Linear-Quadratic Model Predictive Control for Urban Traffic Networks

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Motivation

- Growing demand for transportation.
- Increasing congestion in urban area.
- The need for a real time large scale road traffic control system.

Aims and Outcomes

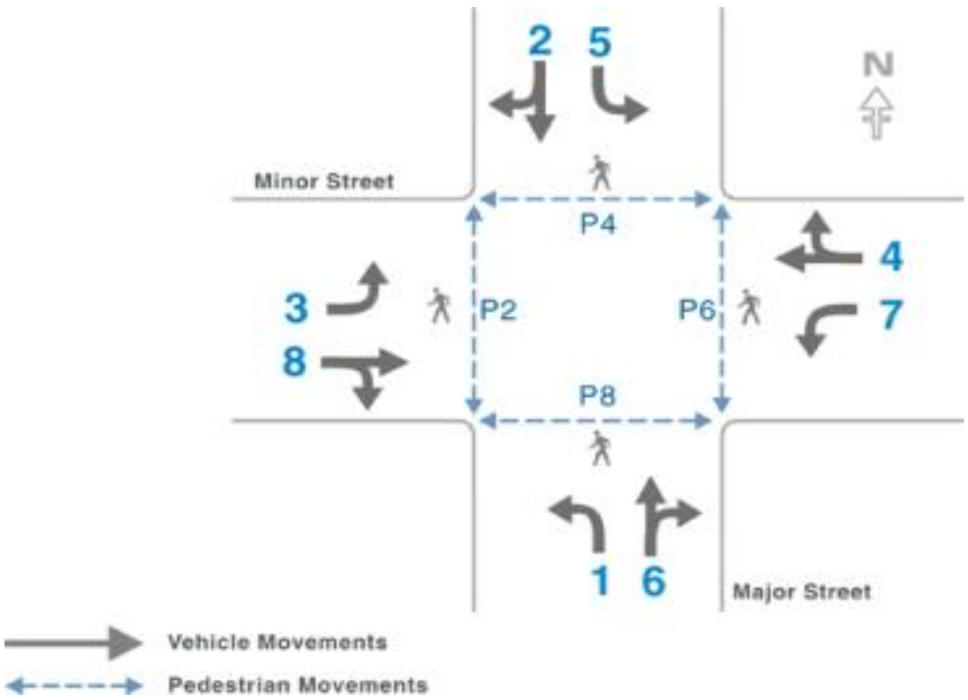
- To develop a better centralized control
- New framework to coordinate the green time split and turning fraction to maximize throughput in urban networks.
  - We explicitly consider travel time and spill-back.
  - The state prediction is linear so the model is suitable for large networks in real time control.
  - Show via simulation that our model is superior in reducing congestion in compare with other control schemes in the studied scenarios.

Outline

- Background and literature review.
- Proposed MPC framework.
  - Predictive model.
  - Optimization problem.
- Simulation and results.

Review of Intersection Control

- Traffic signals at intersections is the main method of control in road traffic networks.
  - ☐ Have big influence on overall throughput.
- Control variables at intersections
  - ☐ Phase combination.
  - ☐ Cycle time.
  - ☐ Split.
  - ☐ Offset.



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Control strategy classification

	Fixed-time	Traffic-responsive
Coordinated	**Fixed-time coordinated** • MAXBAND(John D. C. Little 1966). • TRANSYT(D.I. Robertson 1969).	**Coordinated traffic responsive** • SCOOT(Bretherson 1982) and SCAT(Lowrie 1982). • TUC(C. Diakaki 1999). • Rolling-horizon optimization.
Isolated	**Isolated fixed-time** • SIGSET(Allsop 1971) and SIGCAP(Allsop1976) systems. • G. Importa and G.E. Cantarella(1984).	**Isolated traffic-responsive** • A.J. Miller(1963).

Control strategy classification

Coordinated

Isolated

- SCOOT and SCAT systems.
  - Control splits, offset and cycle time.
  - Use real time measurement to evaluate the effects of changes.
- TUC system.
  - Control splits.
  - Store and forward model: Assume vertical queue, continuous flow.
  - Linear Quadratic Regulator problem.
- Rolling-horizon optimization.

Coordinated traffic responsive

- SCOOT(Bretherson 1982) and SCAT(Lowrie 1982).
- TUC(C. Diakaki 1999).
- Rolling-horizon optimization.

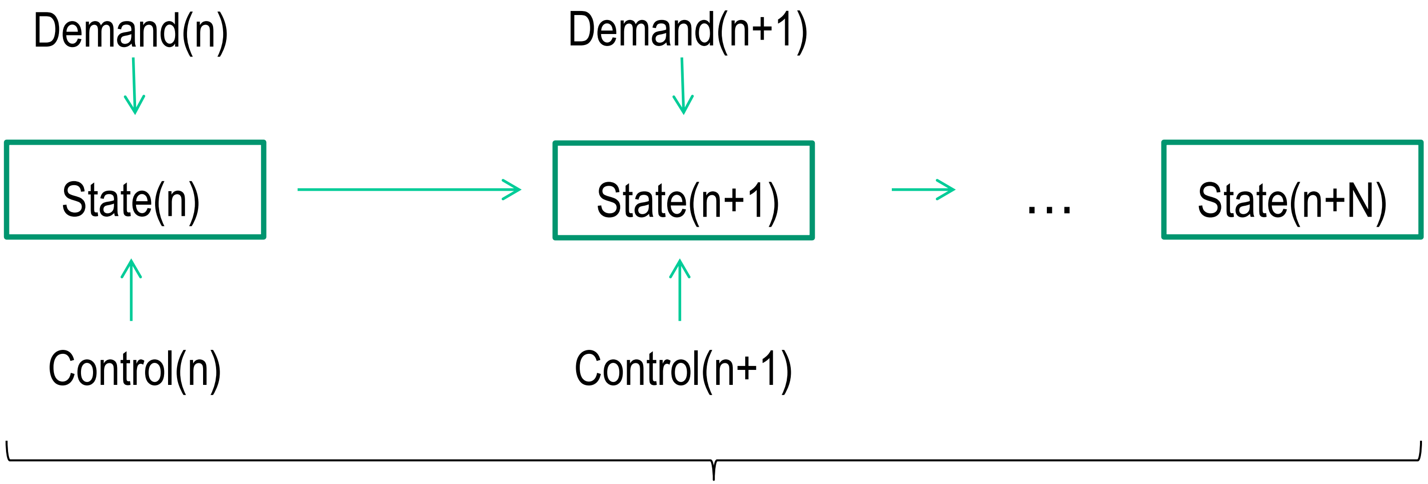
Isolated traffic-responsive

- A.J. Miller(1963).

Fixed-time

Traffic-responsive

Rolling horizon optimization



Optimization over finite horizon N

- OPAC(Gartner (1983)), PRODYN(Henry (1983)), RHODES(Mirchandani (2001)) and CRONOS(Boillot(1992)).

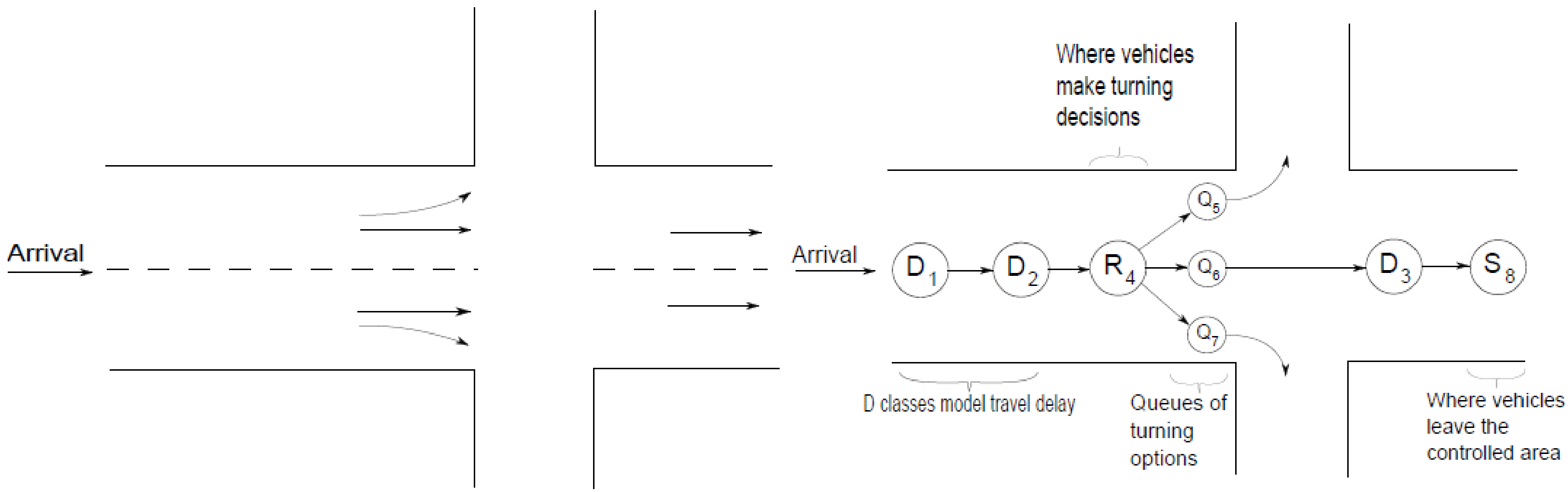
N. H. Gartner, “OPAC: A demand-responsive strategy for traffic signal control,” U.S. Dept. Transp., Washington, DC, Transp. Res. Record 906, 1983.

J.-L. Farges, J.-J. Henry, and J. Tufal, “The PRODYN real-time traffic algorithm,” in Proc. 4th IFAC Symp. Transportation Systems, 1983, pp. 307–312.

S. Sen and L. Head, “Controlled optimization of phases at an intersection,” Transp. Sci., vol. 31, pp. 5–17, 1997.

F. Boillot, J. M. Blosseville, J. B. Lesort, V. Motyka, M. Papageorgiou, and S. Sellam, “Optimal signal control of urban traffic networks,” in Proc. 6th IEE Int. Conf. Road Traffic Monitoring and Control, 1992, pp. 75–79.

Notations



(a) A real network segment

(b) A model network segment

- Time evolves in discrete steps n=0,1,2,…
- Continuous flows of traffic.
- Network state maintains continuous vehicle count in *delay, route, queue* and *sink* classes.

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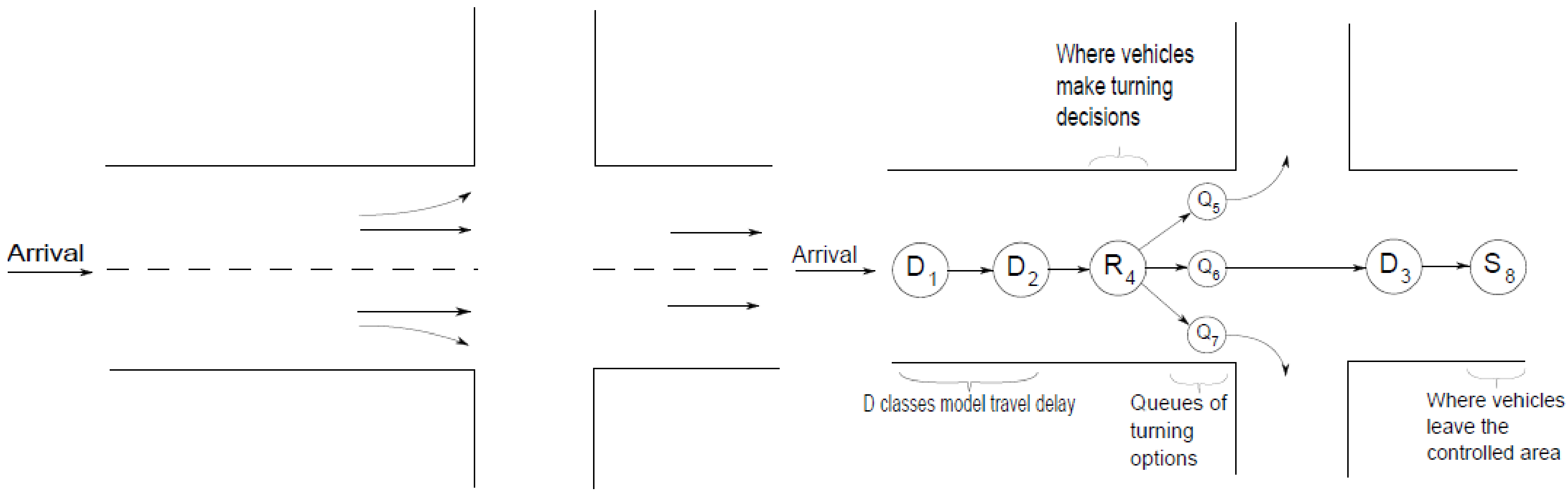
Notations

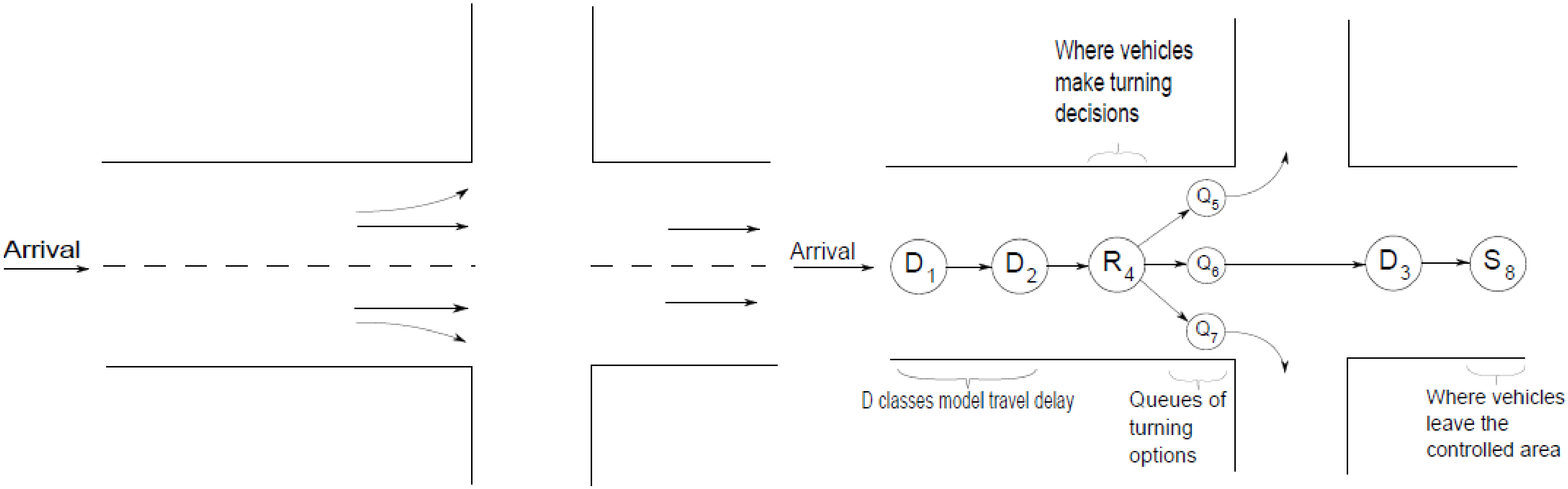


(a) A real network segment

(b) A model network segment

- Vehicles flowing out of a class move to downstream classes though links. We denote the links j=1,…,L.
- Each link is associated with a flow rate f_j, which is the maximum number of vehicles that can go through that link in an unit of time.

Notations



(a) A real network segment

(b) A model network segment

- Control variables: $u_j(n)$ is the fraction of time unit during which link j is active.

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State dynamics

$$x_k(n+1) = x_k(n) + a_k(n) + \sum_{\{j:d_j=k\}} u_j(n) f_j - \sum_{\{j:s_j=k\}} u_j(n) f_j, \quad k \in \mathcal{K}_{D,R,Q,S}\,.$$

- $x_k(n)$ is the queue length of class k at time n
- $a_k(n)$ is exogenous arrival
- $\sum_{\{j:d_j=k\}} u_j(n) f_j$ is the number of vehicles arriving from other classes.
- $\sum_{\{j:s_j=k\}} u_j(n) f_j$ is the number of vehicles leaving class k.

Constraints

- Control constraints:

$$0 \le u_j(n) \le 1, \quad 1 \le j \le J.$$

- Traffic light cycle constraints:

$$u_{L+1+(i-1)*2}(n) + u_{L+1+(i-1)*2+1}(n) \le 1, \quad i \in \mathcal{M}.$$

- Green duration constraints:

$$u_j(n) \le u_{L+1+(i-1)*2}(n)$$

- Flow conflict constraints:

$$\sum_{j \in C^p_{\mathcal{WE}}(i)} u_j(n) \le u_{L+1+(i-1)*2}(n)$$

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Constraints

- Non-negative queue constraints:

$$\sum_{\{j:s_j=k\}} u_j(n) f_j \leq x_k(n), \quad k \in \mathcal{K}_{D,R,Q}.$$

- Capacity constraints:

$$x_k(n+1) \leq c_k,$$

$$x_k(n) + a_k(n) + \sum_{\{j:d_j=k\}} u_j(n) f_j - \sum_{\{j:s_j=k\}} u_j(n) f_j \leq c_k.$$

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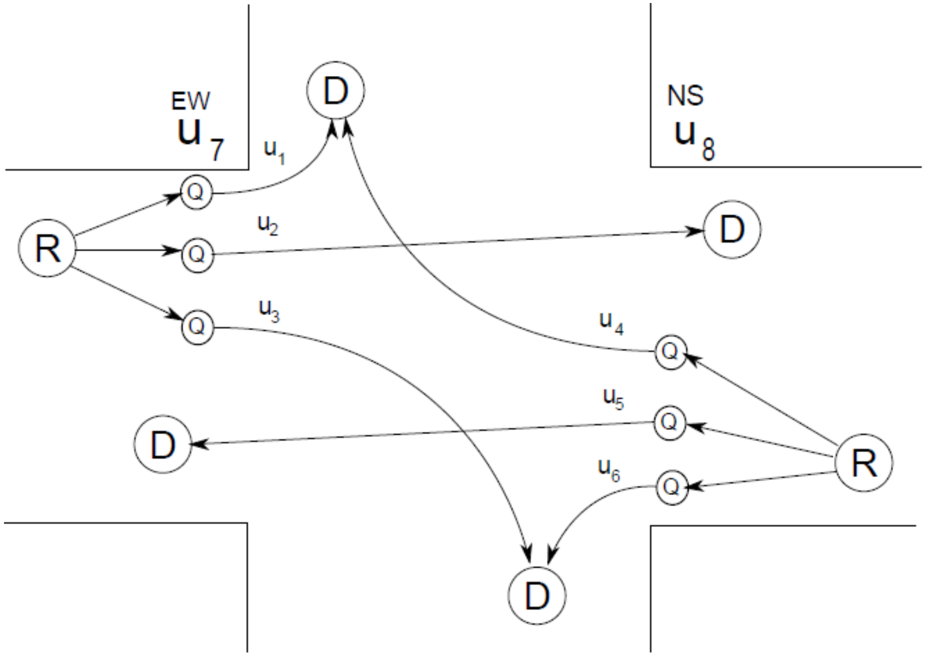
Constraints illustration



- Control constraints:

$$0 \le u_j(n) \le 1, \quad 1 \le j \le J.$$

- Traffic light cycle constraints:

$$u_7 + u_8 \le 1.$$

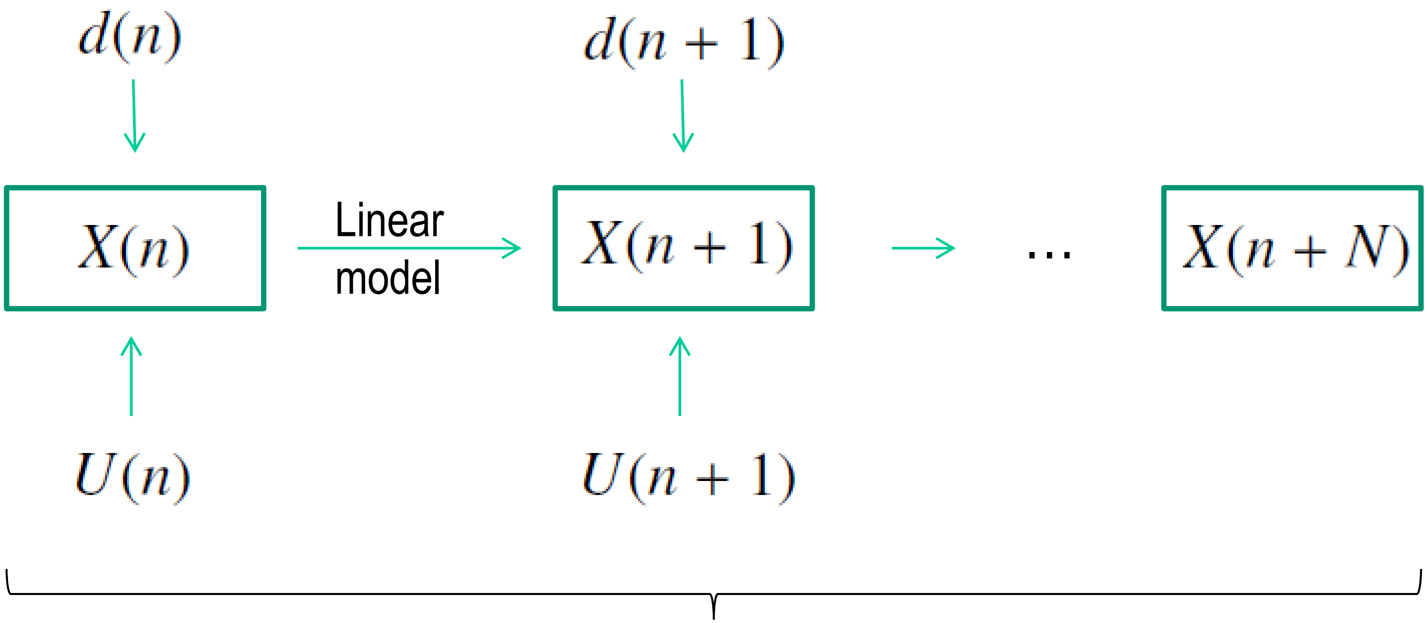
- Green duration constraints:

$$u_j \le u_7, j = 1, \dots, 7.$$

- Flow conflict constraints:

$$u_2 + u_4 \le u_7, u_5 + u + 3 \le u_7$$

The MPC framework



Optimization over finite horizon N

Quadratic Programming

$$\min \sum_{i=n}^{n+N-1} \hat{X}_{D,R,Q}(i+1)' Q \hat{X}_{D,R,Q}(i+1) + RU(i)$$

s.t.

$$F_x \hat{X}_{D,R,Q}(i) + F_u U(i) \leq g(i), \quad i = n, \ldots, n+N-1.$$

- Q is all **1** matrix, positive semi-definite. We minimize the quadratic cost which is square of sum of all queues over horizon N.
- Holding back problem: vehicles are held back even when there is available space in the downstream queues.
- The linear cost $RU(i)$ gives small incentive for the controller to move vehicle forward.

Matrix representation of the QP

$$\min_{\underline{\mathrm{U}}} \quad \underline{\mathrm{U}}'\underline{\mathrm{B}}'\underline{\mathrm{Q}}\,\underline{\mathrm{BU}} + \left(2(X_0'\underline{\mathrm{A}}' + \underline{\mathrm{d}}'\underline{\mathrm{A}}_1')\underline{\mathrm{Q}}\,\underline{\mathrm{B}} + \underline{\mathrm{R}}\right)\underline{\mathrm{U}} + (X_0'\underline{\mathrm{A}}' + \underline{\mathrm{d}}'\underline{\mathrm{A}}_1')\,\underline{\mathrm{Q}}\,(\underline{\mathrm{A}}X_0 + \underline{\mathrm{A}}_1\underline{\mathrm{d}})$$

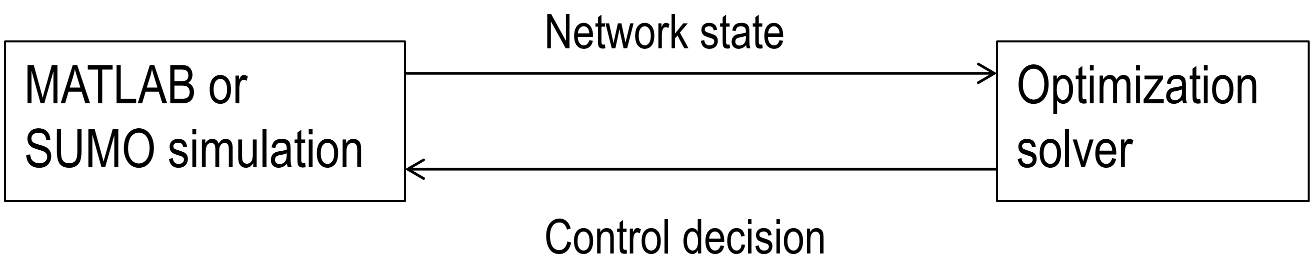
s.t.

$$\begin{bmatrix} F_u\underline{\mathrm{S}}_u^0 \\ F_u\underline{\mathrm{S}}_u^1 + F_x\underline{\mathrm{S}}_x^1\underline{\mathrm{B}} \\ \vdots \\ F_u\underline{\mathrm{S}}_u^{N-1} + F_x\underline{\mathrm{S}}_x^{N-1}\underline{\mathrm{B}} \end{bmatrix} \underline{\mathrm{U}} \leq \begin{bmatrix} g(0) - F_xX_0 \\ g(1) - F_x\underline{\mathrm{S}}_x^1(\underline{\mathrm{A}}X_0 + \underline{\mathrm{A}}_1\,\underline{\mathrm{d}}) \\ \vdots \\ g(N-1) - F_x\underline{\mathrm{S}}_x^{N-1}(\underline{\mathrm{A}}X_0 + \underline{\mathrm{A}}_1\,\underline{\mathrm{d}}) \end{bmatrix}$$

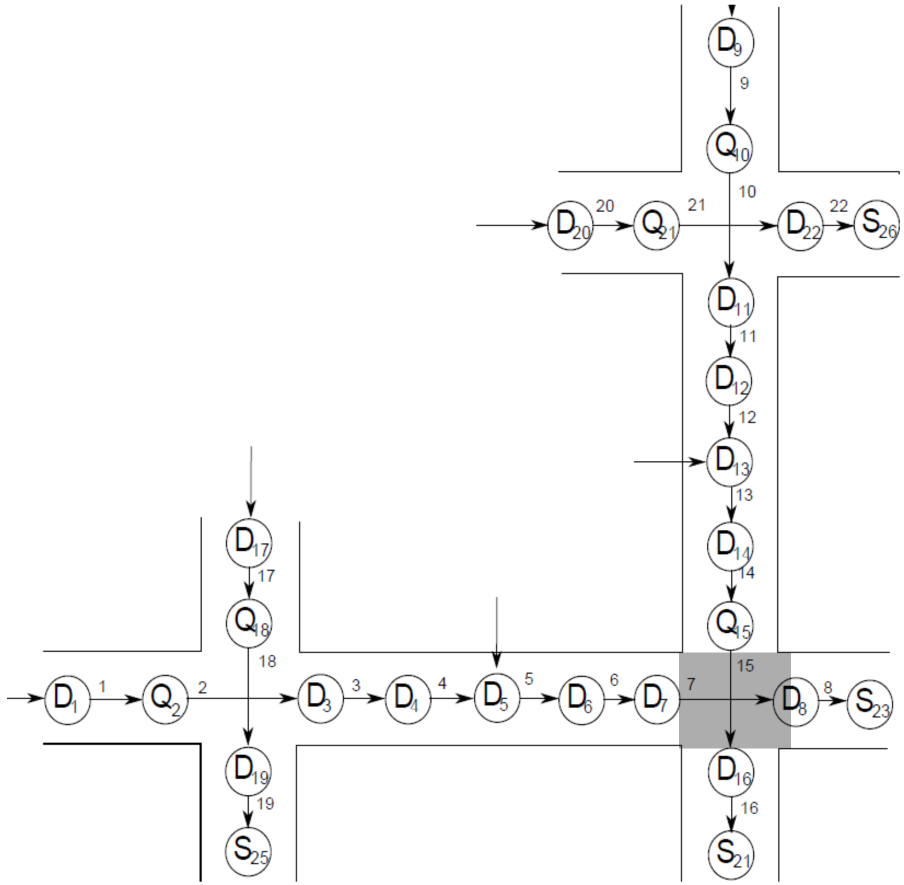
- We prove the above QP is convex but not strictly convex. Thus, there are multiple optimal solutions.

Simulation and results

- We conduct simulations to obtain the performance of our MPC framework.
  - Significantly reduce congestion.



Scenario 1



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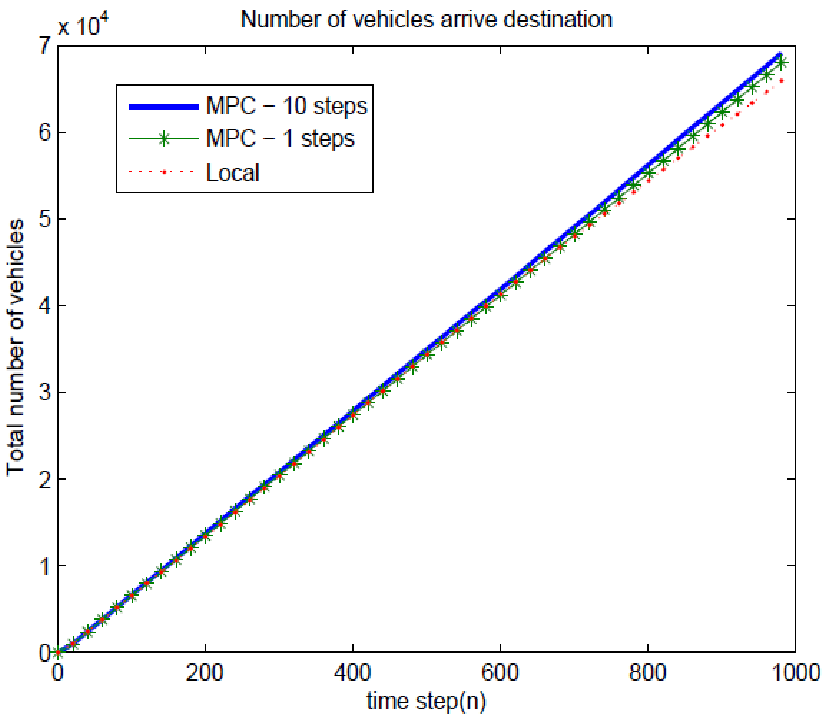
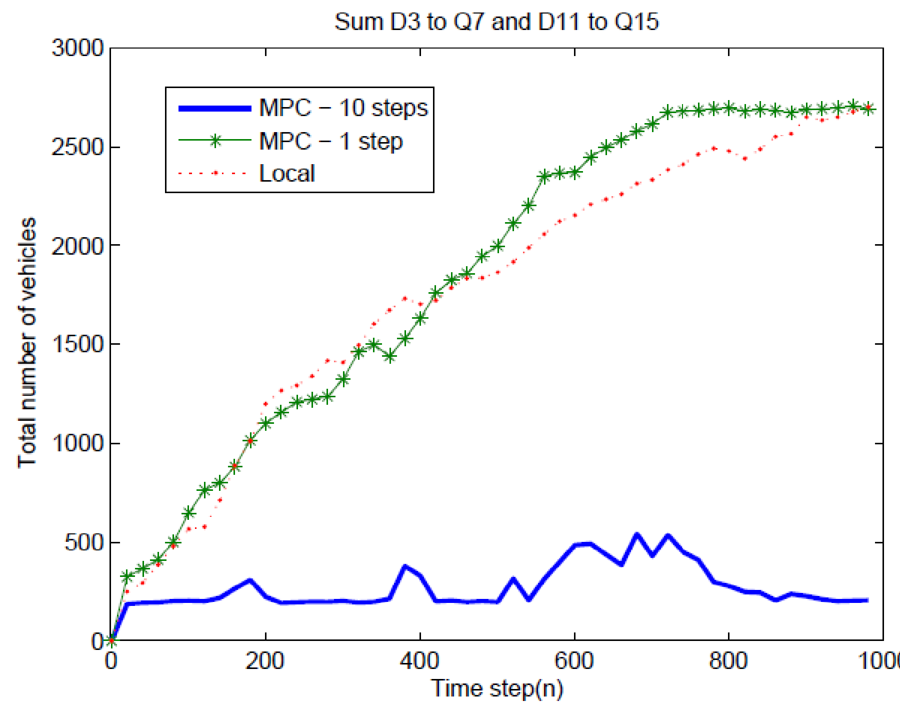
Results for scenario 1



(a) Total vehicles in the network



(b) Queue length at bottleneck intersection

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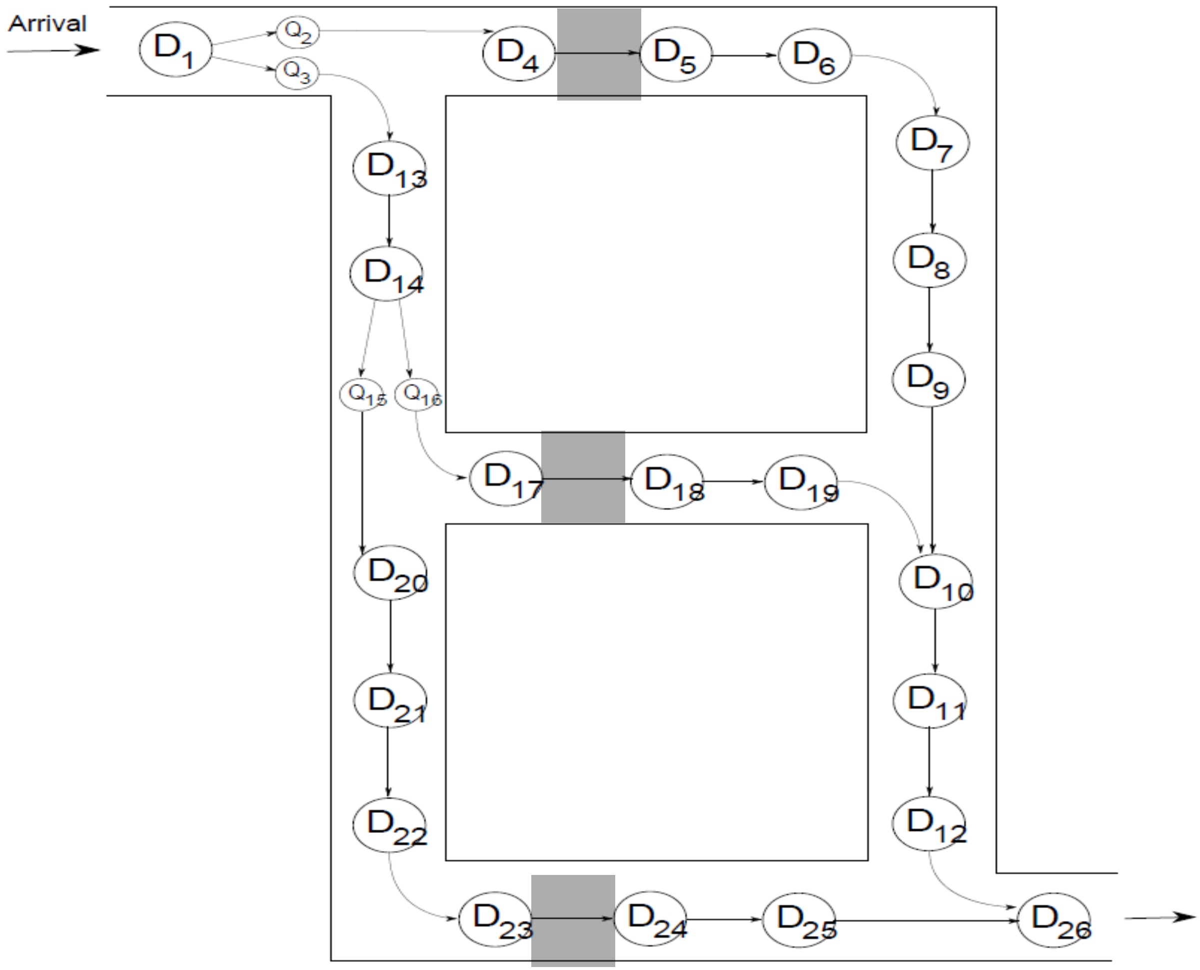
Scenario 2



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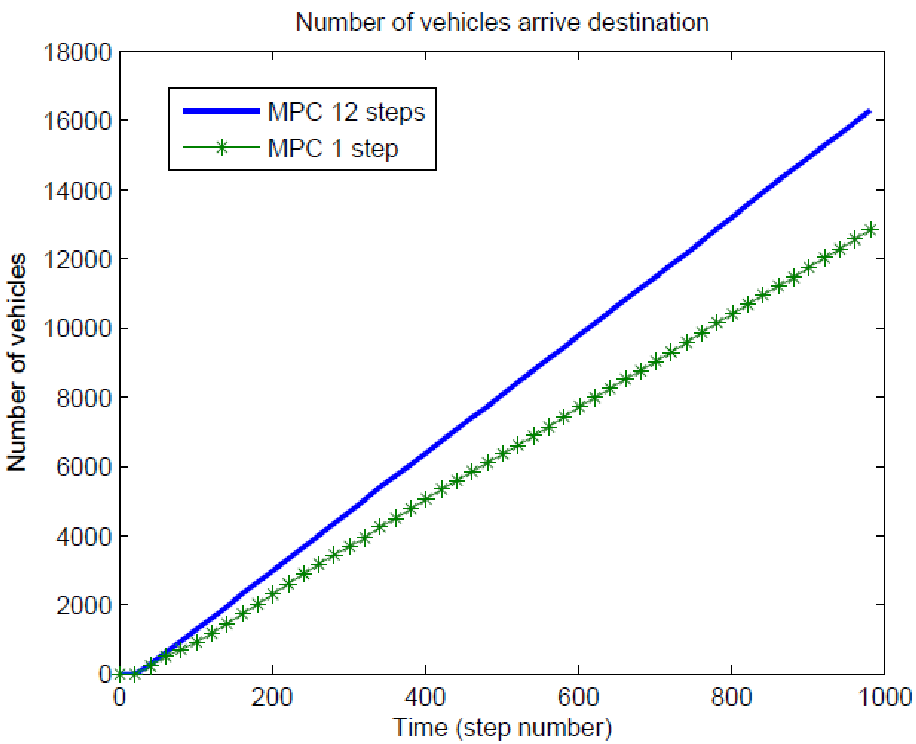
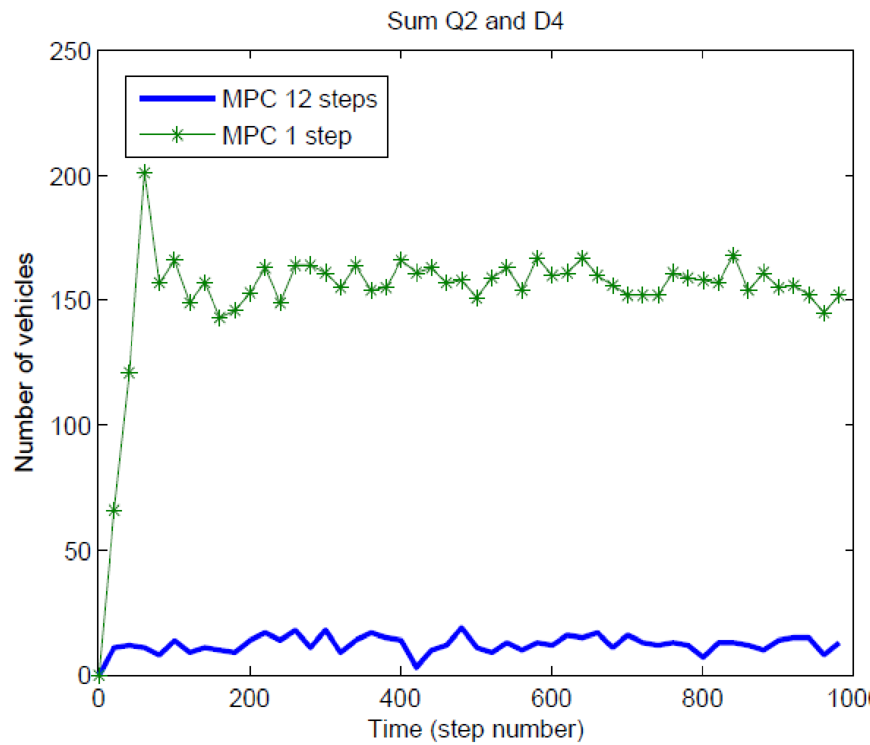
Results for scenario 2



(a) Number of vehicles that arrive destination



(b) Queue at the top bottleneck link

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Conclusion

- We present a general Model Predictive Control framework for centralized traffic signals and route guidance systems aiming to maximize network throughput.

- We explicitly consider travel time, spill back while retain linearity and tractability.

- The numerical experiments show that our proposed scheme may reduce congestion significantly while still achieving better throughput.

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