HOW, WHAT, AND WHY I THINK ABOUT MATHS

Ivanhoe Grammar School

25 November 2014

Norm Do

Monash University

PART 1

in which I tell you about the only topic on which I am the world's foremost expert

ME

PART 2

in which I tell you what maths is about and how school maths is only the tip of the iceberg

WHAT IS MATHS?

What is maths about?

Wikipedia. Maths is about

- studying quantity, structure, space, and change;
- seeking out patterns and formulating new conjectures;
- resolving conjectures by mathematical proof.

Norm. Maths is about

- solving concrete problems;
- playing around with ideas;
- analysing problems beyond their solution.

Maths is more than what you see at school

School maths is like the vocabulary, spelling and grammar of learning English — it allows you to do useful things (like writing letters) and creative things (like writing stories).

The world of maths is limitless — more than 75,000 articles containing new mathematical results appear each year.

At Monash University, you can study subjects on: calculus, real analysis, complex analysis, linear algebra, mathematical modelling, multivariable calculus, discrete mathematics, probability, statistics, partial differential equations, computational mathematics, ordinary differential equations, differential geometry, group theory, number theory, ring theory, field theory, Galois theory, functional analysis, time series, random processes, financial mathematics, fluid dynamics, stochastic processes, graph theory, operations

PART 3

in which I tell you about some maths that you may or may not know and you might have to use your brain

MATHS IS AMAZING

Beautiful solutions

The following boring problem was given to the mathematician Gauss when he was in primary school as punishment.

$$1 + 2 + 3 + \cdots + 98 + 99 + 100 = ???$$

Boring solution.

$$1 + 2 = 3 \qquad 3 + 3 = 6 \qquad 6 + 4 = 10$$

$$10 + 5 = 15 \qquad 15 + 6 = 21 \qquad 21 + 7 = 28$$

$$28 + 8 = 36 \qquad 36 + 9 = 45 \qquad \cdots$$

Maths can be beautiful

Beautiful solution.

| 1	+	2	+	3	+	$\cdots$	+	49	+	50
100	+	99	+	98	+	$\cdots$	+	52	+	51
101	+	101	+	101	+	$\cdots$	+	101	+	101

50 pairs

So the answer is $50 \times 101 = 5050$.

- It's easy to see that the solution is correct.
- The idea behind the solution applies to other problems.
- Look for beautiful solutions — even to boring problems.

Dominoes on a chessboard

- Can you tile an 8 × 8 square with 2 × 1 dominoes?
- Can you tile the square if one corner is removed?
- Can you tile the square if opposite corners are removed?
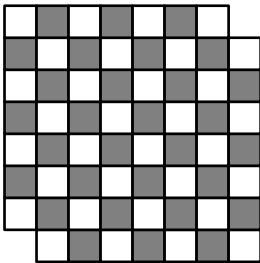
Dominoes occupy one square of each colour — but the mutilated chessboard has fewer black squares than white.

Fibonacci ratios

The next number in the Fibonacci sequence is the sum of the previous two numbers.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, . . .

Let's take consecutive Fibonacci numbers and divide them.

$$1 \div 1 = 1$$
$$2 \div 1 = 2$$
$$3 \div 2 = 1.5$$
$$\vdots$$
$$377 \div 233 = 1.61802575\cdots$$
$$610 \div 377 = 1.61803713\cdots$$
$$987 \div 610 = 1.61803278\cdots$$

Fibonacci ratios

Now let's take some larger examples of Fibonacci numbers.

$$14930352 \div 9227465 = 1.61803398\cdots$$
$$24157817 \div 14930352 = 1.61803398\cdots$$
$$39088169 \div 24157817 = 1.61803398\cdots$$

Why do these numbers get closer and closer to

$$1.61803398\cdots$$

and what exactly is this mysterious number?

And now for something different. . .

Start with the number zero.

$$0$$

Add 1 to your number and then take the square root.

$$1$$

Add 1 to your number and then take the square root.

$$1.41421356\cdots$$

Add 1 to your number and then take the square root.

$$1.55377397\cdots$$

Add 1 to your number and then take the square root.

$$1.59805318\cdots$$

And now for something different. . .

You get the idea. . .

$1.61184775\cdots$	$1.61803232\cdots$
$1.61612120\cdots$	$1.61803347\cdots$
$1.61744279\cdots$	$1.61803382\cdots$
$1.61785129\cdots$	$1.61803393\cdots$
$1.61797753\cdots$	$1.61803397\cdots$
$1.61801654\cdots$	$1.61803398\cdots$
$1.61802859\cdots$	$1.61803398\cdots$

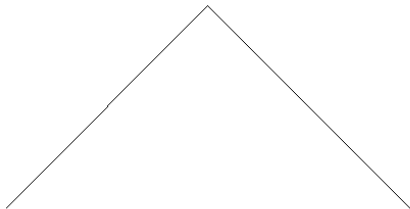SURPRISE! The mysterious number strikes again. . . but why?!

A familiar sequence

Roll a dice many times... what sequence do you get?

Make a new sequence of numbers using the following rule:

- if you see a number for the first time, write a 1;
- if you see a number for the second time, write a 2;
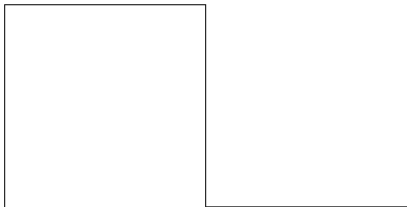- if you see a number for the third time, write a 3; etc.

(Un)folding the dragon

Fold a long strip of paper in half a few times. Then unfold it and make all the folds 90°.
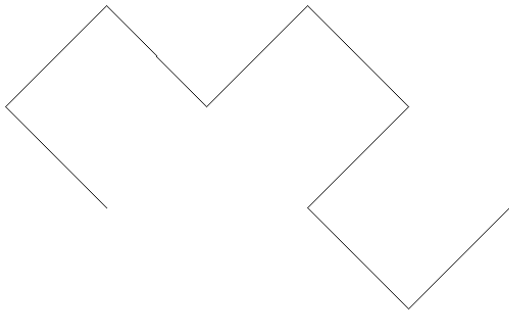
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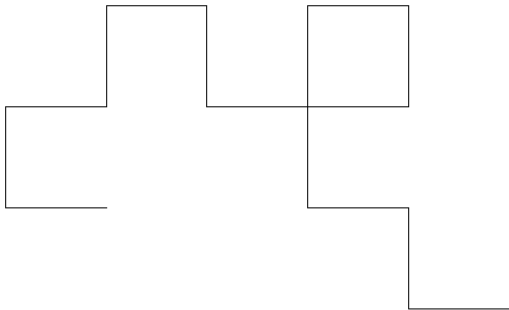
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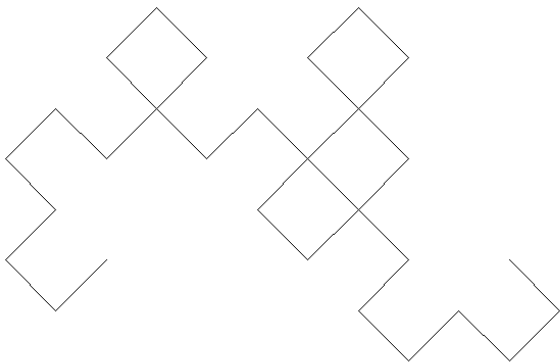
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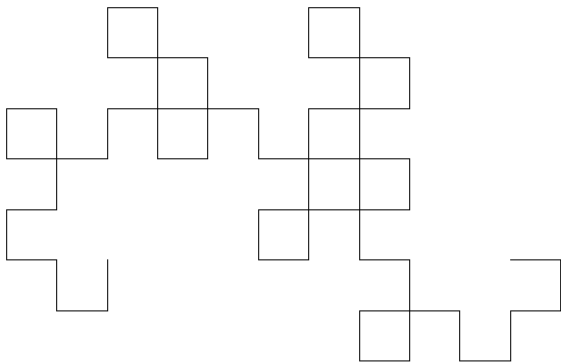
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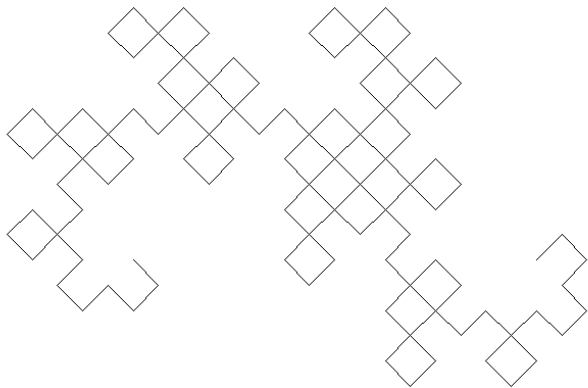
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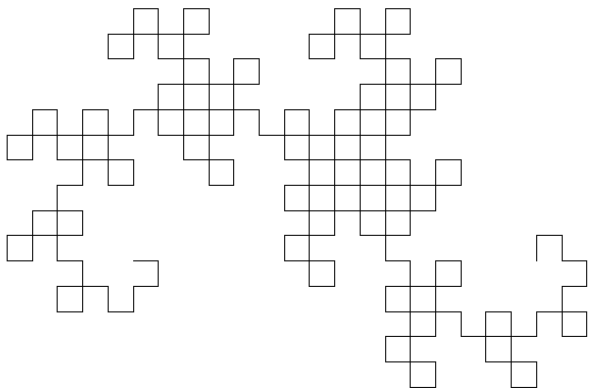
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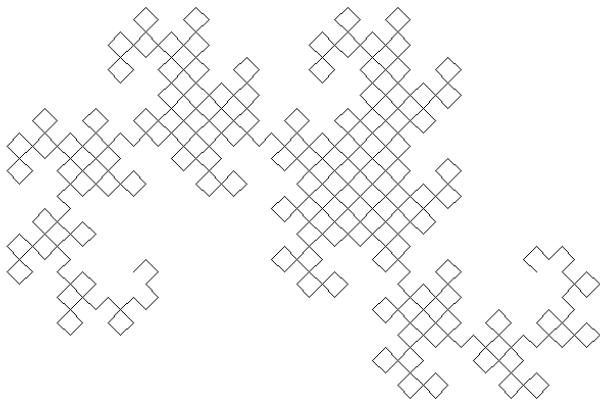
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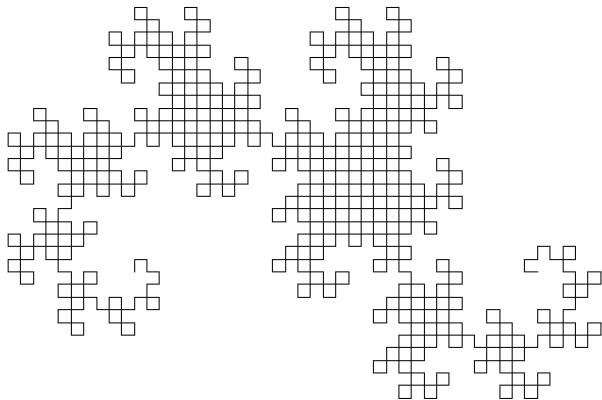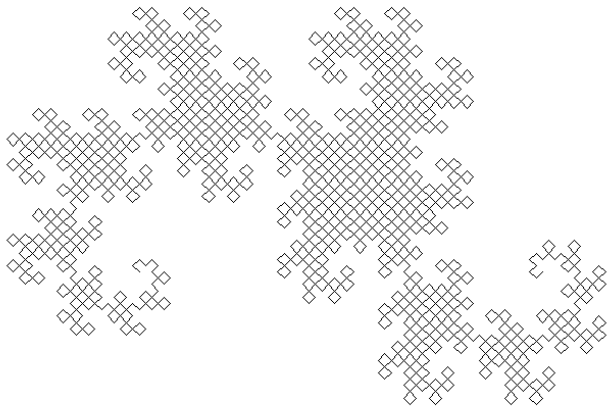
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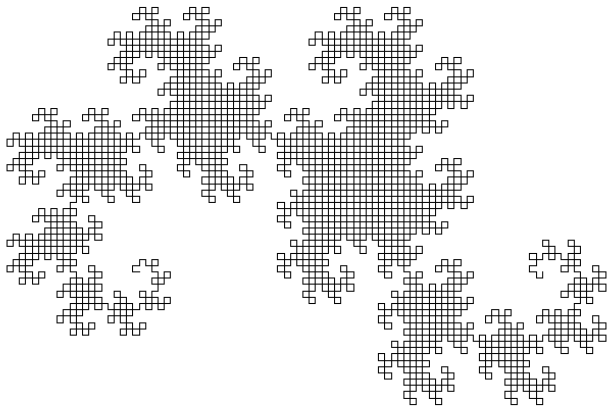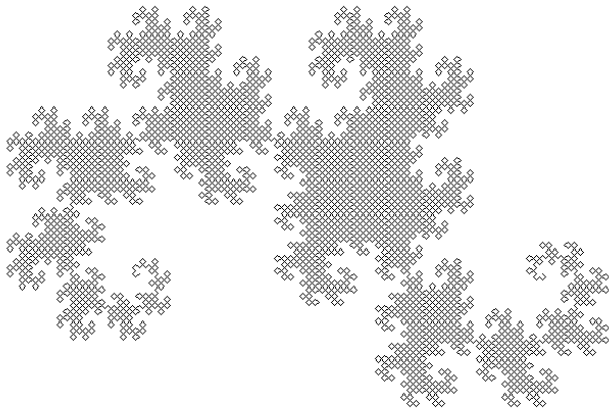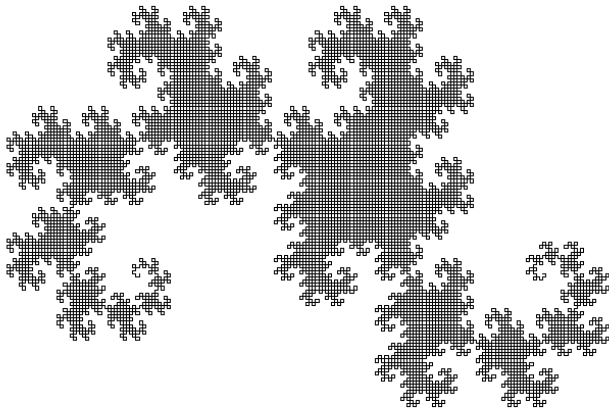
PART 4

in which I tell you about the right way to learn maths to make it easier and more fun

HOW I LEARN MATHS

Maths is difficult. . .

- Andrew Wiles proved Fermat's Last Theorem in 1994 after working on the problem for eight years.

- I have spent the last six months working on a maths problem. . . and haven't solved it yet.

- If maths was easy, it would be boring!

. . . but YOU can do it!

- There is no such thing as a mathematical genius.

- You just need the right attitude, patience, and a passion for challenges.

- Don't be afraid to be wrong!

How to learn ~~maths~~ Chinese

人 man

大 big

口 mouth

日 say

言 word

木 tree

本 beginning

末 end

困 distress

家 home

信 honour

好 good

安 peace

�… quarrel

How to learn maths

Ask why. . . and ask it often.

Things are easy to remember when they make sense — so remember the idea, not just the end result.

Maths was developed by humans for a reason — if you know the reason, then the maths will make more sense.

PART 5

in which I tell you why more people should study more maths and that includes you

MATHS IS USEFUL

Things old people said

The Book of Nature is written in the language of maths.
Galileo Galilei, 1623

*The Black and Scholes formula... is used hundreds of millions of times per day.*
J. F. Price, 1997

The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve.
Eugene Wigner, 1960

I have never done anything useful. No discovery of mine has made... the least difference to the amenity of the world.
G. H. Hardy, 1940

The maths of the internet

- The Google search engine is based on a simple mathematical idea called PageRank.
- Think of every webpage represented by a dot and every link represented by an arrow.
- A page that is linked to by many pages with high PageRank receives a high PageRank itself.
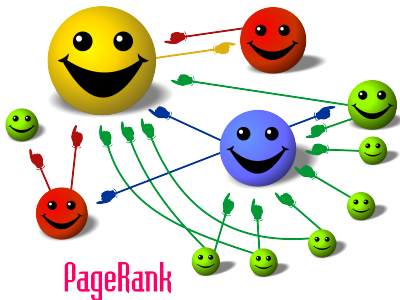
Maths for grown-ups

"The sexy job in the next ten years will be statisticians."
Hal Varian (Google's Chief Economist)

CareerCast.com best jobs of 2014: Ranked by environment,
income, employment outlook, physical demands and stress

200. Lumberjack		4. Actuary
199. Newspaper Reporter		3. Statistician
198. Enlisted Military		2. University Professor
197. Taxi Driver		1. Mathematician

Where you can end up

Maths is a skill, rather than a profession.
People who study maths end up in all sorts of jobs.

- bioinformatician
- biostatistician
- remuneration analyst
- survey programmer
- software developer
- data scientist
- data analyst
- econometrician
- energy forecaster
- credit risk modeller
- financial modeller

- health economics analyst
- statistical analyst
- actuarial analyst
- marketing data analyst
- advertising analyst
- meteorologist
- simulation consultant
- behavioural scientist
- quantitative researcher
- maths teacher
- transport economist

PART 6

in which I tell you about a topic about which I know almost
nothing and hopefully don't embarrass myself

EPISTEMOLOGICAL MUSINGS

Questions without answers

- Is mathematics discovered or created?
- Is mathematics the closest we can get to truth?
- Do we really need pure mathematics?
- To what extent can we rely on mathematical models?
- Can dogs do mathematics?