1. Is it possible to bake a triangular cake and cut it into 2009 congruent triangular pieces?

2. Consider the finite sequences of positive integers

$$S_1 = 1, \qquad S_2 = 1, 2, \qquad S_3 = 1, 1, 2, 3, \qquad S_4 = 1, 1, 1, 2, 1, 2, 3, 4$$

   and if $S_n = s_1, s_2, \ldots, s_m$, then

$$S_{n+1} = 1, 2, \ldots, s_1, 1, 2, \ldots, s_2, \ldots, 1, 2, \ldots, s_m, n.$$

   Prove that if $n > 1$, then the $k$th number in $S_n$ from the left is equal to 1 if and only if the $k$th number in $S_n$ from the right is not equal to 1.

3. Let $a$ and $b$ be positive integers such that $ab + 1$ divides $a^2 + b^2$. Show that

$$\frac{a^2 + b^2}{ab + 1}$$

   is the square of an integer.