BEHAVIOURAL FOUNDATIONS FOR THE TWO-FLUID MODEL

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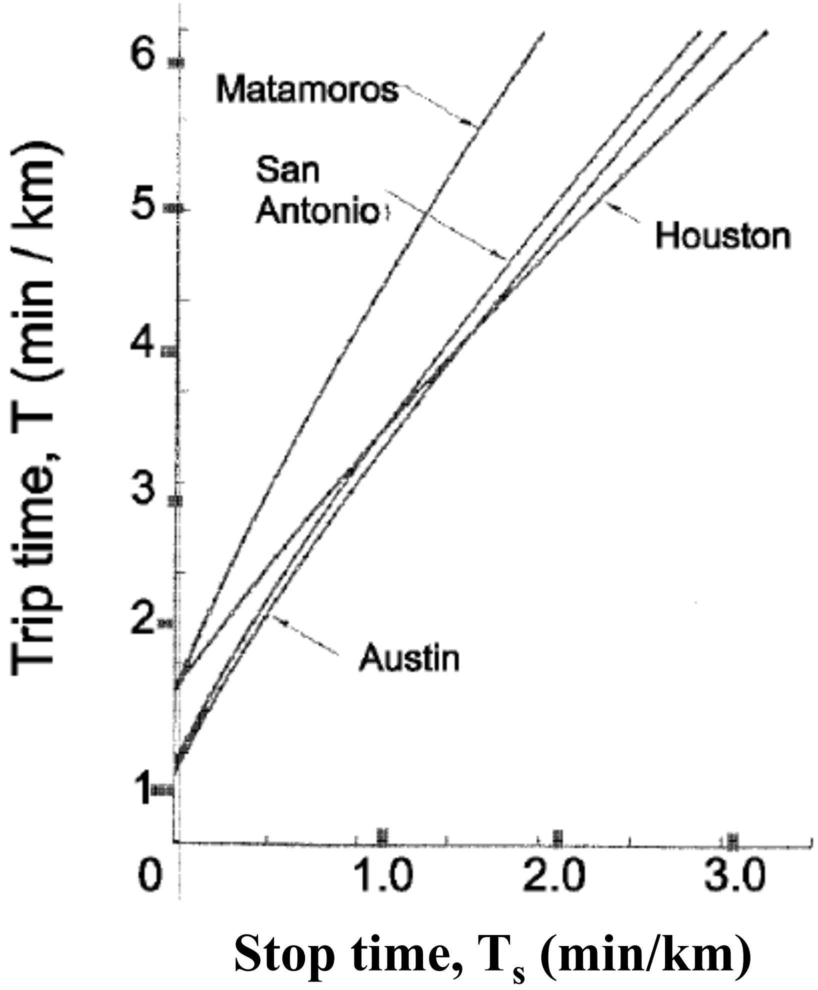
The Two-Fluid Model

The two-fluid model describes the relationship between the running time per mile and travel time per mile of a vehicle in an urban network.

This model was developed based on ‘particle physics’ and lacked the understanding of behavioral significance.

$$v = v_m \left(f_r \right)^n$$

$$T_r = T_m^{\frac{1}{n+1}} T^{\frac{n}{n+1}}$$



The Two-Fluid Model

Two-fluid model has been used to characterize:

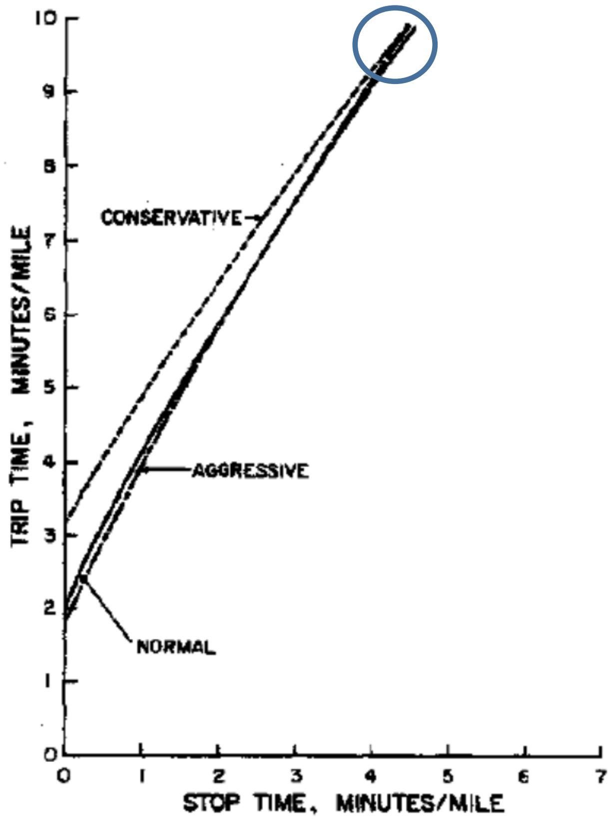
- Traffic flow on urban networks.
  [Herman and Prigogine (1971), Ardekani (1984)].

- Traffic flow on urban arterials.
  [Jones and Wahid (2003)]

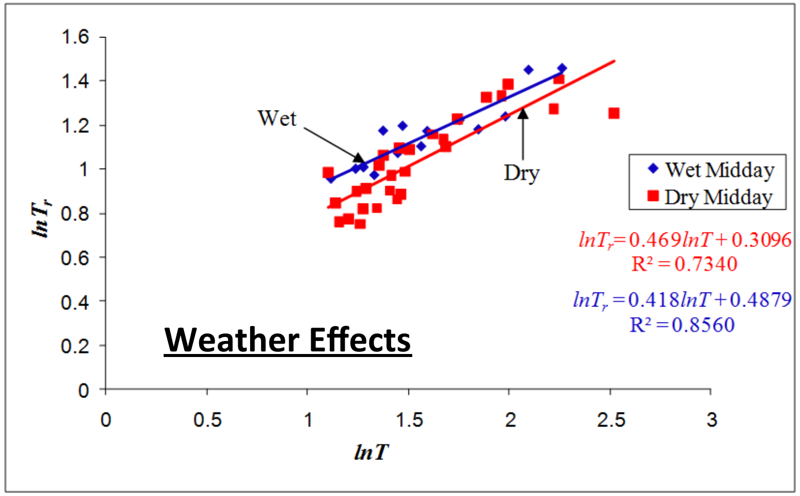
- Individual driver behavior,
  [Herman, Malakhoff and Ardekani (1988)]

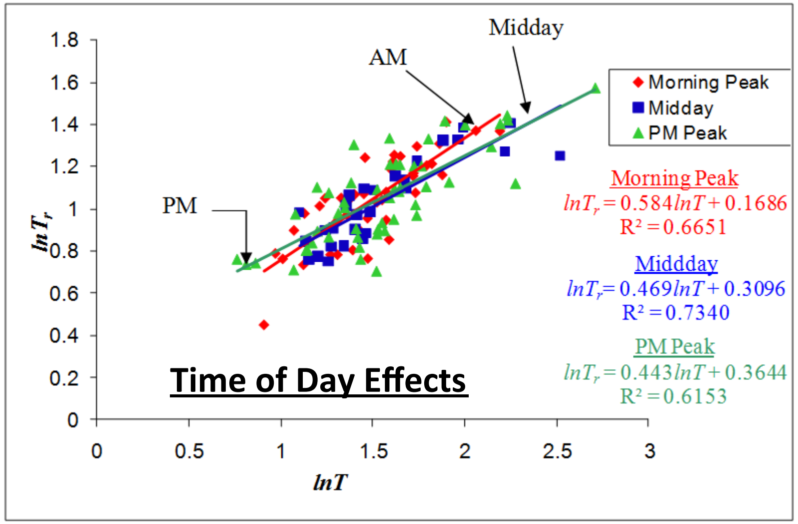
- Safety
  [Dixit et al. (2009)]

Motivation: Two-Fluid Model



(Herman et al., 1988)



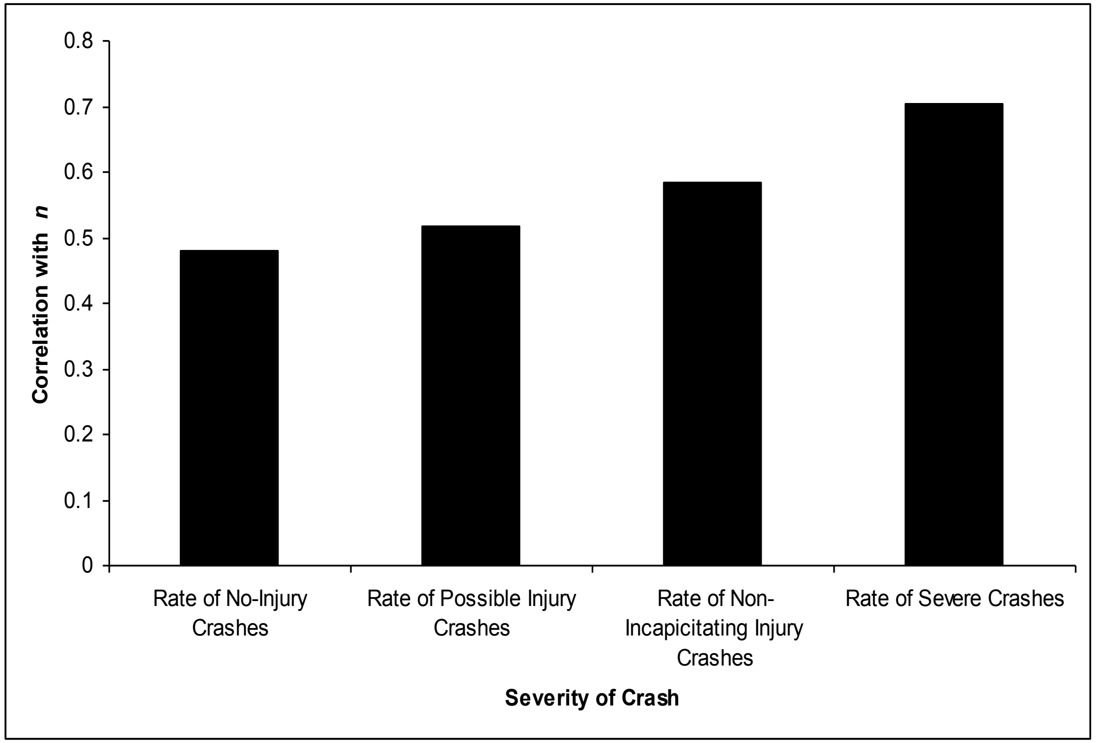


Dixit, V., Gayah, V., and Radwan, E. (2012). "Comparison of Driver Behavior by Time of Day and Wet Pavement Conditions." *J. Transp. Eng.*, 138(8), 1023–1029.

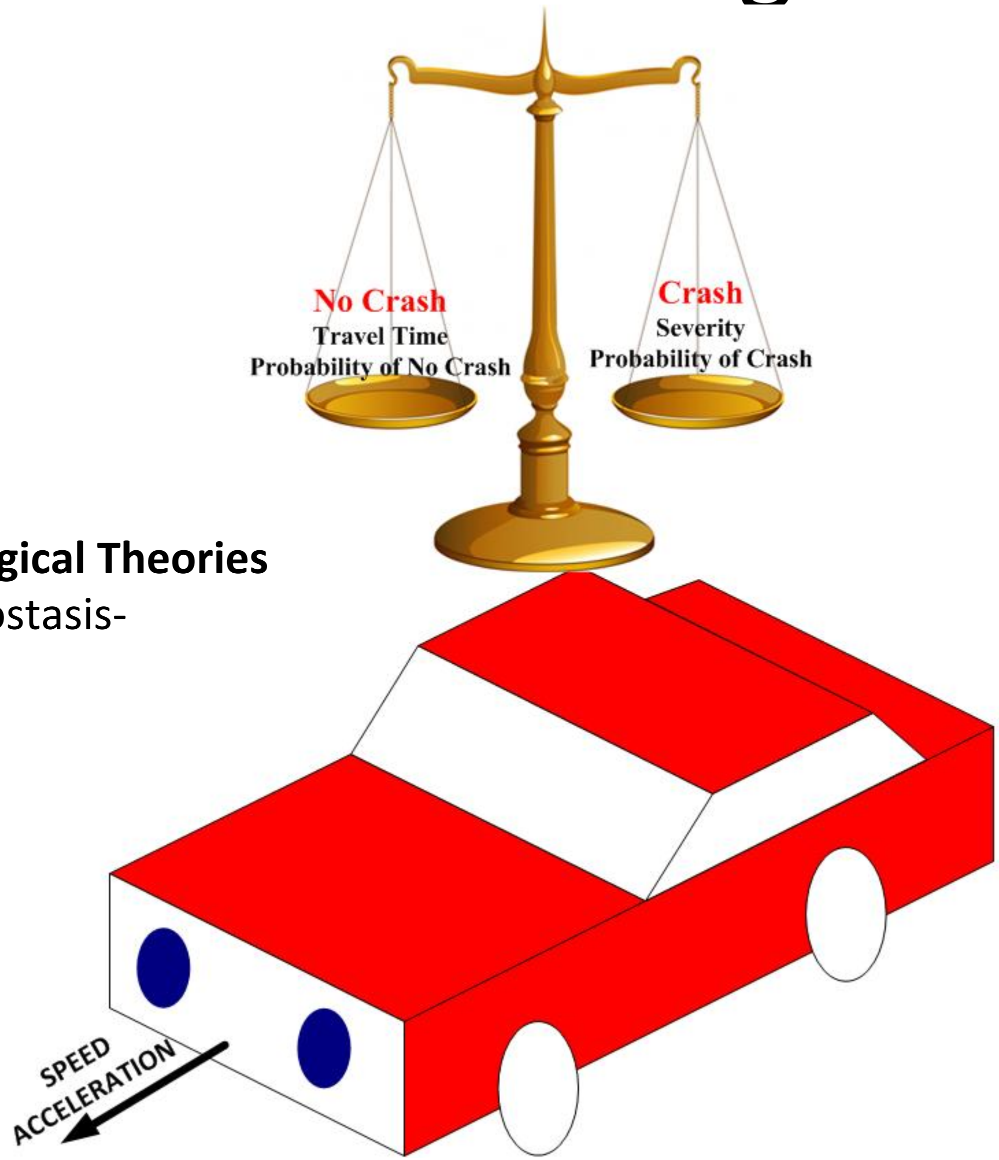
	n	Tm	R_square
n	1	-0.56553	0.40289
		0.144	0.3223
Tm	-0.56553	1	-0.41825
	0.144		0.3024
Speed Limit	-0.07593	-0.444	-0.03445
	0.8582	0.2704	0.9355
Access Management Class	0.23123	0.26308	-0.34474
	0.5816	0.529	0.403
Pavement Condition	0.12589	-0.26976	0.35297
	0.7664	0.5182	0.3911
Number of Lanes	0.00914	-0.02237	-0.59489
	0.9829	0.9581	0.1198
Number of Access per mile	0.41576	0.17059	-0.44511
	0.3056	0.6863	0.2691
Length of Two Way Left Turn Lanes per mile of	0.13363	0.19126	-0.54189
	0.7524	0.65	0.1653
Signal per Mile	0.16437	0.01996	-0.22347
	0.6973	0.9626	0.5947
Average Annual Daily Traffic	-0.4529	0.40385	-0.73589
	0.2598	0.3211	0.0374
Average Annual Daily Traffic per Lane	-0.69923	0.65411	-0.28646
	0.0536	0.0785	0.4916
Total Crash Rate	0.58856	-0.39606	0.19979
	0.1248	0.3314	0.6352
Rear-end Crash Rate	0.67808	-0.509	0.32416
	0.0646	0.1977	0.4334
Angle Crash Rate	0.43606	-0.44034	0.15214
	0.2801	0.2749	0.7191
Side-swipe Crash Rate	0.31856	-0.15132	-0.22677
	0.4419	0.7206	0.5892
Other Type Crash Rate	0.2039	0.21383	0.10578
	0.6282	0.6111	0.8031
Rate of No-Injury Crashes	0.47944	-0.35647	0.41134
	0.2293	0.3861	0.3113
Rate of Possible Injury Crashes	0.51709	-0.35937	0.13975
	0.1894	0.3819	0.7414
Rate of Non-Incapicitating Injury Crashes	0.58491	-0.23365	-0.13684
	0.1278	0.5776	0.7466
Rate of Severe Crashes	0.70317	-0.55848	0.33219
	0.0517	0.1502	0.4214

Correlation

- Negative correlation between T_m and n
- As n increases the correlation with crash severity increases.



Risk of Driving



Descriptive Psychological Theories
Task difficulty Homeostasis-
Risk Homeostatsis

Individual Driving Behavior

- Driving can be described in the form of a state-dependent approach.
  - S = {*crash, no crash*}.
- The utility of being in state "no crash" is associated to reaching the destination as quickly as possible.
- The (dis)utility of being in the state "crash" is the associated to severity of the crash.

Model Assumptions

- Perceived Probability of Crashing $P_{crash} = \alpha (v/v_r)^{\beta}$
  - Conforms with Empirical Findings by Elvik et al. (2004)

$$u(crash) = -w(v_r)^k$$

- Disutility of crashing
  - Related to perceived severity (kinetic energy)

$$u(no\ crash) = v$$

- Utility of Not Crashing
  - Travelling faster

Expected Utility For A Driver

$$EU = P_{no\ crash} \times u(no\ crash) + P_{crash} \times u(crash)$$

$$EU = (1 - \alpha f_r^{\beta})v - \alpha f_r^{\beta} w(v_r)^k$$

Where $f_r = \dfrac{v}{v_r}$

$$EU = (1 - \alpha \left(v_r\right)^{-\beta} \left(v\right)^{\beta})v - \alpha w \left(v\right)^{\beta} (v_r)^{k-\beta}$$

Utility Maximization of Driver

$$\frac{\partial EU}{\partial v_r} = \alpha\beta\left(v_r\right)^{-\beta-1}\left(v\right)^{\beta+1} - \alpha(k-\beta)w\left(v_r\right)^{k-\beta-1}(v)^{\beta} = 0$$

$$\Rightarrow \left(\frac{1}{v_r}\right) = \left(\left(\frac{(k-\beta)w}{\beta}\right)^{\frac{1}{k-1}}\right)^{\frac{k-1}{k}} \left(\frac{1}{v}\right)^{\frac{1}{k}}$$

Solution exists if $k > \beta$

$$\boxed{\frac{\partial P_{crash}}{P_{crash}\partial v_r} = -\frac{\beta}{v_r}}$$

$$\boxed{\frac{\partial u(crash)}{u(crash)\partial v_r} = \frac{k}{v_r}}$$

The marginal rate of change for the perceived disutility

is larger than

the marginal rate of change for the perceived probability to crash.

Utility Maximization of Driver

In order to get the form of the traditional two-fluid model.
To ensure k>1 Substitute *k=(n+1)/n*

Comparing:

$$T_r = \left(\left(\frac{(n+1-\beta n)w}{n\beta}\right)^n\right)^{\frac{1}{n+1}} T^{\frac{n}{n+1}}$$

$$T_r = T_m^{\frac{1}{n+1}} T^{\frac{n}{n+1}}$$

$$\Rightarrow T_m = \left(\frac{(1+n-n\beta)w}{n\beta}\right)^n \quad \Rightarrow \frac{dT_m}{dn} = \left(\frac{-w}{n\beta}\right)\left(\frac{(1+n-n\beta)w}{n\beta}\right)^{n-1}$$

T_m and *n* are negatively correlated

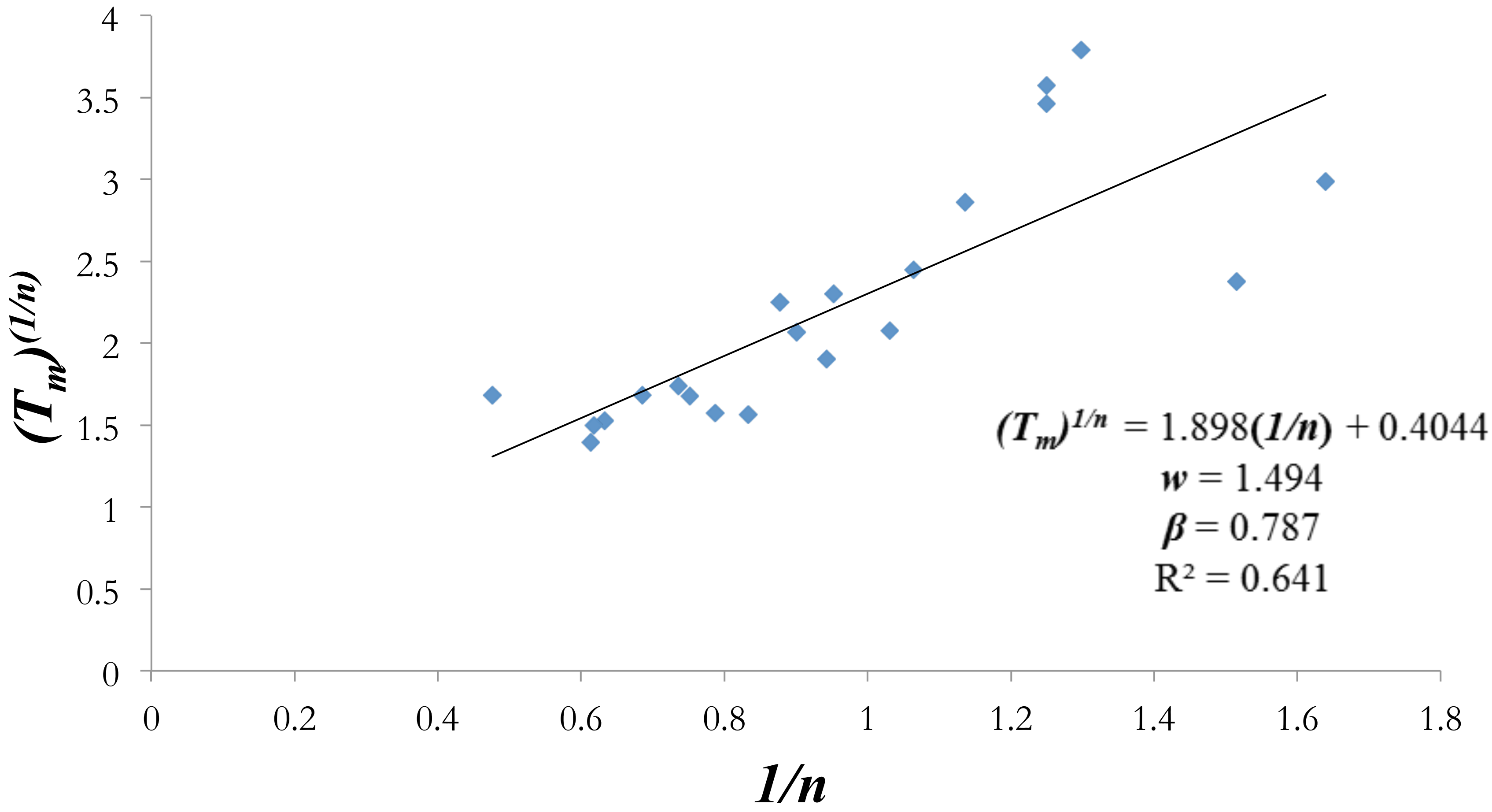
Relationship Between T_m and n

- Using Data from 1983, 1990 and 1991 from the cities of Dallas, Forth Worth, Arlington, Austin, Lubbock, Houston, San Antonio, Albuquerque, Mexico City and Matamoros. (Ardekani, 1981)

- ***T_m* and *n* have a negative correlation of -0.47**

- Test validity of $$\left(T_m\right)^{\frac{1}{n}} = \frac{w}{n\beta} + w\left(\frac{1}{\beta} - 1\right)$$

Empirical Validation
Urban Network Data



Effect of Network Features

severity factor (k)

k=0.75-1.169 (fraction of one-way) ***+0.147*** (#Lanes) ***+0.005*** (Intersection Density) ***+0.502*** (density of actuated signals)

R^2=0.58

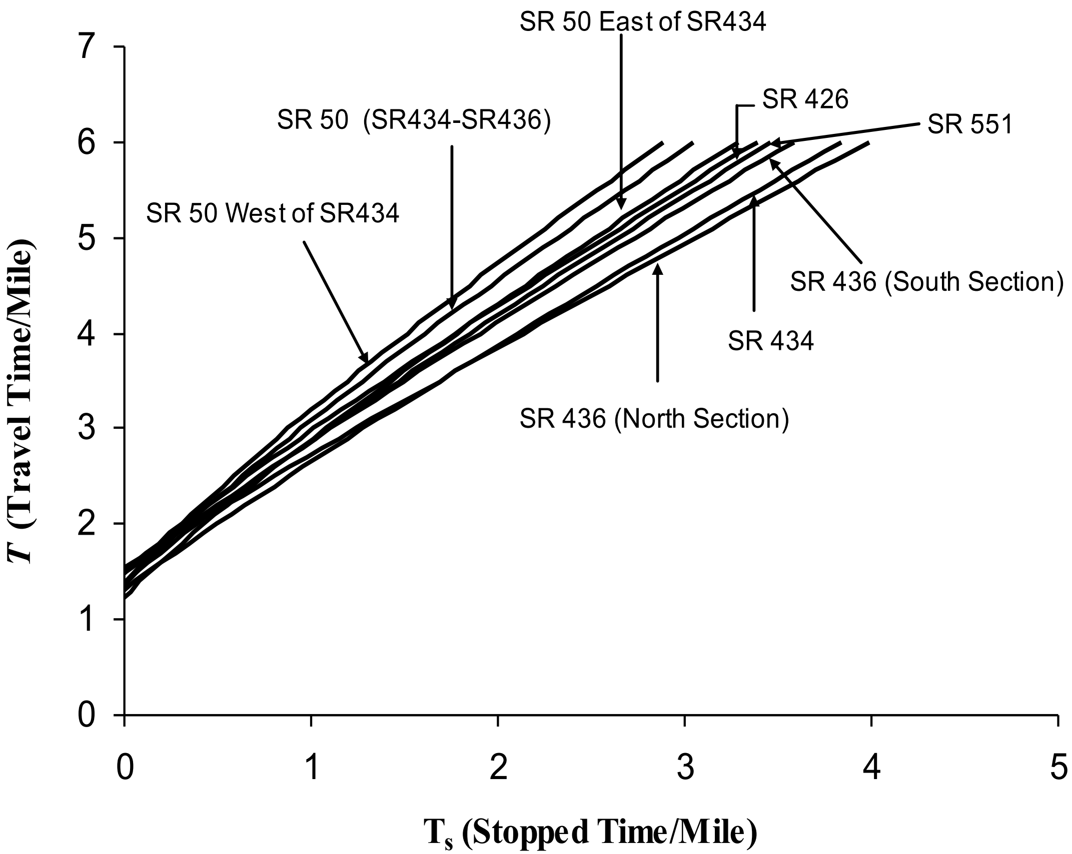
crash likelihood factor (β)

β=1.075-0.295 (fraction of one-way)

$$T_m^{1/n} = \left(\frac{2.081}{1.075-0.295X_2}\right)\left(\frac{n+1}{n}\right)-2.081$$

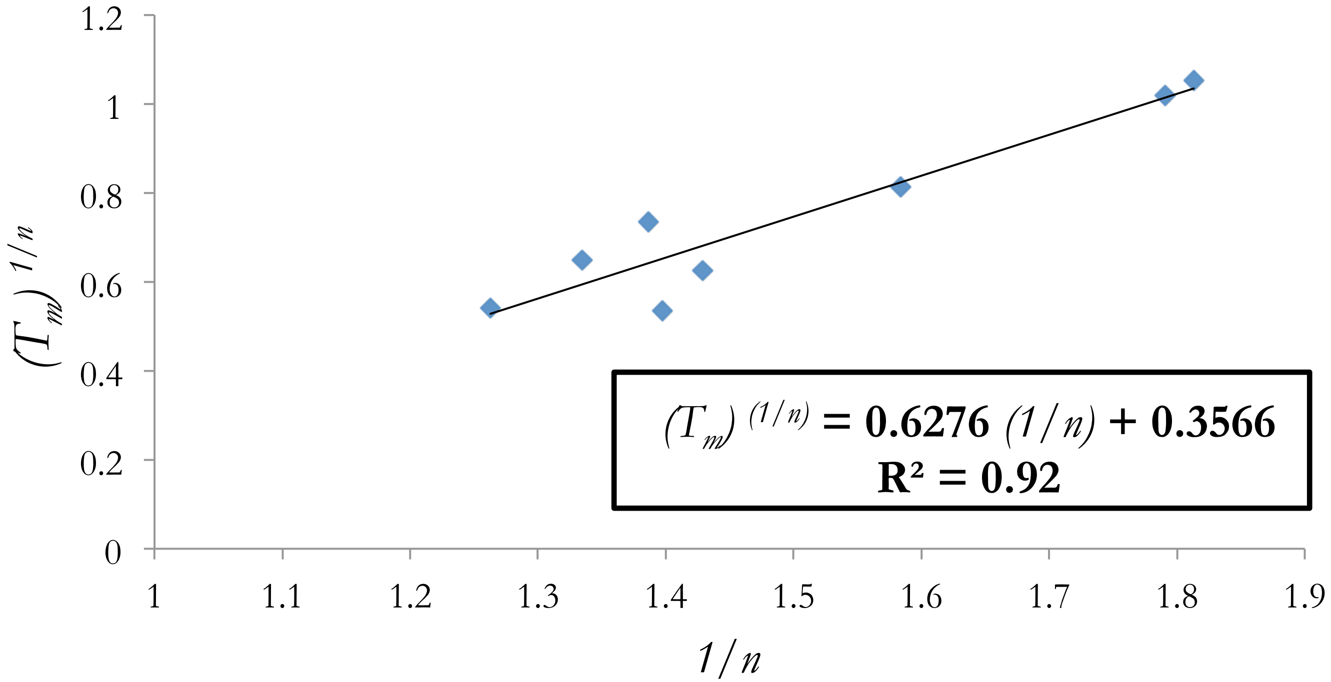
R^2=0.89

Two-Fluid Model Arterials



Number	Road	T_m	n
1	Aloma	1.506595	0.65865
2	SR50(SR434-SR436)	1.362973	1.088119
3	East Colonial	1.242444	0.986097
4	West Colonial	1.382316	1.243662
5	SR434	1.311191	0.49276
6	Semoron	1.47783	0.54012
7	Semoron North	1.551658	0.242082
8	Goldenrod	1.386247	0.697505

Arterial Data (Weak Evidence)



k=-2793.56 (total crash rate) + **5.35** **R^2=0.60**

$T_m^{1/n} = 1.476(n+1/n) - 1.83$ **R^2=0.92**

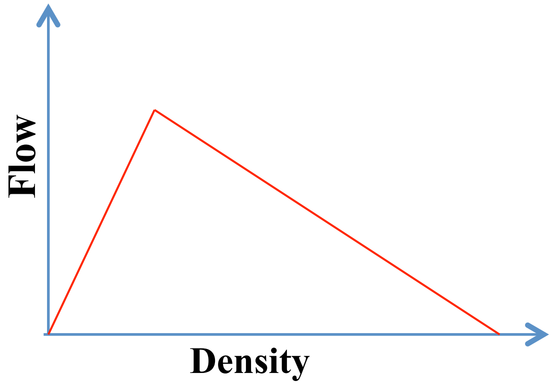
Physics of Traffic Flow

- Models based on physics of particles and fluid
  - Fitting models from fluid dynamics and particle physics to explain traffic behavior
  - Useful for engineering

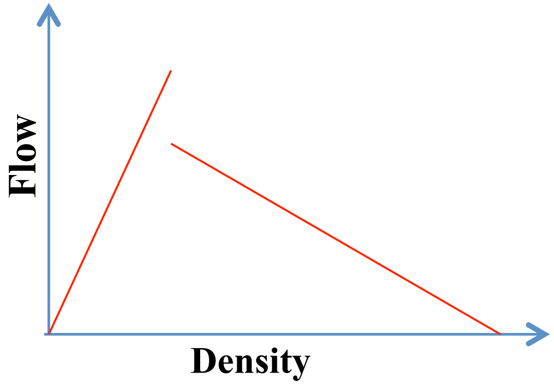
- Models based on explicitly assuming behavior (risk attitudes and Utility Models)
  - Enhances understanding for safety

Example: Fundamental Diagram

Dominance of physics of traffic, with systematic addition of behavioural parameters



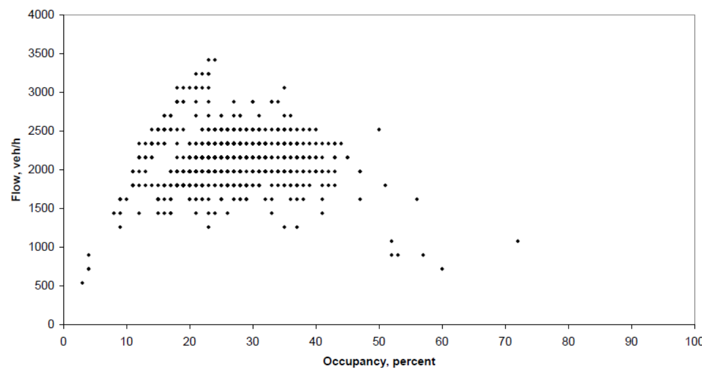
TRADITIONAL THEORY



OBSERVED

Story of the hare and slugs

- *Hares* are aggressive and maintain shorter gaps and therefore greater flows
- In congestion, flows are constrained



Banks, James H.; Amin, Mohammad R.; Cassidy, Michael; Chung, Koohong "Validation of Daganzo's Behavioral Theory of Multi-Lane Traffic Flow" California Partners for Advanced Transit and Highways (PATH), UC Berkley Final Report, 2003

Conclusion

- This study puts the two-fluid model from a behavioral perspective.
- The condition that $k > \beta$ is a necessary condition for the two-fluid model to exist.
  - On freeways this might not exist (the perceived probability to crash might increase at a larger rate than the perceived utility to crash.)
- Evaluation of training and educational programs for new drivers.
- The two-fluid model can be used on corridors to evaluate safety.
- The utility model has the potential of being used to engineer human driving behavior. (Incentives, disincentives and Insurance)