FAMILY NAME ______________________________

GIVEN NAME(S) ______________________________

STUDENT NUMBER ______________________________

DATE	any date
START TIME	any time
FINISH TIME	one hour after the start time
EXAMINER	Norman Do

Instructions

1. Fill in your name and student number clearly in the space provided above.
2. Do not remove any pages from this booklet — all of your writing, even rough work, must be handed in.
3. Calculators, books and notes are not permitted.
4. You are encouraged to spend the first few minutes reading through the problems. The problems are not ordered in terms of difficulty and the number of marks available does not indicate the difficulty of the problem.
5. Please write your rough work and solution in the space provided on the page where the question is printed. If you use up all of that space, then you may write on the back of the page or on one of the pages provided at the back of the booklet. If you use up all of that space as well, then please raise your hand and ask for another sheet of paper.
6. The test booklet should consist of this cover, pages 1 through 6 which contain the questions, and pages 7 through 10 which are blank. Please inform me if your booklet is defective.

1. For each of the following statements, indicate whether it is true or false. If you indicate that a statement is true, then you must explain why it is true and if you indicate that a statement is false, then you must explain why it is false.

   (a) The centroid of a triangle always lies inside the triangle. (Recall that the centroid of a triangle is where the medians meet.)

   (b) Hyperbolic geometry obeys all of Euclid's axioms.

   (c) The composition of a reflection followed by a rotation can only be a reflection.

   (d) There exists a graph with seven vertices of degrees 0, 1, 1, 2, 2, 3, 3.

   (e) It is possible to build a house with exactly six rooms such that each room has exactly five doors and there are exactly three doors which lead outside.

   [$2 + 2 + 2 + 2 + 2 = 10$ *marks*]

2. (a) Prove that the diameter of a circle subtends an angle of $90°$. In other words, if AB is the diameter of a circle and C is a point on the circle, then $\angle ACB = 90°$. (You may only use basic facts about isosceles triangles, the sum of the angles in a triangle, and so on.)

   (b) Start with a cyclic quadrilateral $ABCD$.

   - Let W be the point on BD such that AW is perpendicular to BD.
   - Let X be the point on AC such that BX is perpendicular to AC.
   - Let Y be the point on BD such that CY is perpendicular to BD.
   - Let Z be the point on AC such that DZ is perpendicular to AC.

   Prove that the quadrilateral $WXYZ$ is cyclic.

[3 + 3 = *6 marks*]

3. (a) Fill in each entry of the middle column with the word *direct* or *opposite* and each entry of the right column with the word *yes* or *no*.

ISOMETRY	DIRECT OR OPPOSITE	FIXED POINTS
translation		
rotation		
reflection		
glide reflection		

(b) Suppose that $ABCD$ is a rectangle with the vertices labelled counterclockwise and such that $BC = 2AB$. Let

- G_{CD} denote the glide reflection consisting of a reflection in the line CD followed by a translation which takes C to D;
- T_{DB} denote the translation which takes D to B;
- R_B denote a counterclockwise rotation by 90° about B; and
- M_{AB} denote the reflection in the line AB.

Identify the composition $M_{AB} \circ R_B \circ T_{DB} \circ G_{CD}$.

[2 + 4 = *6 marks*]

4. (a) The following is a *groupoku* puzzle — a Cayley table for the group G, where some of the entries are missing. Use the properties which you know about Cayley tables to fill in all of the missing entries. Give full reasoning only for the first entry of the table that you manage to fill in.

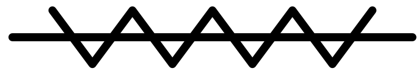
$\cdot$	a	b	c	d	e	f	g	h
a	g	$*$	e	f	h	$*$	d	$*$
b	h	f	$*$	c	$*$	b	$*$	a
c	$*$	a	f	$*$	d	c	h	g
d	f	$*$	b	g	c	d	a	e
e	c	g	$*$	h	$*$	$*$	b	d
f	a	b	c	d	e	f	$*$	h
g	d	e	$*$	$*$	b	g	f	$*$
h	$*$	d	g	b	a	h	c	$*$

(b) In the lectures, we have seen two groups with eight elements — the cyclic group C_8 and the dihedral group D_4. Prove that G is isomorphic to one of these groups by writing down an explicit isomorphism. (You do not have to prove that it is an isomorphism.)

(c) Prove that the cyclic group C_8 and the dihedral group D_4 are not isomorphic.

[2 + 2 + 2 = *6 marks*]

5. (a) Write down the $HVRG$ symbol for the frieze pattern below.



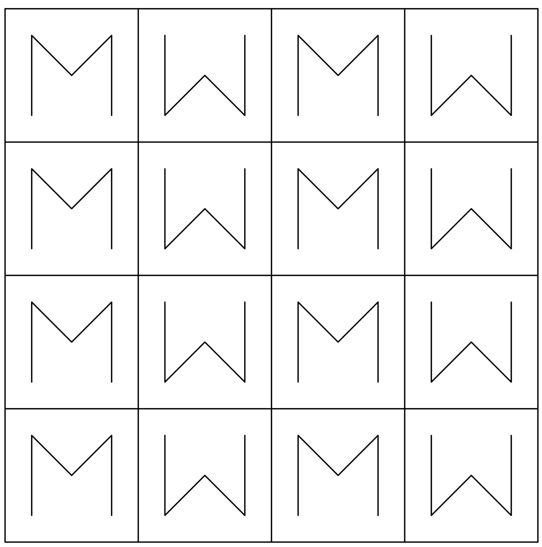
(b) Write down the $HVRG$ symbol for the frieze pattern below.

HXHXHXHXHXHXHXHX

(c) Write down the RMG symbol for the wallpaper pattern pictured below left. Mark clearly on the diagram a point which is the centre of a rotational symmetry of order R, a point where M mirrors meet, and a point where G proper glide axes meet.

(d) Write down the RMG symbol for the wallpaper pattern pictured below right.

(e) Prove that if a symmetry group of a frieze pattern contains a reflection in a vertical mirror and a rotation by 180°, then it must also contain a glide reflection.



[*1 + 1 + 2 + 1 + 2= 7 marks*]

6. Suppose that you have a polyhedron with V vertices, E edges and F faces. You are told that each face is a triangle or a quadrilateral and that four faces meet at every vertex. Furthermore, every triangular face shares an edge with three quadrilateral faces, while every quadrilateral face shares an edge with four triangular faces.
   (a) Use the handshaking lemma applied to the polyhedron to prove that $4V = 2E$.
   (b) Let T be the number of triangular faces and Q be the number of quadrilateral faces of the polyhedron. Use the handshaking lemma applied to the dual of the polyhedron to prove that $E = \frac{3}{2}T + 2Q$.
   (c) Prove the equation $3T = 4Q$.
   (d) Use these equations — along with anything else you know about the polyhedron — to deduce the values of T and Q.

[*1 + 1 + 1 + 2 = 5 marks*]

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