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Modeling of light propagation through biological tissues: a novel approach

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Abstract

An efficient numerical technique for modeling biological tissues using the radiative transfer equation is presented. Time dependence of the transient radiative transfer equation is approximated using Laguerre expansion. Azimuthal angle is discretized using the discrete ordinates method and the resulting set of ordinary differential equations is solved using the Runge-Kutta-Felhberg method.

1 Introduction

Modeling of light propagation through biological tissues is important for many medical applications such as optical tomography for cancer detection [1] and non-invasive detection of diabetes mellitus [2]. Researchers have been working on modeling biological tissues over the last two decades [3],[4].

Light propagation through biological tissues can be modeled using the Radiative Transfer Equation (RTE) [5, 6]. Several numerical models have been developed to solve the RTE over the recent years [7, 8, 9, 10]. These models include techniques for solving the steady state RTE [7], as well as the transient RTE for short pulse propagation [8, 9, 10].

This paper presents an efficient and faster approach for modeling the light propagation through scattering and absorbing media, such as biological tissue, by solving the transient RTE numerically.

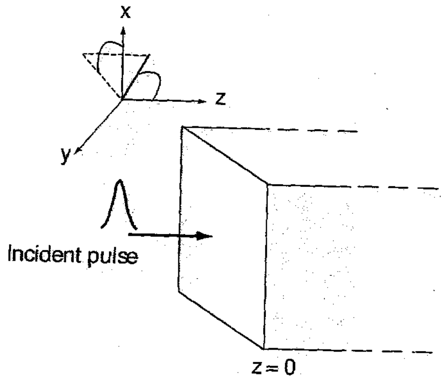

Figure 1. Short light pulse incident on the biological tissue

2 Formulation

Light propagation through biological tissues can be modeled using the transient RTE [8, 10, 11], which is given by,

$$\begin{aligned}&\frac{1}{v}\frac{\partial}{\partial t}I(x,y,z,u,\phi,t)+\xi\frac{\partial}{\partial x}I(x,y,z,u,\phi,t)\\&+\eta\frac{\partial}{\partial y}I(x,y,z,u,\phi,t)+u\frac{\partial}{\partial z}I(x,y,z,u,\phi,t)\\&-\frac{\sigma_s}{4\pi}\int_0^{2\pi}\int_{-1}^{1}P(u',\phi';u,\phi)I(x,y,z,u',\phi',t)du'd\phi'\\&+\sigma_t I(x,y,z,u,\phi,t)=F(x,y,z,u,\phi,t),\qquad(1)\end{aligned}$$

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where $I(z, u, \phi, t)$ is the light intensity, (x, y, z, θ, ϕ) are the standard coordinates, u,ξ and η are direction cosines such that $u = \cos\theta$, $\xi = \sin\theta\cos\phi$, $\eta = \sin\theta\sin\phi$ and t is the time variable; σ_t and σ_s are attenuation and scattering coefficients, respectively. The speed of light in the medium is v, $P(u', \phi'; u, \phi)$ is the phase function and $F(z, u, \phi, t)$ is the source term.

For the problem considered in this paper there are no sources inside the medium, and a short pulse is incident on the boundary of the tissue at $x = 0$, $y = 0$, $z = 0$, $\theta = \theta_0$ and $\phi = \phi_0$ at time $t = 0$. Therefore, the RTE in (1) reduces to,

$$\frac{1}{v}\frac{\partial}{\partial t}I(x,y,z,u,\phi,t) + \mathbf{n}\cdot\nabla I(x,y,z,u,\phi,t)$$
$$-\frac{\sigma_s}{4\pi}\int_0^{2\pi}\int_{-1}^{1}P(u',\phi';u,\phi)I(x,y,z,u',\phi',t)du'd\phi'$$
$$+\sigma_t I(x,y,z,u,\phi,t) = 0, \quad (2)$$

where $\mathbf{n} = \xi\mathbf{i} + \eta\mathbf{j} + u\mathbf{k}$, $\nabla = \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right)$ and $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are the unit vectors along x, y, z axes, respectively.

In this paper it is assumed that there is no initial intensity on the right boundary (at $x = 0$,$y = 0$,$z = 0$) other than the incident. i.e.,

$$I(0,0,0,u_0,\phi_0,t) = I_0 e^{-(\frac{t-t_0}{T})^2}\delta(u-u_0)\delta(\phi-\phi_0). \quad (3)$$

The proposed method involves solving the RTE given by (2) numerically. It reduces the original RTE given by (2) to a set of simple uncoupled differential equations which can then be solved using the Runge-Kutta-Felhberg method. In order to do this, the original equation which contains functions of six independent variables should be reduced to a set of equations which carries functions of only spatial variables, x, y and z.

Without loss of generality, from this point onwards we consider the one dimensional case in order to improve the clarity of the algorithmic formulation by reducing the mathematical complexity of the presentation. However, the same technique can be applied to two and three dimensional cases.

First, the following substitution is used on the RTE and the boundary condition in order to obtain a better approximation with Laguerre polynomials:

$$\tau = t - \frac{z}{vu}, \quad (4)$$

With this substitution the RTE becomes

$$u\frac{\partial}{\partial z}I(z,u,\phi,\tau) + \sigma_t I(z,u,\phi,\tau)$$
$$-\frac{\sigma_s}{4\pi}\int_0^{2\pi}\int_{-1}^{1}P(u',\phi';u,\phi)I(z,u',\phi'0,\tau)du'd\phi' = 0. \quad (5)$$

First, the azimuthal angle is discretized using the discrete ordinates method [7] which results in a set of uncoupled equations of only three independent variables, u, t and z. The next step is to remove the time dependence by expanding the intensity using Laguerre polynomials. This operation will result in the following set of equations:

$$u\frac{\partial}{\partial z}B_n(z,u,\phi_r) + \sigma_t B_n(z,u,\phi_r)$$
$$-\frac{\sigma_s}{4\pi}\sum_{j=1}^{L}w_j^{\phi}\int_{-1}^{1}P(u',\phi_j;u,\phi_r)B_n(z,u',\phi_j)du' = 0, \quad (6)$$

where $n = 0, \ldots, N$. In (6) $B_n(z, u, \phi_r)$ represents the n^{th} Laguerre coefficient and w_j^{ϕ} is the gaussian weight for ϕ.

Then the azimuthal angle, θ, is discretized using the Gaussian quadrature and the following set of uncoupled equations is obtained;

$$u\frac{\partial}{\partial z}B_n(z,u_i,\phi_r) + \sigma_t B_n(z,u_i,\phi_r)$$
$$-\frac{\sigma_s}{4\pi}\sum_{j=1}^{L}w_j^{\phi}\sum_{k=1}^{K}w_k^{u}P(u_k,\phi_j;u_i,\phi_r)B_n(z,u_k,\phi_j) = 0,(7)$$

where w_k^u are the Gaussian weights for u. Now there are $K \times L$ number of uncoupled equations corresponding to each quadrature point, (u_i, ϕ_r). This set of equations can be represented in matrix format as follows:

$$\mathbf{M}\frac{\partial}{\partial z}\mathbf{B}_n = \left(\frac{\sigma_s}{4\pi}\mathbf{P}_m\mathbf{W} - \sigma_t\right)\mathbf{B}_n \quad (8)$$

where $\mathbf{B}_n = [B_n(z, u_i, \phi_r)]_{K\times L}$. If the boundary condition is simplified using the same operations (3) reduces to

$$\mathbf{B}_n(z{=}0) = \begin{cases} C_k & ;u{=}u_0,\ \phi{=}\phi_0 \quad (9a) \\ 0 & ;u{\neq}u_0,\ \phi{\neq}\phi_0 \quad (9b) \end{cases}$$

where $C_k \approx \sum_{j=1}^{N} w_j f(t_j - z/(vu)) L_k(\tau)$ and w_j are the Laguerre weights. Thus, (8) can be solved using the Runge-Kutta-Fehlberg (RKF) method.

3 Results and Discussion

The figures below were obtained from the above algorithm. Without loss of generality we have obtained these results for the one dimensional problem. The Henyey-Greenstein phase function [12] was used for the simulation where g is the asymmetry factor.

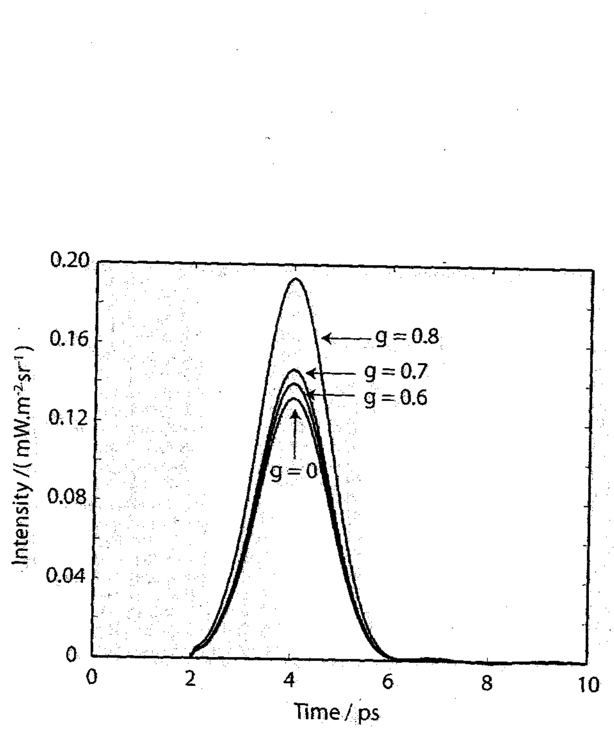

Figure 2. Intensity at z = 1 for different asymmetry factor (g) values

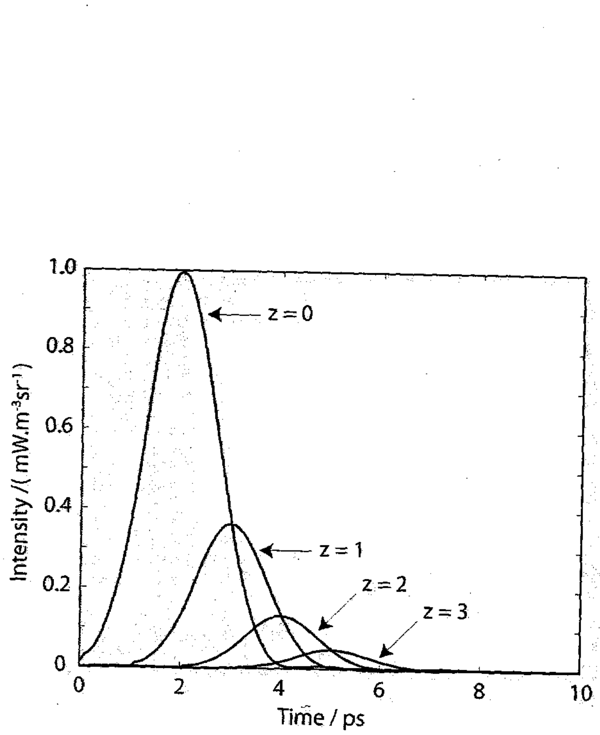

Figure 3. Forward intensity at different locations for isotropic (g = 0) scattering

Figure 2 shows the variation of intensity with time at $z = 1$ with varying g. The graphs corresponds to $g = 0.8$, $g = 0.7$, $g = 0.6$ and $g = 0$. Other parameters such as the scattering coefficient and the absorption coefficient were kept constant for all the three graphs. The condition $g = 0$ corresponds to the isotropic scattering case while $g = 0.8$ represents strong forward scattering. This is illustrated by the above four graphs.

Figure 3 shows the variation of the forward intensity at different locations, that is, corresponding to different z values, with a constant asymmetry factor, $g = 0$. It can be clearly seen from this figure that the intensity reduces with increasing distance due to scattering and absorption. Also, the pulse is shifted in time as shown.

4 Conclusion

This paper introduces a novel numerical model to solve the transient radiative transfer equation for modeling biological tissues. It has several advantages over the existing methods. Laguerre polynomials, which are causal, are used to represent the time dependence. Therefore, it is not necessary to impose causality explicitly. Since it is possible to approximate a short pulse using a few number of Laguerre polynomials, this model is capable of producing accurate results with relatively low computational power and in less time when compared with most of the existing methods which uses time marching techniques such as Cranck-Nicholson to propagate the pulse. Also, it is anticipated that the extension to layered media will not increase the computational requirements considerably, as is the case with many of the other models.

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