MODELING CHAOTIC DROPLET ROTATION THROUGH HIGH-FREQUENCY ACOUSTIC EXCITATION

James Friend* & Leslie Yeo*
*MicroNanophysics Research Laboratory, Monash University, Melbourne, 3800, Australia

INTRODUCTION

The application of microfluidics to biomedicine has been touted [1] as a means to deliver improved and inexpensive medical care. Though there are many methods for transporting fluids within these devices [2], from the passive capillary forces that draw a fluid along a hydrophilic channel to the actively driven forces like electrophoresis, none would appear to be as powerful as surface acoustic wave (SAW) irradiation [3].Using a piezoelectric substrate with an interdigital electrode (IDT) [4] deposited upon its surface, a traveling strain or *Rayleigh* [5] wave may be generated on the substrate surface from the IDT. As the wave passes, a point on the surface will move in a transversely-polarized elliptical fashion at less than 10 nm but with velocity of nearly 1 m/s. At 10 MHz, the accelerations of the surface easily exceed one million m/s^2, and so researchers have moved beyond the original application of SAW in telecommunications [6] to consider what SAW can do in microfluidics [3, 7, 8].

In particular, a rapid particle concentration method in sessile droplets and confined fluid chambers has been developed [9] using asymmetric surface wave propagation on a substrate upon which the droplet is placed. The SAW generates a combination of forces upon suspended particles and the fluid droplet itself via diffraction to drive rapid collection or dispersion of the particles, far faster than other currently available methods due to the large convective velocities achieved using the device. Strangely, a droplet will also rotate if placed *completely* in the path of the SAW (*see* Fig. 1); *there should be no torque on the droplet due to the symmetric SAW radiation*. Further, several transition regions exist with respect to IDT input power, from local particle collection at <1 mW, particle collection and dispersion *blinking* phenomena at intermediate (1–5 mW) power, to complete particle collection toward the center of the droplet at high (>5 mW) power, evidently representing a Hopf bifurcation with O(2) symmetry [10]. Though the fluid dynamics behavior in particle collection through localized and acoustically-driven agglomeration, followed by shear vortex migration has been verified [9], the reasons for the rotation of a droplet completely immersed in SAW irradiation was left unexplained.

MODEL AND ANALYSIS

The exposure of the droplet to very high frequency (10 MHz or more) pressure oscillations due to the SAW appears to cause *fast harmonic excitation* [10] of the droplet's rotation, with symmetry breaking in rotation from an initial bifurcation appearing with respect to power and induced by an asymmetric agglomeration of particles within the droplet that reflect the incoming acoustic radiation appearing in the fluid from the SAW. A model of the behavior has been constructed which provides results phenomenologically similar to the experimental results. Using Fig. 1(e), assume there is a pressure placed on the fluid droplet with some distribution $p = p(r, z, t) = \boldsymbol{\tau}(r, z, t)\cdot\boldsymbol{e}_x$, with the unit vector $\boldsymbol{e}_x$ defined along the direction of acoustic radiation. For this early model we ignore the azimuthal distribution of pressure and its effect on the droplet rotation, choosing instead to consider only the pressure applied to a cross sectional area A perpendicular to the acoustic radiation direction. The net torque on the droplet about its central axis is then $\boldsymbol{M}_p = \int_{r^-}^{r^+}\int_0^{z_r=c+\sqrt{b^2-r^2}} pr\,\mathrm{d}z\,\mathrm{d}r\,\boldsymbol{e}_z$. The parameters a, b, and c are defined as the radius of the droplet at the surface, the radius of curvature of the droplet, and the

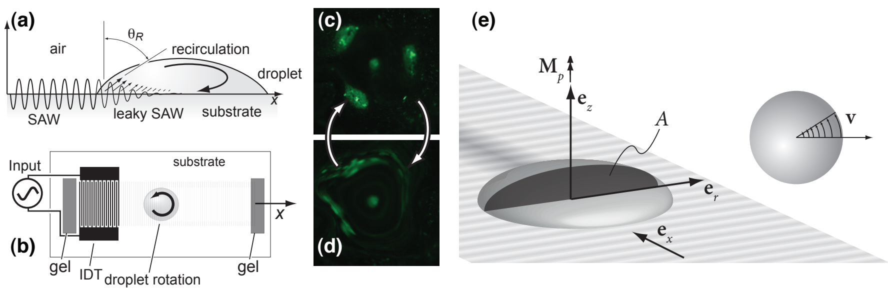

Figure 1. As the (a) SAW propagates underneath a fluid droplet, a portion of the SAW enters the droplet as a longitudinal sound wave at the *Rayleigh angle*, 23 deg. for LN and water. With sufficient power, the sound wave is strong enough to cause density variations out of phase with the pressure in the sound wave, causing *acoustic streaming* [11] and meridonal recirculation. Curiously, (b) a droplet placed entirely in the path of the SAW will also rotate, against expectations, and will furthermore exhibit particle (c) agglomeration and (d) dispersion behavior that alternates over time in a regular yet unpredictable fashion. To explore this phenomenon a crude model has been developed with (e) a simple geometry of the wave-droplet interaction.

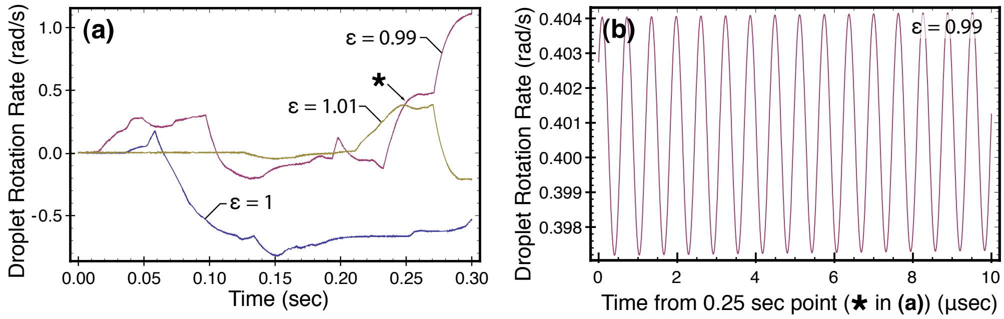

Figure 2. Computational results: (a) droplet rotation rate versus time for three slightly different droplet diameters (here a, the droplet diameter, is replaced throughout by εa; $\varepsilon = 0.99, 1, 1.01$ causes a $\pm 1\%$ change in the droplet diameter). The rotation of the droplet is driven by the high-frequency oscillation, and the direct effect of the oscillation may be seen if (b) one closely examines the result over a very short period of time, here at the point marked with a star in the original result (a).

distance of the center of curvature beneath the surface, respectively, presuming the droplet's cross-sectional shape may be defined as a cut of a circular shape. If we *assume* the solution for the motion of the droplet is the typical azimuthal flow $\boldsymbol{v} = r\dot{\theta}\boldsymbol{e}_\theta$, $\boldsymbol{M} = M\boldsymbol{e}_z = \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{H}$, where $\boldsymbol{H} = I\,v/r\,\boldsymbol{e}_z = I\,\dot{\theta}\,\boldsymbol{e}_z$. The quantity I represents the net moment of inertia of the droplet. Now part of $\boldsymbol{M}$ is $\boldsymbol{M}_p$, but there are other components; perhaps we may model the viscous drag resisting the rotation of the droplet at the fluid-solid interface as a torque opposing the rotation that is proportional to the droplet rotation speed. Since $\boldsymbol{v} = r\dot{\theta}$ where $\dot{\theta} = \dot{\theta}(t)$ and r is an autonomous parameter, the angular velocity defines the behavior of the system and indeed may be used to define the drag torque $\boldsymbol{M}_D = -\boldsymbol{e}_z\, C_D\dot{\theta}$ where C_D is the drag coefficient. After some work on describing an equivalent moment of inertia for the droplet, I, and including a time constant, t_D, for the effect of drag, the equation for the droplet rotation is

$$I\ddot{\theta} = \frac{1}{5}mr^2\ddot{\theta} = \frac{\rho\pi}{5}a^4h\ddot{\theta} = -\frac{\rho\pi}{5t_D}a^4h\dot{\theta} + \frac{1}{2}P_0\left[a\left(\sqrt{b^2-a^2}+2c\right) + b^2\mathrm{Arcsin}\frac{a}{b}\right].$$

This equation is entirely linear unless $P_0(t)$, the pressure *difference* from one side of the droplet to the other, is a nonlinear function of time. This equation is similar to others that exhibit chaotic behavior when P_0 is itself complex [12]. The acoustic pressure due to the incident SAW appears on both sides of the droplet in equal amounts, and so cannot contribute to the development of an applied moment on the droplet. However, in particle-laden droplets, there is a difference in pressure from one side of the droplet to the other proportional to the difference in area fraction of particles from one side to the other, $P_0 = [\mathcal{C}(-\theta_i) - \mathcal{C}(\pi - \theta_i)]\,F_p$, where $\mathcal{C}(\cdot)$ is the particle concentration and F_p is the acoustic force on *one* particle [13], $F_{p,DC} \sim 2\pi\rho|u|^2$. Traditionally, this acoustic force is treated as a time-constant expression [14], yet under the unique conditions presented by SAW the time-varying component cannot be ignored, giving $F_p(t) \sim F_{p,DC} + 2\pi\rho|u|^2\cos\omega t$.

RESULTS AND CONCLUSIONS

With a water droplet 1 mm in diameter and 250 μm in height containing 1 μm polystyrene particles at a 0.1% weight concentration atop a SAW device operating at 10 MHz, rotation is quite complex and notably sensitive to the droplet diameter as shown in Fig. 2. Using a crude model of SAW irradiation of a particle-laden droplet with fast harmonic excitation, the rapid and inharmonic rotation of the droplet observed in past experiments has been demonstrated.

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