1. There are 25 stones sitting in a pile next to a blackboard. You are allowed to take a pile and divide it into two smaller piles of size a and b, but then you must write the number $a \times b$ on the blackboard. You continue to do this until you are left with 25 piles, each with one stone. What is the maximum possible sum of the numbers written on the blackboard?

2. Let X be a set consisting of one hundred integers whose sum is 1. What is the maximum possible number of subsets of X which have positive sum?

3. Consider the function
$$f(x) = \frac{1}{\lfloor x \rfloor + 1 - \{x\}}$$
and the sequence defined by $a_0 = 0$ and $a_{n+1} = f(a_n)$ for all non-negative integers n. Prove that the sequence $a_0, a_1, a_2, \ldots$ contains every positive rational number exactly once.

4. Let E denote the set of non-negative integers less than 10^{2009} whose digits sum to an even number. Let O denote the set of non-negative integers less than 10^{2009} whose digits sum to an odd number. Take the numbers in E, raise them all to the one hundredth power, and then let X be the sum of these numbers. Take the numbers in O, raise them all to the one hundredth power, and then let Y be the sum of these numbers. Prove that $X = Y$.