

## The connection

Here we use the results of `metric.cdbp` and `metric-inv.cdbp` to compute the metric connection  $\Gamma_{bc}^a(x)$ . We use the standard metric compatible connection

$$\Gamma_{bc}^a(x) = \frac{1}{2}g^{ad}(g_{dc,b} + g_{bd,c} - g_{bc,d})$$

```
::KeepHistory(false).
::PostDefaultRules( @@collect_terms!(%), @@sumflatten!(%) ).
```

```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#,v#}::Indices.
```

```
\nabla_{\#}::PartialDerivative.
\partial_{\#}::PartialDerivative.
```

```
g_{a b}::Metric.
g^{a b}::Metric.
\delta^{a}_{b}::KroneckerDelta.
```

```
R_{a b c d}::RiemannTensor.
R^{a}_{b c d}::RiemannTensor.
R^{a}_{b c}{}^d::RiemannTensor.
R^{a}_{b}{}^c{}^d::RiemannTensor.
```

```
# --- imported from metric.lib -----
```

```
metric:="import metric.lib metric":
@run(metric){"/Users/leo/local/sh/cdbfile"}:
```

```
dmatrix:="import metric.lib dmatrix":
@run(dmatrix){"/Users/leo/local/sh/cdbfile"}:
```

```
# --- imported from metric-inv.lib -----
```

```
inverse:="import metric-inv.lib metric":
@run(inverse){"/Users/leo/local/sh/cdbfile"}:
```

```
@print["g_{ab}(x)"]="~@(metric)~"+\BigO{\eps^6}";
@print["g^{ab}(x)"]="~@(inverse)~"+\BigO{\eps^6}";
@print["g_{ab,c}(x)"]="~@(dmatrix)~"+\BigO{\eps^6}";
```

# =====  
#  $g_{ab}(x)$ ,  $g^{ab}(x)$  and  $g_{ab,c}(x)$   
# =====

$$g_{ab}(x) = \left( g_{ab} - \frac{1}{3} R_{acbd} x^c x^d - \frac{1}{6} \nabla_c R_{adb e} x^c x^d x^e + \frac{2}{45} R_{acde} R_{bf dg} x^c x^e x^f x^g - \frac{1}{20} \nabla_{cd} R_{aeb f} x^c x^d x^e x^f + \frac{1}{45} R_{acde} \nabla_f R_{bg dh} x^c x^e x^f x^g x^h + \frac{1}{45} R_{bcde} \nabla_f R_{ag dh} x^c x^e x^f x^g x^h - \frac{1}{90} \nabla_{cde} R_{afb g} x^c x^d x^e x^f x^g \right) + \mathcal{O}(\epsilon^6)$$

$$g^{ab}(x) = \left( g^{ab} + \frac{1}{3} x^c x^d R^a{}_c{}^b{}_d + \frac{1}{6} x^c x^d x^e \nabla_c R^a{}_d{}^b{}_e + \frac{1}{15} x^c x^d x^e x^f R^a{}_{cdg} R^b{}_{efg} + \frac{1}{20} x^c x^d x^e x^f \nabla_{cd} R^a{}_e{}^b{}_f + \frac{1}{30} x^c x^d x^e x^f x^g R^a{}_{cdh} \nabla_e R^b{}_{fgh} + \frac{1}{30} x^c x^d x^e x^f x^g R^b{}_{cdh} \nabla_e R^a{}_{fgh} + \frac{1}{90} x^c x^d x^e x^f x^g \nabla_{cde} R^a{}_f{}^b{}_g \right) + \mathcal{O}(\epsilon^6)$$

$$g_{ab,c}(x) = \left( -\frac{1}{3} R_{acbd} x^d - \frac{1}{3} R_{adbc} x^d - \frac{1}{6} \nabla_c R_{adb e} x^d x^e - \frac{1}{6} \nabla_d R_{acbe} x^d x^e - \frac{1}{6} \nabla_d R_{aeb c} x^d x^e + \frac{2}{45} R_{acde} R_{bf dg} x^e x^f x^g - \frac{2}{45} R_{adce} R_{bf eg} x^d x^f x^g + \frac{2}{45} R_{adef} R_{bceg} x^d x^f x^g - \frac{2}{45} R_{adef} R_{bgce} x^d x^f x^g - \frac{1}{10} \nabla_{cd} R_{aeb f} x^d x^e x^f - \frac{1}{20} \nabla_{de} R_{acbf} x^d x^e x^f - \frac{1}{20} \nabla_{de} R_{afb c} x^d x^e x^f + \frac{1}{45} R_{acde} \nabla_f R_{bg dh} x^e x^f x^g x^h - \frac{1}{45} R_{adce} \nabla_f R_{bg eh} x^d x^f x^g x^h + \frac{1}{45} R_{adef} \nabla_c R_{bge h} x^d x^f x^g x^h + \frac{1}{45} R_{adef} \nabla_g R_{bce h} x^d x^f x^g x^h - \frac{1}{45} R_{adef} \nabla_g R_{bhce} x^d x^f x^g x^h + \frac{1}{45} R_{bcde} \nabla_f R_{ag dh} x^e x^f x^g x^h - \frac{1}{45} R_{bdce} \nabla_f R_{age h} x^d x^f x^g x^h + \frac{1}{45} R_{bdef} \nabla_c R_{age h} x^d x^f x^g x^h + \frac{1}{45} R_{bdef} \nabla_g R_{ace h} x^d x^f x^g x^h - \frac{1}{45} R_{bdef} \nabla_g R_{ahce} x^d x^f x^g x^h - \frac{1}{30} \nabla_{cde} R_{afb g} x^d x^e x^f x^g - \frac{1}{90} \nabla_{def} R_{acbg} x^d x^e x^f x^g - \frac{1}{90} \nabla_{def} R_{agbc} x^d x^e x^f x^g \right) + \mathcal{O}(\epsilon^6)$$

```
# =====
#   compute gcb,a + gac,b - gab,c
# =====
```

```
tmp:= dg_{c b a} + dg_{a c b} - dg_{a b c}:
```

```
@substitute!(%)(dg_{a b c} -> @(dmetric)):
```

```
@distribute!(%): @prodsort!(%): @rename_dummies!(%): @canonicalise!(%):
```

```
@print["2g_{dc}(x)\Gamma^d_{ab}(x)="\~@(tmp)~"+\BigO{\eps^6}"];
```

$$\begin{aligned}
2g_{dc}(x)\Gamma^d_{ab}(x) = & \left( \frac{2}{3} R_{acbd}x^d - \frac{1}{6} \nabla_a R_{bdce}x^d x^e + \frac{1}{3} \nabla_d R_{acbe}x^d x^e - \frac{4}{45} R_{acde} R_{bf dg} x^e x^f x^g - \frac{2}{45} R_{adce} R_{bf dg} x^e x^f x^g - \frac{2}{45} R_{adbe} R_{cf dg} x^e x^f x^g - \frac{1}{10} \nabla_{ad} R_{becf} x^d x^e x^f \right. \\
& + \frac{1}{10} \nabla_{de} R_{acbf} x^d x^e x^f - \frac{2}{45} R_{acde} \nabla_f R_{bg dh} x^e x^f x^g x^h - \frac{1}{45} R_{adce} \nabla_f R_{bg dh} x^e x^f x^g x^h + \frac{1}{45} R_{cdef} \nabla_a R_{bge h} x^d x^f x^g x^h - \frac{1}{45} R_{cdef} \nabla_g R_{aeb h} x^d x^f x^g x^h \\
& - \frac{1}{45} R_{adbe} \nabla_f R_{cg dh} x^e x^f x^g x^h + \frac{1}{45} R_{bdef} \nabla_a R_{cge h} x^d x^f x^g x^h - \frac{2}{45} R_{bdef} \nabla_g R_{ace h} x^d x^f x^g x^h - \frac{1}{45} R_{bdef} \nabla_g R_{ace h} x^d x^f x^g x^h - \frac{1}{30} \nabla_{ade} R_{bf cg} x^d x^e x^f x^g \\
& + \frac{1}{45} \nabla_{def} R_{acbg} x^d x^e x^f x^g + \frac{2}{3} R_{adb c} x^d - \frac{1}{6} \nabla_b R_{adce} x^d x^e + \frac{1}{3} \nabla_d R_{aeb c} x^d x^e - \frac{2}{45} R_{adbe} R_{cf eg} x^d x^f x^g - \frac{4}{45} R_{adef} R_{bce g} x^d x^f x^g - \frac{2}{45} R_{adef} R_{bec g} x^d x^f x^g \\
& - \frac{1}{10} \nabla_{bd} R_{aecf} x^d x^e x^f + \frac{1}{10} \nabla_{de} R_{afbc} x^d x^e x^f - \frac{1}{45} R_{adbe} \nabla_f R_{cge h} x^d x^f x^g x^h + \frac{1}{45} R_{adef} \nabla_b R_{cge h} x^d x^f x^g x^h - \frac{2}{45} R_{adef} \nabla_g R_{bce h} x^d x^f x^g x^h \\
& - \frac{1}{45} R_{adef} \nabla_g R_{bech} x^d x^f x^g x^h - \frac{2}{45} R_{bcde} \nabla_f R_{ag dh} x^e x^f x^g x^h - \frac{1}{45} R_{bdce} \nabla_f R_{ag dh} x^e x^f x^g x^h + \frac{1}{45} R_{cdef} \nabla_b R_{age h} x^d x^f x^g x^h \\
& - \frac{1}{45} R_{cdef} \nabla_g R_{ahbe} x^d x^f x^g x^h - \frac{1}{30} \nabla_{bde} R_{afcg} x^d x^e x^f x^g + \frac{1}{45} \nabla_{def} R_{agbc} x^d x^e x^f x^g + \frac{1}{6} \nabla_c R_{adbe} x^d x^e + \frac{2}{45} R_{adce} R_{bf eg} x^d x^f x^g + \frac{2}{45} R_{adef} R_{bgce} x^d x^f x^g \\
& + \frac{1}{10} \nabla_{cd} R_{aebf} x^d x^e x^f + \frac{1}{45} R_{adce} \nabla_f R_{bge h} x^d x^f x^g x^h - \frac{1}{45} R_{adef} \nabla_c R_{bge h} x^d x^f x^g x^h + \frac{1}{45} R_{adef} \nabla_g R_{bhce} x^d x^f x^g x^h + \frac{1}{45} R_{bdce} \nabla_f R_{age h} x^d x^f x^g x^h \\
& \left. - \frac{1}{45} R_{bdef} \nabla_c R_{age h} x^d x^f x^g x^h + \frac{1}{45} R_{bdef} \nabla_g R_{ahce} x^d x^f x^g x^h + \frac{1}{30} \nabla_{cde} R_{afbg} x^d x^e x^f x^g \right) + \mathcal{O}(\epsilon^6)
\end{aligned}$$

```

# =====
#   compute  $\Gamma_{bc}^a(x)$ 
# =====

gamma:= 1/2 ( dg_{d c b} + dg_{b d c} - dg_{b c d} ) invmetric^{d a}:

@substitute!(%)(dg_{a b c} -> @(dmetric)):
@substitute!(%)(invmetric^{a b} -> @(inverse)):
@distributed!(%): @eliminate_metric!(%): @prodsort!(%): @rename_dummies!(%): @canonicalise!(%):

# =====
#    $g_{ab}$  is accurate to terms including  $\mathcal{O}(s^5)$  but  $g_{ab,c}$  are accurate only to terms  $\mathcal{O}(s^4)$ 
#   so we must truncate  $\Gamma_{bc}^a$  at  $\mathcal{O}(s^4)$ 
# =====

# gamma(x) = term00 + x^d term01 + x^d x^e term02 + x^d x^e x^f term03 + ...

# term01 = 1! gamma_{,d}      at x=0, symmetrised over {d}
# term02 = 2! gamma_{,de}     at x=0, symmetrised over {d,e} = ( de + ed )/2!
# term03 = 3! gamma_{,def}    at x=0, symmetrised over {d,e,f} = ( def + edf + dfe + ...)/3!

# --- extract the terms, one by one, then rebuild the series

poly:=@(gamma):
@distributed!(%):

x^{a}::Weight(label=xterms,value=1).

term00:=@(poly): @keep_weight!(term00){xterms}{0}:
term01:=@(poly): @keep_weight!(term01){xterms}{1}:
term02:=@(poly): @keep_weight!(term02){xterms}{2}:
term03:=@(poly): @keep_weight!(term03){xterms}{3}:
term04:=@(poly): @keep_weight!(term04){xterms}{4}:

# =====
#   force all indices on  $R$  to be downstairs
# =====

@substitute!(term00)(R^{a}_{b c d} -> R_{a b c d},
                    R_{a}^{b}_{c d} -> R_{a b c d},
                    R^{a}_{b c}{}^d -> R_{a b c d},
                    R_{a b}{}^{c}{}_d -> R_{a b c d},

```

```

R_{a b c}^{d} -> R_{a b c d},
R^{a}_{b}c^{d} -> R_{a b c d},
R^{a}_{b}c^{d}_{e} -> R_{a b c d},
\nabla^{a}_{R_{b c d e}} -> \nabla_{a}\{R_{b c d e}\},
\nabla^{a}_{f}\{R_{b c d e}\} -> \nabla_{a f}\{R_{b c d e}\},
\nabla^{a}_{f g}\{R_{b c d e}\} -> \nabla_{a f g}\{R_{b c d e}\}):

```

```

@substitute!(term01)(R^{a}_{b c d} -> R_{a b c d},
R^{a}_{b}c^{d} -> R_{a b c d},
R^{a}_{b c}^{d} -> R_{a b c d},
R_{a b}^{c}_{d} -> R_{a b c d},
R_{a b c}^{d} -> R_{a b c d},
R^{a}_{b}c^{d} -> R_{a b c d},
R^{a}_{b}c^{d}_{e} -> R_{a b c d},
\nabla^{a}_{R_{b c d e}} -> \nabla_{a}\{R_{b c d e}\},
\nabla^{a}_{f}\{R_{b c d e}\} -> \nabla_{a f}\{R_{b c d e}\},
\nabla^{a}_{f g}\{R_{b c d e}\} -> \nabla_{a f g}\{R_{b c d e}\}):

```

```

@substitute!(term02)(R^{a}_{b c d} -> R_{a b c d},
R^{a}_{b}c^{d} -> R_{a b c d},
R^{a}_{b c}^{d} -> R_{a b c d},
R_{a b}^{c}_{d} -> R_{a b c d},
R_{a b c}^{d} -> R_{a b c d},
R^{a}_{b}c^{d} -> R_{a b c d},
R^{a}_{b}c^{d}_{e} -> R_{a b c d},
\nabla^{a}_{R_{b c d e}} -> \nabla_{a}\{R_{b c d e}\},
\nabla^{a}_{f}\{R_{b c d e}\} -> \nabla_{a f}\{R_{b c d e}\},
\nabla^{a}_{f g}\{R_{b c d e}\} -> \nabla_{a f g}\{R_{b c d e}\}):

```

```

@substitute!(term03)(R^{a}_{b c d} -> R_{a b c d},
R^{a}_{b}c^{d} -> R_{a b c d},
R^{a}_{b c}^{d} -> R_{a b c d},
R_{a b}^{c}_{d} -> R_{a b c d},
R_{a b c}^{d} -> R_{a b c d},
R^{a}_{b}c^{d} -> R_{a b c d},
R^{a}_{b}c^{d}_{e} -> R_{a b c d},
\nabla^{a}_{R_{b c d e}} -> \nabla_{a}\{R_{b c d e}\},
\nabla^{a}_{f}\{R_{b c d e}\} -> \nabla_{a f}\{R_{b c d e}\},

```

```

\nabla^{a}_{f g}\{R_{b c d e}\} -> \nabla_{a f g}\{R_{b c d e}\}):

@substitute!(term04)(R^{a}_{b c d} -> R_{a b c d},
  R_{a}^{b}_{c d} -> R_{a b c d},
  R^{a}_{b c}^{d} -> R_{a b c d},
  R_{a b}^{c}_{d} -> R_{a b c d},
  R_{a b c}^{d} -> R_{a b c d},
  R^{a}_{b}^{c}_{d} -> R_{a b c d},
  R^{a}_{b}^{c}_{d} -> R_{a b c d},
  \nabla^{a}\{R_{b c d e}\} -> \nabla_{a}\{R_{b c d e}\},
  \nabla^{a}_{f}\{R_{b c d e}\} -> \nabla_{a f}\{R_{b c d e}\},
  \nabla^{a}_{f g}\{R_{b c d e}\} -> \nabla_{a f g}\{R_{b c d e}\}):

# =====
#   now we have the leading terms, tidy up and print
# =====

{x^{a},R_{a b c d},\nabla_{a}\{R_{b c d e}\}}::SortOrder.

@prodsort!(term00): @rename_dummies!(%): @canonicalise!(%);
@prodsort!(term01): @rename_dummies!(%): @canonicalise!(%);
@prodsort!(term02): @rename_dummies!(%): @canonicalise!(%);
@prodsort!(term03): @rename_dummies!(%): @canonicalise!(%);
@prodsort!(term04): @rename_dummies!(%): @canonicalise!(%);

```

$$term00 := 0$$

$$term01 := -\frac{1}{3}x^d R_{abcd} - \frac{1}{3}x^d R_{acbd}$$

$$term02 := -\frac{1}{12}x^d x^e \nabla_b R_{adce} - \frac{1}{6}x^d x^e \nabla_d R_{abce} - \frac{1}{12}x^d x^e \nabla_c R_{adbe} - \frac{1}{6}x^d x^e \nabla_d R_{acbe} + \frac{1}{12}x^d x^e \nabla_a R_{bdce}$$

$$\begin{aligned}
term03 := & -\frac{4}{45}x^d x^e x^f R_{adeg} R_{bgcf} + \frac{2}{45}x^d x^e x^f R_{abdg} R_{cefg} + \frac{1}{45}x^d x^e x^f R_{adbg} R_{cefg} - \frac{1}{20}x^d x^e x^f \nabla_{bd} R_{aecf} \\
& - \frac{1}{20}x^d x^e x^f \nabla_{de} R_{abcf} - \frac{4}{45}x^d x^e x^f R_{adeg} R_{bfce} + \frac{2}{45}x^d x^e x^f R_{acdg} R_{befg} + \frac{1}{45}x^d x^e x^f R_{adcg} R_{befg} \\
& - \frac{1}{20}x^d x^e x^f \nabla_{cd} R_{aebf} - \frac{1}{20}x^d x^e x^f \nabla_{de} R_{acbf} - \frac{1}{45}x^d x^e x^f R_{agbd} R_{cefg} - \frac{1}{45}x^d x^e x^f R_{agcd} R_{befg} + \frac{1}{20}x^d x^e x^f \nabla_{ad} R_{becf}
\end{aligned}$$

$$\begin{aligned}
term04 := & -\frac{2}{45} x^d x^e x^f x^g R_{bhcd} \nabla_e R_{afgh} - \frac{1}{60} x^d x^e x^f x^g R_{adeh} \nabla_b R_{cfgh} - \frac{2}{45} x^d x^e x^f x^g R_{adeh} \nabla_f R_{bhcg} + \frac{1}{45} x^d x^e x^f x^g R_{abdh} \nabla_e R_{cfgh} + \frac{1}{90} x^d x^e x^f x^g R_{adbh} \nabla_e R_{cfgh} \\
& + \frac{1}{90} x^d x^e x^f x^g R_{cdeh} \nabla_b R_{afgh} + \frac{1}{45} x^d x^e x^f x^g R_{cdeh} \nabla_f R_{abgh} + \frac{1}{90} x^d x^e x^f x^g R_{cdeh} \nabla_f R_{agbh} - \frac{1}{60} x^d x^e x^f x^g \nabla_{bde} R_{afcg} - \frac{1}{90} x^d x^e x^f x^g \nabla_{def} R_{abcg} \\
& - \frac{2}{45} x^d x^e x^f x^g R_{bdch} \nabla_e R_{afgh} - \frac{1}{60} x^d x^e x^f x^g R_{adeh} \nabla_c R_{bfgh} - \frac{2}{45} x^d x^e x^f x^g R_{adeh} \nabla_f R_{bgch} + \frac{1}{90} x^d x^e x^f x^g R_{bdeh} \nabla_c R_{afgh} \\
& + \frac{1}{45} x^d x^e x^f x^g R_{bdeh} \nabla_f R_{acgh} + \frac{1}{90} x^d x^e x^f x^g R_{bdeh} \nabla_f R_{agch} + \frac{1}{45} x^d x^e x^f x^g R_{acdh} \nabla_e R_{bfgh} + \frac{1}{90} x^d x^e x^f x^g R_{adch} \nabla_e R_{bfgh} \\
& - \frac{1}{60} x^d x^e x^f x^g \nabla_{cde} R_{afbg} - \frac{1}{90} x^d x^e x^f x^g \nabla_{def} R_{acbg} - \frac{1}{36} x^d x^e x^f x^g R_{adeh} \nabla_h R_{bfcg} - \frac{1}{90} x^d x^e x^f x^g R_{ahbd} \nabla_e R_{cfgh} - \frac{1}{90} x^d x^e x^f x^g R_{bdeh} \nabla_a R_{cfgh} \\
& - \frac{1}{90} x^d x^e x^f x^g R_{bdeh} \nabla_f R_{ahcg} - \frac{1}{90} x^d x^e x^f x^g R_{ahcd} \nabla_e R_{bfgh} - \frac{1}{90} x^d x^e x^f x^g R_{cdeh} \nabla_a R_{bfgh} - \frac{1}{90} x^d x^e x^f x^g R_{cdeh} \nabla_f R_{ahbg} + \frac{1}{60} x^d x^e x^f x^g \nabla_{ade} R_{bfcg}
\end{aligned}$$

```

# --- raise index {a}

Gamma00:= @(term00):

Gamma01:= @(term01)) g^{a u1}:
@distributed! (%): @eliminate_metric! (%):
tmp:=@(Gamma01) \delta^{a}_{u1}: @distributed! (%): @eliminate_kr! (%): @rename_dummies! (%):
Gamma01:= @(tmp):

Gamma02:= @(term02) g^{a u1}:
@distributed! (%): @eliminate_metric! (%):
tmp:=@(Gamma02) \delta^{a}_{u1}: @distributed! (%): @eliminate_kr! (%): @rename_dummies! (%):
Gamma02:= @(tmp):

Gamma03:= @(term03)) g^{a u1}:
@distributed! (%): @eliminate_metric! (%):
tmp:=@(Gamma03) \delta^{a}_{u1}: @distributed! (%): @eliminate_kr! (%): @rename_dummies! (%):
Gamma03:= @(tmp):

Gamma04:= @(term04)) g^{a u1}:
@distributed! (%): @eliminate_metric! (%):
tmp:=@(Gamma04) \delta^{a}_{u1}: @distributed! (%): @eliminate_kr! (%): @rename_dummies! (%):
Gamma04:= @(tmp):

# =====
# drop the dependence on x^a, this is risky, here are two possible traps

```

```

#
# (i) A_{a} x^a + B_{e} x^e followed by x^a -> 1 would produce an invalid expression
# (ii) setting x^ -> 1 introduces new free indices, but we can't be certain what they might be
# this might be a problem if we later wished to reintroduce the x^a
#
# two solutions (i) don't set x^a -> 1 or (ii) use @prodsort, @rename_dummies to ensure
# all terms have the same indices for each x^a
# I have used {x^a,...}::SortOrder to ensure all the x^a appear to the left of any other objects
# and thus, after @prodsort, @rename_dummies the indices on the x^a can be inferred from the
# known free indices (the free indices will take values from the start of the index set, the
# x^a will follow immediately after). to see this in action, print out the results of the
# above @prodsort, @rename_dummies, @canonicalise commands
# =====

@canonicalise!(Gamma00): @substitute!(Gamma00)(x^{a} -> 1): @sym!(Gamma00){_b,_c}; "Gamma00.del"
@canonicalise!(Gamma01): @substitute!(Gamma01)(x^{a} -> 1): @sym!(Gamma01){_b,_c}; "Gamma01.del"
@canonicalise!(Gamma02): @substitute!(Gamma02)(x^{a} -> 1): @sym!(Gamma02){_b,_c}; "Gamma02.del"
@canonicalise!(Gamma03): @substitute!(Gamma03)(x^{a} -> 1): @sym!(Gamma03){_b,_c}; "Gamma03.del"
@canonicalise!(Gamma04): @substitute!(Gamma04)(x^{a} -> 1): @sym!(Gamma04){_b,_c}; "Gamma04.del"

# =====
# for the printed version we can suppress the explicit symmetry on (bc)
# i.e. print (bc)( R_{abcd} ) rather than (1/2) ( R_{abcd} + R_{acdbd} )
# =====

{u^{a},\nabla_{a b c}{R_{p q r s}},\nabla_{a b}{R_{p q r s}},\nabla_{a}{R_{p q r s}},R_{p q r s}}::SortOrder.

tmp01:=@(Gamma01) u^b u^c: @distribute!(tmp01): @prodsort!(%): @sumsort!(%): @rename_dummies!(%):
tmp02:=@(Gamma02) u^b u^c: @distribute!(tmp02): @prodsort!(%): @sumsort!(%): @rename_dummies!(%):
tmp03:=@(Gamma03) u^b u^c: @distribute!(tmp03): @prodsort!(%): @sumsort!(%): @rename_dummies!(%):
tmp04:=@(Gamma04) u^b u^c: @distribute!(tmp04): @prodsort!(%): @sumsort!(%): @rename_dummies!(%):

@canonicalise!(tmp01): @substitute!(tmp01)(u^{a} -> 1):
@canonicalise!(tmp02): @substitute!(tmp02)(u^{a} -> 1):
@canonicalise!(tmp03): @substitute!(tmp03)(u^{a} -> 1):
@canonicalise!(tmp04): @substitute!(tmp04)(u^{a} -> 1):

@print["\Btag{01}\gamma^{a}_{(bc)d}="~@(tmp01)~"\Etag{01}"];
@print["\Btag{02}\gamma^{a}_{(bc)de}=\killL~@(tmp02)~"\killR\Etag{02}"];

```



```
@print["\Btag{03}\gamma^{a}_{(bc)def}=\killL"~@(tmp03)~"\killR\Etag{03}"];
@print["\Btag{04}\gamma^{a}_{(bc)defg}=\killL"~@(tmp04)~"\killR\Etag{04}"];
```

```
# =====
#   the connection
# =====
```

$$\Gamma_{bc}^a(x) = \gamma_{bcd}^a x^d + \gamma_{bde}^a x^d x^e + \gamma_{bcdef}^a x^d x^e x^f + \gamma_{bcdefg}^a x^d x^e x^f x^g + \mathcal{O}(\epsilon^6)$$

$$\gamma_{(bc)d}^a = \frac{2}{3} R^a{}_{bdc}$$

$$\gamma_{(bc)de}^a = \frac{1}{12} \nabla^a R_{dbec} + \frac{1}{6} \nabla_b R^a{}_{dec} + \frac{1}{3} \nabla_d R^a{}_{bec}$$

$$\gamma_{(bc)def}^a = \frac{1}{20} \nabla^a{}_d R_{ebfc} + \frac{1}{10} \nabla_{db} R^a{}_{efc} + \frac{1}{10} \nabla_{de} R^a{}_{bfc} - \frac{4}{45} R^a{}_{bdg} R_{ecfg} - \frac{2}{45} R^a{}_{dbg} R_{ecfg} + \frac{8}{45} R^a{}_{deg} R_{fbcg} - \frac{2}{45} R^a{}_{gdb} R_{ecfg}$$

$$\begin{aligned} \gamma_{(bc)defg}^a = & \frac{1}{60} \nabla^a{}_{de} R_{fbgc} + \frac{1}{30} \nabla_{deb} R^a{}_{fgc} + \frac{1}{45} \nabla_{def} R^a{}_{bgc} - \frac{1}{45} \nabla^a R_{fbgh} R_{dceh} - \frac{1}{45} \nabla_b R^a{}_{fgh} R_{dceh} + \frac{1}{30} \nabla_b R_{fcgh} R^a{}_{deh} + \frac{4}{45} \nabla_e R^a{}_{fgh} R_{dbch} - \frac{2}{45} \nabla_e R_{fbgh} R^a{}_{cdh} \\ & - \frac{1}{45} \nabla_e R_{fbgh} R^a{}_{dch} - \frac{1}{45} \nabla_e R_{fbgh} R^a{}_{hdc} - \frac{2}{45} \nabla_f R^a{}_{bgh} R_{dceh} - \frac{1}{45} \nabla_f R^a{}_{gbh} R_{dceh} - \frac{1}{45} \nabla_f R^a{}_{hgb} R_{dceh} + \frac{4}{45} \nabla_f R_{gbch} R^a{}_{deh} - \frac{1}{36} \nabla_h R_{fbgc} R^a{}_{deh} \end{aligned}$$

```
# =====  
#   export the connection terms Gamma* (these are *not* the generalised connections)  
# =====  
  
com:="open connection.lib":  
@run(com){"/Users/leo/local/sh/cdbfile"}:  
  
com:="export connection.lib Gamma00.del":  
@run(com){"/Users/leo/local/sh/cdbfile"}:  
  
com:="export connection.lib Gamma01.del":  
@run(com){"/Users/leo/local/sh/cdbfile"}:  
  
com:="export connection.lib Gamma02.del":  
@run(com){"/Users/leo/local/sh/cdbfile"}:  
  
com:="export connection.lib Gamma03.del":  
@run(com){"/Users/leo/local/sh/cdbfile"}:  
  
com:="export connection.lib Gamma04.del":  
@run(com){"/Users/leo/local/sh/cdbfile"}:
```