

Here are some basic problems for you to try.

1. Consider the sequence of positive integers defined by  $a_0 = 0$ ,  $a_1 = 1$ , and for every positive integer  $n$ ,  $a_{n+1}$  is the smallest positive integer larger than  $a_n$  such that no three distinct terms in the set  $\{a_0, a_1, \dots, a_{n+1}\}$  form an arithmetic progression. What is the value of  $a_{2009}$ ?
2. How many functions  $f : \mathbb{N}_+ \rightarrow \mathbb{N}_+$  are there which satisfy the equation  $f(f(x)) = 3x$ ? What happens if we change the domain and codomain to  $\mathbb{Q}_+$ ? What happens if we change the domain and codomain to  $\mathbb{R}_+$ ?
3. A function  $f : \mathbb{N}_+ \rightarrow \mathbb{N}_+$  is defined by  $f(1) = 1$ ,  $f(3) = 3$ , and for every positive integer  $n$ ,
  - $f(2n) = n$ ;
  - $f(4n + 1) = 2f(2n + 1) - f(n)$ ; and
  - $f(4n + 3) = 3f(2n + 1) - 2f(n)$ .

Determine the number of positive integers less than or equal to 2009 for which  $f(n) = n$ .