1. For a fixed value of n, choose any subset of n integers from the set $\{1, 2, 3, \ldots, 2n\}$. Now arrange them in increasing order to obtain the sequence $a_1 < a_2 < \cdots < a_n$ and arrange the remaining numbers in decreasing order to obtain the sequence $b_1 > b_2 > \cdots > b_n$. Determine all possible values of the expression
$$|a_1 - b_1| + |a_2 - b_2| + \cdots + |a_n - b_n|.$$

2. A highway is being built between two cities which are 100 kilometres apart. In the first month, one kilometre of the highway is built. If X kilometres of the highway have been built by the start of a given month, then $\frac{1}{X^{100}}$ more kilometres of highway are built during that month. Will the highway construction ever be finished?

3. Below is part of a table with infinitely many rows and infinitely many columns. The entry in the top left corner is 0 and every other square is labelled with the smallest non-negative integer which does not appear in that row to the left of it or in that column above it.

0	1	2	3	4	5	
1	0	3	2	5	4	
2	3	0	1	6	7	
3	2	1	0	7	6	
4	5	6	7	0	1	
5	4	7	6	1	0	

   By completing the table, or otherwise, determine which number occurs in the 123rd column and the 456th row.

4. (a) A number of ants are distributed around a narrow circular path. At one particular instant, each of the ants chooses a particular direction and begins to walk along the path. The ants all walk at a constant speed and, when two ants meet, they both instantly change directions and continue walking at the same speed. Prove that at some later moment, each ant will be in its starting location.

   (b) There are n ants crawling along a thin stick one metre in length. Each ant crawls at one metre per minute and, when two ants meet, they both instantly change directions and continue walking at the same speed. Furthermore, when an ant meets an end of the stick, it instantly turns around and continues walking at the same speed.

      i. Prove that after two minutes, each ant will be in its starting location.

      ii. Under what conditions is each ant in its starting location after one minute?

   (c) Three ants find themselves on the hour, minute and second hands of an analog clock at precisely twelve o'clock noon. Whenever one hand overtakes another, the two corresponding ants instantly swap positions. Twelve hours later, each of the ants has travelled a whole number of revolutions around the clock. Which ant has travelled the most number of times around?