds up on the Putnam (or some other competition) that is false or meaningless as stated (B2,'02), (B3,'00), possibly (A3,'05). More often, a problem shows up where it's not clear what is required for a correct answer (B4,'98), (B2,'95), not really (A1,'96), but this one merits discussion. [I wouldn't count problems that have yes/no answers and don't state explicitly that you have to justify which is correct — of course you must; this is the Putnam.] And — most irritatingly — you get things that are either standard, or known if you have just read the right textbook. (A5,'98) is not the only one, but it's all I could find tonight.
4. Functional equations. Every other Putnam has one of these. Frequently, they amount to solving a difference equation, of the Fibonacci type. The Fibonacci sequence itself shows up every once in a while. (This could be two separate projects.) (B4,'00), (B3,'63), (A5,'02), sort of, (B5,'01), (A6,'99), (A4,'98), (A6,'97), (B1,'96)

5. Floors and ceilings. As anyone who has participated in any math competition knows, the *floor* of a real number x , $\lfloor x \rfloor$ is the biggest integer $\leq x$; the *ceiling* $\lceil x \rceil$ is the smallest integer $\geq x$. They show up all the time in competition math (well, floors do). (B4,'98), (B4,'97), (A5,'96), (B6,'95), (A5,'89), etc. A variation is the nearest integer to a given number (B3,'01); or the distance to the nearest integer. (B1,'97)
6. Geometry/trigonometry. Unlike some other math competitions, the Putnam rarely has straight synthetic geometry problems. But they show up once in a while. More often, you see problems involving trigonometry. (B4,'00), (B4,'04), (A3,'03), (A2,'02), (A4,'01), (A5,'00), (B1,'99), (B5,'99), (A6,'98), (A5,'85), (A3,'86), (B5,'93) — a gorgeous extended riff on the cosine law, etc. I think there are several projects here.
7. Swindles. The following is excerpted from the “Official Dictionary of Loveys-Speak.”

Swindle (*n.*) An easy problem disguised as a difficult one. Something that takes two lines to do once you find the kick-yourself-in-the-arse-obvious trick that you missed.

Note that a swindle is not just an easy problem, particularly not a grind-it-out boring problem (A1,'91). (Of course, if you get an A1,'91, you *must* spend the 15 or 20 minutes needed to do it. I mean, it's oatmeal, but it's *good* for you.) A swindle is also not a problem that becomes routine if you catch the really clever trick involved. Those are classic Putnam problems; a swindle needs something that any idiot could notice in his sleep — it's just that you didn't. The classic swindle is (B1,'88), but see also (A4,'64), (A2,'02), (A5,'91), (A2,'04), (B1,'87), (B1,'92), (B4,'62).

I have some other ideas, but this will do as a start.