1. One hundred coins of various denominations lie in a row on a table. Alex and Bree alternately take a coin from either end of the row. Alex goes first which means that Bree will take the last coin on the table. Show that Alex can guarantee to end up with at least as much money as Bree.

2. There are 15 left shoes and 15 right shoes jumbled up and placed in a row. Show that there must be 10 consecutive shoes consisting of 5 left shoes and 5 right shoes.

3. Prove the following interesting identity for every integer n greater than 1, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

$$\lfloor \sqrt{n} \rfloor + \lfloor \sqrt[3]{n} \rfloor + \cdots + \lfloor \sqrt[n]{n} \rfloor = \lfloor \log_2 n \rfloor + \lfloor \log_3 n \rfloor + \cdots + \lfloor \log_n n \rfloor$$

4. You are given three infinite lists of numbers: A, B, and C. List A contains all numbers of the form 10^k, where k is a positive integer, written in base 10. Lists B and C contain the same numbers written in base 2 and base 5, respectively. Prove that for every integer $n > 1$, there is exactly one number appearing in lists B or C which has exactly n digits.

A	B	C
10	1010	20
100	1100100	400
1000	1111101000	13000
⋮	⋮	⋮