CHAPTER FOUR

4. NEAR-SURFACE SOIL MOISTURE PROFILE MEASUREMENT

In order to use remote sensing data to make updates of a hydrologic model for estimation of the spatial distribution and temporal variation of soil moisture profiles, using either the hard-update or Kalman-filter assimilation schemes (section 3.3) within an operational system, it is: (i) essential to have knowledge of the soil depth for which the remote sensing observations of near-surface soil moisture content relate (observation depth); and (ii) desirable to have a relationship between the remote sensing observations and the soil moisture profile over this depth. As there are no appropriate passive microwave satellite systems currently available for soil moisture studies, and active microwave satellite systems have a spatial resolution that is more applicable to hydrologic and agricultural applications, these relationships are required for the active microwave observations.

A literature review of observation depth relationships for active microwave remote sensing and an exploratory study indicated the need for work in this area, in order to provide anything better than a "rule of thumb" estimate for the radar observation depth. Thus, a theoretical model for estimation of the observation depth is developed in this chapter.

Other than the regression based approach of Bruckler *et al.* (1988) (section 3.4.1), all empirical and semi-empirical backscattering models (section 2.4.5.2) have been developed for a single soil moisture value in the top 2 to 5 cm soil layer. Furthermore, until recently all theoretical backscattering models (section 2.4.5.3) have been formulated as a function of the dielectric constant of the soil at the air-soil interface, and have not accounted for the effects of volume scattering in the near-surface soil layer. At the present time, the only theoretical model available for active microwave remote sensing, which accounts for volume scattering due to a dielectric profile in the near-surface soil, is the Modified Integral Equation Model of Fung *et al.* (1996).

4.1 THE MODIFIED INTEGRAL EQUATION MODEL

In order to extract information on the near-surface soil moisture profile from remote sensing observations, the Integral Equation Model (IEM) can be applied in conjunction with a modified set of reflection coefficients (section 2.4.5.3), termed the Modified IEM. These modified reflectivities (2.67) take into account volume scattering by the application of a physical dielectric gradient in the theoretical model (Fung *et al.*, 1996). This varying dielectric profile is approximated by an exponential relationship, which increases the relative dielectric constant from 1 in air to some "infinity depth" value $\varepsilon_{r_{s}}$, through a transition rate factor *m* (2.68). Fung *et al.* (1996) have suggested that an appropriate value for *m* is around 12 cm⁻¹, independent of observation frequency.

Figure 4.1 shows the variable dielectric profile used in derivation of the modified reflectivities, for an infinity depth relative dielectric constant of 20, with transition rate factors of 12 cm⁻¹ and 1 cm⁻¹. This figure illustrates the physical meaning of m in the exponential relationship governing the dielectric profile of the Modified IEM. The value for m of 12 cm⁻¹ suggests that the infinity depth dielectric constant value occurs at a depth of around 3 mm.



Figure 4.1: Illustration of the dielectric profile imposed by the Modified IEM for the transition rate factor m of 12 cm⁻¹ proposed by Fung *et al.* (1996) and for m with a value of 1 cm⁻¹.

A variable dielectric profile over a depth of 3 mm limits application of the Modified IEM, as this does not provide any additional information on the nearsurface soil moisture profile than the surface scattering models. Nor does it provide information on the near-surface soil moisture profile that would be of use in the fields of hydrology, agriculture or meteorology. Furthermore, it is believed that the thickness of the soil layer that can significantly effect the backscattering response (radar observation depth) is of the order of a few tenths of the free space wavelength at normal moisture contents (Schmugge, 1985; Engman and Chauhan, 1995; see also section 2.4.6). It has also been noted that the radar observation depth is a function of the soil moisture content, with observation depths being greater for drier soil conditions than for wet soil conditions (Bruckler et al., 1988; Engman and Chauhan, 1995). In addition to soil moisture content and free space wavelength, the radar observation depth has been noted to be a function of incidence angle, wave polarisation, surface roughness and vegetation cover (Arya et al., 1983). Hence, under average soil moisture conditions, one could expect an observation depth of 3 to 6 cm for 1 GHz and 3 to 6 mm for 10 GHz. Thus for low observation frequencies, one may theoretically expect to gain some valuable information about the near-surface soil moisture profile.

4.1.1 A VARIABLE TRANSITION RATE FACTOR

As the transition rate factor in the modified reflection coefficients governs the depth over which a varying dielectric profile is imposed on the theoretical backscattering model, it is proposed that the transition rate factor m should be a function of the radar observation depth. In this way, a varying dielectric profile may be imposed over the radar observation depth. Given that radar observation depth has been noted as a function of observation frequency, soil moisture content, incidence angle and wave polarisation, one would expect the value assigned to the transition rate factor m to also be a function of these factors. This chapter proposes a variation on the Modified IEM, by using a variable transition rate factor to emulate a variable dielectric profile over the radar observation depth.

The new method for evaluating the modified reflection coefficients is as follows: (i) Determine the radar observation depth d as a function of observation frequency, soil moisture content, incidence angle and wave polarisation;

(ii) evaluate from (2.68) an appropriate value of the transition rate factor *m*, such that the exponential relationship for the dielectric profile gives $\varepsilon_r(z) = \varepsilon_{r_{\infty}}$ at z = d; and (iii) assign to $\varepsilon_{r_{\infty}}$ the dielectric constant value at the radar observation depth. The reasoning for the third assumption is that so far as the radar is concerned, it can only "see" as deep as the radar observation depth. Therefore it is unable to differentiate whether the soil moisture (dielectric) profile is constant for deeper depths, or if in fact it has some different profile.

In order to apply this variable transition rate factor, it is necessary to have an estimate of the radar observation depth. Hence, an observation depth model is required. Preliminary investigations using the Modified IEM in conjunction with the proposed variable transition rate factor and an observation depth of $d = \lambda_o/10$, where λ_o is the free space wavelength, were performed. The results of this investigation suggested that a frequency dependent *m* value was feasible, and gave reasonable backscattering simulations when compared with European Microwave Signature Laboratory (EMSL) (section 4.4.1) observations.

4.2 REVIEW OF THEORETICAL RADAR OBSERVATION DEPTH RELATIONSHIPS

The difficulty that arises with implementation of the variable transition rate factor *m* in the Modified IEM is in finding a realistic relationship between the radar observation depth, observation frequency, soil moisture content, wave polarisation and incidence angle. Ulaby *et al.* (1982) have proposed a relationship for radar penetration depth δ_p as a function of observation frequency and soil moisture content, by considering the power of an electromagnetic wave incident upon a soil surface. This relationship defines the penetration depth as the depth in the soil at which the transmitted wave power just below the soil surface has diminished to the proportion 1/e (ie. 37%). By ignoring scattering in the soil medium, Ulaby *et al.* (1982) arrived at the relationship given in (4.1), for mineral soils having $\varepsilon_n''/\varepsilon_n' < 0.1$.

$$\delta_{p} \cong \frac{\lambda_{o} \sqrt{\varepsilon_{r}'}}{2\pi \varepsilon_{r}''} \tag{4.1},$$



Figure 4.2: Comparison of penetration depth as defined by Ulaby *et al.* (1982) for moisture contents of 5% v/v (dash-dot line) and 40% v/v (dashed line) with the empirical relationship of $d = \lambda_c/10$ (solid line).

where λ_o is the free space wavelength, ε_r' is the real component of the near-surface soil relative dielectric constant and ε_r'' is the imaginary component of the nearsurface soil relative dielectric constant. This relationship has been commonly used to approximate the radar observation depth.

Ulaby *et al.* (1982) note that the values of δ_p given by (4.1) are somewhat larger than those for real soil conditions, as the formulation does not take into account losses due to scattering in the soil medium. Equation (4.1) and the empirical relationship of $d = \lambda_q/10$ are compared in Figure 4.2 for both dry and wet soil. Figure 4.2 shows that the relationship proposed by Ulaby *et al.* (1982) gives penetration depths significantly greater than the reported observation depth of a few tenths of a wavelength, apart from the case of a wet soil. Therefore, either the losses due to scattering in the soil are significant and need to be included in the formulation, or the *penetration depth* δ_p as defined by Ulaby *et al.* (1982) is not equivalent to the *observation depth d.* It is felt that the latter is most likely the correct interpretation, given that the penetration depth is defined as the depth at which an incident wave has reduced to 36% of its power. If the incident wave has lost 74% of its power in reaching this depth, then it is unlikely that it will have sufficient power to reach the surface again, let alone influence the backscattering wave by a significant amount. Hence, (4.1) is not applicable for estimating the radar observation depth.

An alternative method for evaluating penetration depth is given by Bruckler *et al.* (1988), which uses the same power loss criterion as Ulaby *et al.* (1982). However, in this formulation the soil is treated as a stratified profile, with transmission and reflection at each layer interface, and amplitude attenuation throughout each layer. The result of using this model is essentially the same as using the penetration depth model of Ulaby *et al.* (1982), except that it accounts for variation in the dielectric profile.

The only other attempt to relate radar observation depth to observation frequency and soil moisture content that can be found in literature is given by Troch *et al.* (1996). In this case, the backscattering signal received by the radar antenna was considered as a superposition of a surface scattering term and a volume scattering term. The basis for this hypothesis was that comparison between multi-frequency observations and the standard IEM did not explain the observed oscillations (see Figure 2.20). These oscillations were believed by Troch *et al.* (1996) to be caused by surface scattering and volume scattering waves moving into and out of phase. Therefore the observation depth d was determined through the phase shift between the surface and volume scattered waves.

The phase shift ϕ due to a wave travelling through a different medium is given as a function of frequency *f* and incidence angle ϑ_i through the relationship

$$\phi = 4\pi R \frac{f}{c} \tag{4.2},$$

where *c* is the propagation velocity of an electromagnetic wave in the medium, and $R = d/\cos \vartheta_i$. By letting the phase difference $(\phi_1 - \phi_2)$ equal 2π , and using the relationship that $c = c_o /\sqrt{\varepsilon_r}$, where c_o is the propagation velocity in a vacuum $(2.997925 \times 10^{10} \text{ cm s}^{-1})$ and ε_r is the relative dielectric constant of the medium, Troch *et al.* (1996) arrived at the relationship

$$\Delta f = \frac{c_o \cos \vartheta_i}{2d\sqrt{\varepsilon_r}} \tag{4.3}.$$



Figure 4.3: Comparison of the penetration depth (dashed line) as defined by Ulaby *et al.* (1982) with the observation depth (solid line) as determined by Troch *et al.* (1996) (Troch *et al.*, 1996).

The observation depth d in (4.3) was then evaluated by Troch *et al.* (1996) through evaluation of Δf from a Hilbert transformation of the difference in backscattering predicted by the standard surface scattering IEM and observed EMSL data. The radar observation depth results from this analysis are compared against the penetration depth equation of Ulaby *et al.* (1982) in Figure 4.3. This comparison shows the observation depth as determined by the methodology of Troch *et al.* (1996) to be significantly deeper than that predicted by the penetration depth equation of Ulaby *et al.* (1982), especially in the low frequency region. It is therefore concluded that the theoretical approach of Troch *et al.* (1996) is also inconsistent with the experimental values noted in literature.

An explanation for the large observation depths predicted by Troch *et al.* (1996) might be that the observed oscillations in the EMSL data were not caused entirely by the interaction of surface and volume scattering waves.

Mancini and Rosso (1997) have presented multi-frequency backscattering results of surfaces both with and without a metallic coating, so that the effect of volumetric scattering on backscattering could be investigated. The results of Mancini and Rosso (1997) indicate that surface scattering alone also contains oscillations. Hence, only a part of the observed oscillations is likely to be due to volume scattering. If these oscillations from scattering at the soil surface could be taken into account in evaluating the phase change, then the observation depth evaluated using this approach would most likely be significantly reduced. Furthermore, the assumption made in this method for evaluating the phase change is that volume scattering only occurs from the deepest dielectric discontinuity that can return a detectable signal to the surface. However, dielectric discontinuities are suggested by Ulaby *et al.* (1982) to be randomly distributed within the soil, and hence volume scattering would be expected from all depths down to the radar observation depth. Thus, the observed phase change would be the integration of volume scattered waves from all depths, with the surface scattered wave.

4.3 A NEW OBSERVATION DEPTH RELATIONSHIP

As the theoretical approaches presented in the literature for evaluating the radar observation depth do not give comparable results with published values from experimental work (eg. Bruckler and Witono, 1989), an alternative radar observation depth relationship was required. Two alternative radar observation depth relationships are presented below.

The total backscattering coefficient from the soil may be considered as a surface reflection summed with a sub-surface reflection(s) that is altered in phase and attenuated in amplitude due to the thickness of the radar observation depth layer and the dielectric properties of that layer (Sadeghi *et al.*, 1984). Hence, the first radar observation depth model proposes to estimate the radar observation depth proposes to estimate the radar observation depth proposes to estimate the radar observation depth proposes to estimate the radar observation.

4.3.1 A SEMI-EMPIRICAL PHASE CHANGE MODEL

This semi-empirical phase change radar observation depth model considers the soil as a stratified dielectric medium, consisting of a two-layer soil profile as shown in Figure 4.4. A similar two-layer model has been used by Sadeghi *et al.* (1984). The two soil layers are considered to be homogeneous



Figure 4.4: Schematic representation of the phase change for volume and surface scattering waves from a soil having: a) a drying profile; and b) a wetting profile.

layers with different dielectric properties, resulting in a dielectric discontinuity at the interface between the upper and lower soil layers. The electromagnetic field in the air layer consists of the incident component, a reflected component due to the air-soil interface, and a reflected component due to the additional interface between the two soil layers. The dielectric discontinuity in the soil profile is assumed to occur at the radar observation depth.

The analogy considered by this model is that of a glass plate coated with a thin transparent medium of lower refractive index, for reducing unwanted reflections. The laws of optical physics for a light wave incident on a thin film may be summarised as follows: "When reflection occurs from an interface beyond which the medium has a lower index of refraction, the reflected wave undergoes no phase change; when the medium beyond the interface has a higher index, there is a phase change of π . The transmitted wave does not experience a change of phase in either case" (Halliday and Resnick, 1978).

The analogy between the optical situation and that of a stratified soil lies in the relationship between a material's refractive index n and its relative dielectric constant ε_r through $n = \sqrt{(\mu_r \varepsilon_r)} \approx \sqrt{\varepsilon_r}$, for soil. Although Figure 4.4 indicates homogeneous specular reflecting surfaces at the layer interfaces, the phase changes indicated should be the same for backscattering from diffuse reflecting surfaces. Ignoring for the moment the effect of the dielectric constant of the underlying medium on the phase change of a reflected wave, the phase change ϕ of the transmitted wave exiting at the soil surface may be expressed by (4.2), with $R = d/\cos \vartheta_s$, where ϑ_s is the soil transmission angle rather than the incidence angle, and $c = c_o /\sqrt{\varepsilon_r}$ as before. Rearranging (4.2) in terms of d yields the following expression for radar observation depth in terms of the phase change ϕ ,

caused by the wave passing through a medium of different dielectric value,

$$d = \phi \, \frac{c_o}{f} \, \frac{\cos \vartheta_s}{4\pi \sqrt{\varepsilon_r}} \tag{4.4}.$$

Using Snell's law for refraction at a plane surface, the transmission angle of the soil may be evaluated in terms of the incidence angle ϑ_i and the relative dielectric constant of the soil ε_r by the relationship

$$\vartheta_s = \arcsin\left[\frac{\sin\vartheta_i}{\sqrt{\varepsilon_r}}\right]$$
(4.5).

As the dielectric constant is a complex number, the transmission angle may be approximated from (4.5) by using only the real part of the dielectric constant for low loss media such that $\varepsilon_r //\varepsilon_r << 1$. However, if these conditions are not satisfied, the real angle of transmission should be found using (4.10) (Ulaby *et al.*, 1981).

In order to evaluate the radar observation depth from (4.4), knowledge of the phase change ϕ caused solely by a change of wavespeed when travelling through the soil is required. As the relative dielectric constant of the soil is always greater than that of air ($\varepsilon_{r_{air}} = 1$), the surface scattering wave will always undergo a phase change of π upon reflection. However, the phase change of the volume scattering wave upon reflection at the dielectric discontinuity is dependent on the soil dielectric properties. Therefore, for a wetting profile there will be no phase change upon reflection at the dielectric discontinuity, as the underlying soil will have a lower dielectric value (refractive index) than that of the near-surface soil layer. For a drying profile, the reverse situation exists and yields a phase change of π (see Figure 4.4).

To summarise the above discussion, the *phase difference* (phase change caused by the dielectric discontinuity plus the phase change caused by the wave passing through a medium of different dielectric value) between the surface scattered and volume scattered waves of a drying profile will be approximately ϕ , while for a wetting profile will be approximately $\phi-\pi$. Thus for maximum amplitude amplification to occur between the surface scattered and volume scattered waves under consideration, a phase difference of 2π , 4π , 6π ... is required, while for maximum amplitude reduction, a phase difference of π , 3π , 5π ... is necessary. Therefore, if we were to have some knowledge of the phase difference between the surface scattered wave and the volume scattered wave from the dielectric discontinuity at the radar observation depth (ie. amplitude amplification or amplitude reduction), a value for the phase change of the volume scattered wave may be obtained for evaluation of the observation depth.

Comparisons of the observation depth relationship in (4.4) are made with the empirical relationship of $d = \lambda_0/10$ in Figure 4.5 and Figure 4.6, for ϕ equal to



Figure 4.5: Comparison of observation depth as evaluated from the phase change model for $\phi = 2\pi$ at soil moisture contents of 5% v/v (open symbols) and 40% v/v (closed symbols) at incidence angles of 11° (circles), 23° (squares) and 35° (triangles), with the empirical relationship of $d = \lambda_o/10$ (crosses).



Figure 4.6: Comparison of observation depth as evaluated from the phase change model for $\phi = \pi$ at soil moisture contents of 5% v/v (open symbols) and 40% v/v (closed symbols) at incidence angles of 11° (circles), 23° (squares) and 35° (triangles), with the empirical relationship of $d = \lambda_d/10$ (crosses).

 2π and π respectively. Figure 4.5 indicates an over-estimation of the observation depth for the dry soil condition when compared with the empirical relationship, while Figure 4.6 indicates an under-estimation for the wet soil condition. This would perhaps suggest that destructive interference (amplitude reduction as a result of wave addition) occurs between the surface scattered wave and the volume scattered wave from the observation depth. Hence, a value of $\phi = \pi$ for drying profiles and $\phi = 2\pi$ for wetting profiles may be a reasonable assumption. This is opposite to that suggested for passive microwave, which has been proposed to result in constructive interference (amplitude amplification as a result of wave addition) as a result of the surface layer of soil behaving as a thin film (Choudhury *et al.*, 1979; Schmugge and Choudhury, 1981). It may also be observed from Figure 4.5 and Figure 4.6 that the observation depth given by this relationship is relatively insensitive to the incidence angle.

This semi-empirical phase change model has many weaknesses, as dielectric discontinuities are located randomly within the soil (Ulaby *et al.*, 1982). Thus, the resulting backscattered wave is the summation of the surface scattered wave, and volume scattered waves from dielectric discontinuities at various depths between the soil surface and the radar observation depth. In addition, the

volume scattered waves include multiple scattering within the soil volume, and are attenuated in wave amplitude as a function of distance travelled in the soil. Thus, the volume scattered wave from the dielectric discontinuity at the radar observation depth would only have a small wave amplitude in comparison to volume scattered waves from other depths, and would thus have the least influence on the total backscattering. Hence, the only way in which this model can be used is to consider the total volume scattering as an effective volume scattered wave, which has an effective phase change.

Given that there is no theoretical basis for suggesting destructive interference occurs between the effective volume scattered wave that combines with the surface scattered wave, the effective phase change ϕ must be determined empirically for all soil moisture conditions and observation frequencies. A possible way of empirically evaluating ϕ may be to undertake multi-frequency radar observations of a soil at different soil moisture contents, with a metallic plate buried at varying depth.

4.3.2 A THEORETICAL AMPLITUDE ATTENUATION MODEL

Given the inherent weaknesses in the above phase change radar observation depth model, through imposing the observation depth by an effective phase change, an alternative radar observation depth relationship was investigated. This second radar observation depth model proposed, is a theoretical model that compares the amplitude of a volume scattered wave with that of the surface scattered wave.

The theoretical basis for this observation depth model, is that when the amplitude of a volume scattered wave E_v falls below some proportion of the surface scattered wave E_s , it is no longer making a detectable contribution to the total backscattering of the dielectric medium. Hence, the maximum depth from which a volume scattered wave is returned to the surface, such that it just satisfies the limit of E_v/E_s may be considered as the radar observation depth. In order to apply this philosophy, a method for estimating the amplitude of the surface scattered and volume scattered waves is required. This problem can be considered in two simplistic ways, as illustrated in Figure 4.7.



Figure 4.7: Schematic representation of incident, reflected and transmitted waves in: a) a single soil layer of varying thickness; and b) multiple soil layers of constant thickness.

The representation shown in Figure 4.7a, again treats the soil as a stratified dielectric medium consisting of two dielectric layers, with the intermediate boundary representing a dielectric discontinuity in the soil. In this case, the amplitude of the surface scattered and transmitted waves are evaluated at the air-soil interface. The transmitted wave then passes through the near-surface soil layer until it reaches a dielectric discontinuity in the soil with attenuated amplitude. Upon reaching this dielectric discontinuity, the wave is again transmitted and reflected. The reflected wave then passes back through the near-surface soil layer until it reaches the air-soil interface with further amplitude attenuation. At the air-soil interface, the wave undergoes reflection and transmission for a third time. The amplitude of this transmitted (volume scattered) wave, and the layer thickness incrementally increased until the appropriate ratio between the surface scattered and volume scattered waves is achieved.

The alternative approach, illustrated in Figure 4.7b, is to discretise the soil profile into a number of thin layers. This was the approach used by Bruckler *et al.* (1988) for evaluating the penetration depth. In this instance, the path of the volume scattered wave under consideration is taken firstly for the surface layer as described above. If the ratio between the volume scattered wave and the surface scattered wave is considered too large, the wave is allowed to proceed through the second layer before reflecting back to the surface. The amplitudes of the surface and volume scattered waves are again compared, and the number of layers increased until the limit of detection is reached.

In order to evaluate the amplitudes of the surface scattered and volume scattered waves, the following relationships from Ulaby *et al.* (1981) can be used, where the phasor form of the incident (\mathbf{E}_i) , reflected (\mathbf{E}_r) and transmitted (\mathbf{E}_i) waves for horizontal polarisation are,

$$\mathbf{E}_{i} = \hat{\mathbf{y}} E_{i} \exp\left[-i(k_{x1}x - k_{z1}z)\right]$$
(4.6a)

$$\mathbf{E}_{r} = \hat{\mathbf{y}}R_{h}E_{i}\exp\left[-i(k_{x1}x + k_{z1}z)\right]$$
(4.6b)

$$\mathbf{E}_{t} = \hat{\mathbf{y}} T_{h} E_{i} \exp[-i(k_{x2}x - k_{z2}z)]$$
(4.6c).

These equations describe both the amplitude and phase of the incident, reflected and transmitted waves respectively, with the coefficient representing the amplitude, and the exponent representing the phase. In the following text, the subscript 1 refers to the *incident layer* and the subscript 2 refers to the *transmitted layer* (as indicated in Figure 4.7).

For vertical polarisation the phasor forms of the waves are identical to those above, except that the horizontal reflection and transmission coefficients (R_h and T_h) are replaced by the vertical reflection and transmission coefficients (R_v and T_v) respectively. The reflection and transmission coefficients are

$$R_{h} = \frac{\eta_{2} \cos \vartheta_{1} - \eta_{1} \cos \vartheta_{2}}{\eta_{2} \cos \vartheta_{1} + \eta_{1} \cos \vartheta_{2}}$$
(4.7a)

$$R_{\nu} = \frac{\eta_1 \cos \vartheta_1 - \eta_2 \cos \vartheta_2}{\eta_1 \cos \vartheta_1 + \eta_2 \cos \vartheta_2}$$
(4.7b)

$$T_{h} = \frac{2\eta_{2}\cos\vartheta_{1}}{\eta_{2}\cos\vartheta_{1} + \eta_{1}\cos\vartheta_{2}}$$
(4.7c)

$$T_{v} = \frac{2\eta_{1}\cos\vartheta_{1}}{\eta_{1}\cos\vartheta_{1} + \eta_{2}\cos\vartheta_{2}}$$
(4.7d),

where

$$1 + R = T \tag{4.8},$$

and the intrinsic impedance η is given by

$$\eta_1 = \eta_o \sqrt{\frac{\mu_{r_1}}{\varepsilon_{r_1}}} \tag{4.9a}$$

$$\eta_2 = \eta_o \sqrt{\frac{\mu_{r_2}}{\varepsilon_{r_2}}}$$
(4.9b),

with $\eta_o = 376.7 \ \Omega$ being the impedance of free space, $\mu_r \approx 1$ for air and soil and $\varepsilon_r \approx 1+i0$ for air.

In evaluating the reflection and transmission coefficients, η_1 and $\cos \vartheta_1$ are taken to be real, so that the law for conservation of energy is maintained. The reason for this is that the imaginary component relates to the losses and any loss of energy in the incident wave before it reaches the boundary does not enter into the reflection or transmission (Ulaby *et al.*, 1981).

As the transmitted wave is defracted upon transmission from one medium to that of another medium having different refractive index, the refraction angle in the transmission layer may be evaluated from Snell's law. However, as the dielectric constant of the transmission layer is complex, the problem of finding the transmission angle is more involved. After Ulaby *et al.* (1981), the *real angle of transmission* is

$$\tan \vartheta_{2} = \frac{\sqrt{2}k_{1}\sin\vartheta_{1}}{\sqrt{\left(p^{2}+q^{2}\right)^{\frac{1}{2}}+q}}$$
(4.10),

$$p = 2\alpha\beta \tag{4.11a}$$

$$q = \beta^2 - \alpha^2 - k_1^2 \sin \vartheta \tag{4.11b},$$

with α being the attenuation constant and β being the phase constant given by

$$\alpha = k_2 \left[\frac{1}{2} \left(\sqrt{1 + \tan^2 \delta} - 1 \right) \right]^{\frac{1}{2}}$$
(4.12a)

$$\beta = k_2 \left[\frac{1}{2} \left(\sqrt{1 + \tan^2 \delta} + 1 \right) \right]^{\frac{1}{2}}$$
(4.12b),

where $\tan \delta = \frac{\varepsilon_{r_2}''}{\varepsilon_{r_2}'}$ is known as the loss tangent, and *k* is the wave number. The

wave number is

$$k_1 = \frac{2\pi f}{c_o} \sqrt{\mu_{r_1} \varepsilon_{r_1}}$$
(4.13a)

$$k_2 = \frac{2\pi f}{c_a} \sqrt{\mu_{r_2} \varepsilon_{r_2}}$$
(4.13b),

given that $c_o = 2.997925 \times 10^{10}$ cm s⁻¹ is the propagation velocity of an electromagnetic wave in a vacuum and *f* is the observation frequency in Hz.

The amplitude attenuation factor a for the transmission layer may then be evaluated as

$$a_{2} = \exp\left\{\frac{1}{\sqrt{2}}\left[\left(p^{2} + q^{2}\right)^{\frac{1}{2}} - q\right]^{\frac{1}{2}}.(-dz)\right\}$$
(4.14),

where dz is the layer thickness.

Thus for an incident wave amplitude of E_{p} the reflected and transmitted wave amplitudes immediately at the layer interface are given by

$$E_r = \left| R \right| E_i \tag{4.15a}$$

$$E_t = |T| E_i \tag{4.15b},$$

where R and T are the reflection and transmission coefficients depending on polarisation. The attenuated wave amplitudes for the reflected and transmitted waves after passing through the layer are evaluated by

$$E_r' = a_1 \cdot E_r \tag{4.16a}$$

$$E_t' = a_2 \cdot E_t$$
 (4.16b).

This process is explained a little more clearly by reference to the single layer illustration given in Figure 4.8.

A major weakness of this theoretical observation depth model is the assumption of homogeneous specular reflecting surfaces at the layer interfaces. However, Ulaby *et al.* (1982) notes that volume scattering is caused mainly by dielectric discontinuities within a volume, and that the spatial locations of



Figure 4.8: Schematic illustration of the single layer radar observation depth model.

discontinuities are generally random. Thus the scattered waves within the soil volume may be expected to be in all directions. Therefore, if the soil surface is considered as lambertian (ie. scattering occurs equally in all directions), evaluation of the radar observation depth using the amplitude attenuation model with specular reflecting surfaces will yield the same results. The effect of smoother soil surfaces is to increase the observation depth, as the proportion of surface scattering in the backscattering direction is less, and hence a smaller contribution from volume scattering will still yield a significant (but not necessarily detectable) influence on the total backscattering from the soil. In the case of a specular reflecting surface at the air-soil interface, backscattering will be entirely from volume scattering. In this case, the amplitude of the volume scattered wave from the dielectric discontinuity at the radar observation depth is compared with the amplitude of the volume scattered wave from a dielectric discontinuity just below the air-soil interface.

In estimating the region of validity for the assumption of a lambertian soil surface, the following relationships given by Ulaby *et al.* (1982) may be used as a guide. This is not to say that the observation depth model proposed cannot be used for other surface roughness conditions, just that observation depth may be underpredicted for smoother surfaces. The advice given by Ulaby *et al.* (1982) is that a surface may be considered as smooth (ie. specular) if it has $k_o \sigma < 0.2$ and as very rough (ie. lambertian) if it has $k_o \sigma > 1$, where k_o is the free space wave number given by $k_o = 2\pi/\lambda_o$, λ_o is the free space wavelength and σ is the rms height of surface variations (see also section 2.4.5). These relationships are illustrated in Figure 4.9.

The difficulty associated with evaluation of this theoretical model for radar observation depth is in deciding: (i) whether to use a single or multi-layer approach; (ii) what limit to place on the ratio of volume scattered wave amplitude to surface scattered wave amplitude; and (iii) in the case of the multi-layer approach, what layer thicknesses to use.

Given that volume scattering is considered to be governed by the presence of dielectric discontinuities in the soil (Ulaby *et al.*, 1982), the single layer approach appears to be more defensible than the multi-layer approach. In this



Figure 4.9: Regions for considering surfaces as specular, diffuse or lambertian with respect to observation frequency and root mean square height of surface variations.

instance the layer thickness represents the depth to the deepest dielectric discontinuity that can return a volume scattered wave having an amplitude large enough to make a detectable influence on the total backscattering coefficient. Thus, the remaining question is what limit to place on the ratio of volume scattering wave amplitude to surface scattering wave amplitude.

This question can be answered by considering the sensitivity of backscattering coefficient to soil moisture content and to the sensor calibration accuracy, through an error analysis of the backscattering equation (Fung, 1994)

$$\sigma_{dB}^{o} = 10 \log_{10} \left[4\pi \cos \vartheta_i \left(\frac{E_R^2}{E_i^2} \right) \right]$$
(4.17),

where σ_{dB}^{o} is the backscattering coefficient in dB, ϑ_i is the incidence angle (degrees), E_i is the incident wave amplitude, $E_R = E_s + E_v$ is the returned wave amplitude, E_s is the surface scattered wave amplitude and E_v is the volume scattered wave amplitude. Differentiation of the backscattering equation with respect to the returned wave amplitude, yields the relationship

$$\frac{\partial \sigma_{dB}^{\circ}}{\partial E_{R}} = \frac{20}{E_{R} \log_{e} [10]}$$
(4.18).

Given that surface scattering is generally considered to govern the amplitude of the backscattered wave (Ulaby *et al.*, 1978), it may be assumed that $E_R \approx E_s$. Hence, any error in E_R is due to neglecting E_v , and therefore $\partial E_R \approx E_v$. Thus, the following relationship between the ratio of volume scattering to surface scattering and backscattering coefficient sensitivity is obtained.

$$\frac{E_v}{E_s} = \partial \sigma_{dB}^o \frac{\log_e [10]}{20} \tag{4.19}$$

In order to evaluate the required ratio between volume and surface scattering from (4.19), a value for the backscattering coefficient sensitivity is required. As the radar observation depth model is based on the limit at which the effect from volume scattered waves on the total backscattering can no longer be detected, this backscattering sensitivity $\partial \sigma_{dB}^{o}$ may be imposed through either the sensitivity of backscattering to soil moisture content, or through the calibration accuracy of the sensor. In the first instance, Altese et al. (1996) indicate a 1 dB change in backscattering for a 2% change in volumetric soil moisture content at low soil moisture content, and a 0.15 dB change in backscattering coefficient for a 2% change in volumetric moisture content at high soil moisture content. For the latter case, literature would suggest an absolute radiometric calibration accuracy of between 1 and 2 dB (Ulander et al., 1991; van Zyl et al., 1993; Zink et al., 1993; Dubois et al., 1995a; Sarabundi et al., 1995). Therefore, calibration accuracy would appear to govern the observable influence of volume scattering on surface scattering. In the event that calibration accuracy was to exceed the sensitivity of soil moisture content variation on backscatter, sensitivity to soil moisture content would be the controlling variable. Hence, a backscattering sensitivity of about 1.5 dB would appear appropriate, which yields a ratio of volume to surface scattering of 0.17.

4.4 APPLICATION OF THE MODELS

Both the theoretical radar observation depth model (based on amplitude attenuation) and the Modified IEM with a variable transition rate factor were tested using data collected in the EMSL experiments.

4.4.1 EUROPEAN MICROWAVE SIGNATURE LABORATORY

The aims of experiments undertaken at the EMSL, Ispra, Italy, include the evaluation of surface scattering models and estimation of soil moisture profiles using multi-frequency polarimetric data for different types of soils and surface roughness (Nesti *et al.*, 1995). The objective of the experiment was to generate a number of data sets under controlled conditions. The data sets consist of soil moisture and temperature profiles in a sandy soil using TDR probes and thermocouples, in combination with mono-static (send and receive the signal from the same location), multi-frequency (1 to 10 GHz), polarimetric (*hh*, *hv*, *vh*, *vv*) radar measurements at three incidence angles (11°, 23° and 35°) (Mancini *et al.*, 1995).

The sample under test was contained in a 2 m diameter cylinder of 0.4 m depth, placed in the centre of a 20 m diameter temperature controlled measurement chamber (Figure 4.10). Soil moisture profiles were generated with a series of irrigation and drying out steps. Four irrigation steps were applied to increase the near-surface soil moisture content from 10% v/v to 35% v/v. The total water discharge was monitored by an electromagnetic flow meter, and the net volume of water applied was calculated by subtracting the measured runoff. For drying, 35 ceramic infra-red resistances were used, each having a 25 cm \times 10 cm radiation area. The radiators were operated at a distance of 20 cm from the soil and had a total power output of 14 kW (Mancini and Troch, 1995). Soil moisture measurements were made at depths of 2.5, 5, 10, 15, and 25 cm and soil temperature (Mancini *et al.*, 1995). The surface of the soil target was shaped by means of a mould with specific roughness characteristics. Three types of surface roughness were planned (Coppo *et al.*, 1995):



Figure 4.10: Exploded view of the EMSL (http://www.ei.it/landmines/landmines/sai/AT1.html).

- Relatively smooth, Gaussian correlation function 60 mm correlation length, rms height 4 mm.
- Medium rough, mixed correlation function 30 mm correlation length, rms height 9 mm.
- iii) Very rough, Gaussian correlation function 60 mm correlation length, rms height 25 mm.

At the time of using this data, experiments for only the first and third surface roughness had been undertaken.

The soil was filled into the cylinder in a series of layers and compressed to achieve a homogeneous bulk density of $\rho_b = 1.3 \text{ g cm}^{-3}$. The average soil porosity for the smooth surface was $\phi = 0.570 \text{ v/v}$ and for the rough surface $\phi = 0.588 \text{ v/v}$ (Wütherich, 1997). The soil textural composition of the soil used in the two experiments is given in Table 4.1.

For the first type of surface roughness, backscattering observations were made for frequencies in the range from 1.0 to 10.0 GHz in steps of 11.25 MHz.

Surface	%	% Silt (g/g) 2-50 μm	% Sand (g/g) > 50 μm
Smooth	8.5	27.7	63.8
Very Rough	4.3	36.0	59.8

Table 4.1: Textural composition of the soil used in EMSL experiments 1 and 3.

For the third type of surface roughness, backscattering observations were also made in the range of 0.5 to 5.0 GHz (Mancini *et al.*, 1996).

Calibration of the radar used at the EMSL was evaluated as a two-fold process, namely evaluation of an additive part and evaluation of a multiplicative part. The additive part was evaluated by performing an empty room measurement (a measurement in the same conditions and with the same measurement parameters but without the target) and eliminated by subtraction from the target measurement. In contrast, the multiplicative part was evaluated by making use of the measured scattering matrix of a reference object, whose theoretical response was known (ie. a metallic disc or sphere) (Nesti *et al.*, 1994).

The data used in this evaluation was that from a drying out step (average soil moisture content of 9% v/v), as the Modified IEM is only valid for a drying profile. Moreover, data collected from the relatively smooth surface experiment has not been used as it acts as a specular surface for the frequency range of 1 to 2 GHz and a diffuse surface from 2 to 10 GHz (see Figure 4.9). However, data collected from the very rough surface experiment has been used as it corresponds with the assumption of the radar observation depth model that the surface is lambertian. It may be seen from Figure 4.9 that the very rough surface should act as a diffuse surface for the frequency range of 0.5 to 2 GHz and as a lambertian for frequencies higher than 5 GHz, validity conditions of the IEM are violated (see section 2.4.5.2.2). Hence simulations given here are only for the frequency range of 0.5 to 5 GHz.

4.4.2 EVALUATION OF THE RADAR OBSERVATION DEPTH MODEL

Evaluation of the theoretical radar observation depth model (based on amplitude attenuation) has been performed by comparing the observation depth estimated from the theoretical model, with the radar observation depth range suggested in literature. In evaluating the radar observation depth, an exponential drying profile was fitted to the observed soil dielectric constant profile. The dielectric constant for the near-surface soil layer was taken as the dielectric constant at mid-layer depth and the dielectric constant for the deep soil layer was taken as the dielectric constant at half the near-surface soil layer depth below the dielectric discontinuity.

The graphs of radar observation depth given in Figure 4.11 indicate that the theoretical amplitude attenuation model provides an estimate of the radar observation depth that is in the range of values reported in literature, having a value of slightly less than one-tenth of the free space wavelength at low frequencies and slightly greater than one-quarter of the free space wavelength at higher frequencies. Although the literature suggests that the radar observation depth is between one-tenth and one-quarter of the free space wavelength, it is not necessarily a constant proportion of the wavelength for a given sensor configuration and soil condition. In fact, soil moisture generally increases with depth for a drying profile. Hence, as the observation depth is increased the effective soil moisture content is also increased, thus the observed decrease in proportion of wavelength at low frequencies. Furthermore, an average soil moisture content of 9% v/v is rather dry, and under an exponential drying profile would be very dry near the soil surface. Thus, radar observation depths presented here, particularly for the higher observation frequencies, should be close to the maximum radar observation depth.

The observation depth results also indicate that vv polarisation yields a greater observation depth than *hh* polarisation for the same surface roughness, sensor configuration and soil moisture content. It can also be seen that for *hh* polarisation, radar observation depth decreased with increasing incidence angle, while for vv polarisation the radar observation depth increased for an increase in incidence angle. This would indicate that using a sensor with vv polarisation and a



Figure 4.11: Radar observation depths calculated from the theoretical amplitude attenuation model for a drying step of the very rough surface EMSL experiment.

large incidence angle would maximise the radar observation depth for a given observation frequency.

While evaluation of the radar observation depth using the amplitude attenuation model requires an estimate of the near-surface soil moisture profile, this is not considered to be a major limitation to application. In the situation that profile soil moisture is being modelled by a hydrologic model and backscattering observations are for updating of the hydrologic model, then the radar observation depth may be estimated from the hydrologic model estimate of soil moisture profile. Alternatively, the radar observation depth model and Modified IEM may be used simultaneously to retrieve both the near-surface soil moisture profile and the radar observation depth.

4.4.3 SIMULATIONS USING THE MODIFIED INTEGRAL EQUATION MODEL

Backscattering simulations using the Modified IEM with a frequency dependent transition rate factor have been evaluated, using the radar observation depth estimate obtained with the theoretical amplitude attenuation model. These simulations are compared with simulations from the Modified IEM for m equal to 12 cm⁻¹, the standard IEM, and EMSL data (Figure 4.12).

The simulations given in Figure 4.12 using the Modified IEM with a variable transition rate factor show good agreement with the EMSL data. Furthermore, the simulations are better than those using the standard IEM and Modified IEM with *m* equal to 12 cm^{-1} in some instances.

Volume scattering is generally seen as an addition to surface scattering, the sum of both being higher or nearly equal (if volume scattering is low compared to surface scattering) to surface scattering alone. This does not appear to be the case in the simulations presented here. A reason for this may be that while volume scattering is an addition to surface scattering, it depends on the phase of both the surface scattered and volume scattered waves as to whether this results in an increased or decreased wave amplitude. Furthermore, simulations for the standard IEM and Modified IEM with m equal to 12 cm⁻¹ were for the dielectric constant measured at 2.5 cm depth (as is the standard procedure). As the backscattering coefficient increases with moisture content and simulations were for a drying profile, the use of a greater dielectric constant in the surface scattering model results in a greater backscattering coefficient than the combined surface and volume scattering model with a lower dielectric constant at the air-soil interface. This is because surface scattering generally dominates the total backscattering from the dielectric medium (Ulaby et al., 1978). The good agreement between the three models at low frequency is likely to be a result of the Modified IEM with variable transition rate factor having a greater contribution



Figure 4.12: Comparison of backscattering simulations from the Modified IEM with variable transition rate factor against the Modified IEM with transition rate factor m equal to 12 cm⁻¹, standard IEM and EMSL data from a drying step of the very rough surface experiment.

from volume scattering and a greater infinity depth dielectric constant than for the higher frequencies. The poor agreement at 5 GHz for all three backscattering models at incidence angles of 11° and 23° is likely to be a result of the IEM validity conditions being too loose.

A limitation of the Modified IEM is that it is only applicable to nearsurface drying (dielectric) profiles that conform to the exponential model in (2.68). Furthermore, the exponential model used to describe the drying (dielectric) profile imposes a transition in the dielectric profile of the near-surface air layer (see Figure 4.1), in addition to the near-surface soil layer, whose thickness also increases as a function of the transition rate factor. Hence, as the observation depth is increased, the thickness of this transition layer in the air is also increased.

While the near-surface soil moisture profile assumes a drying profile relatively quickly after a drying event, the imposition of an exponential profile and a transition layer in the near-surface air layer is undesirable. Hence the Modified IEM might be improved by replacing the modified reflection coefficients with reflection coefficients from an approximation to the Riccati equation (see Ulaby *et al.*, 1981, pp. 84). The use of these reflection coefficients would eliminate the transition in the near-surface air layer, and would allow the specification of any dielectric profile desired over the radar observation depth. This would allow application of the Modified IEM to non-drying and non-exponential near-surface soil moisture profiles. In the situation that soil moisture profiles are being modelled and backscattering observations of near-surface soil moisture are for updating of the hydrologic model, the soil dielectric profile shape applicable for evaluation of the Riccati equation may be estimated from the hydrologic model.

4.5 CHAPTER SUMMARY

The amplitude attenuation model developed for estimating the radar observation depth was found to give comparable results to those presented in literature. Furthermore, the radar observation depth was found to be greater for vv polarisation than *hh* polarisation, and was found to increase with increasing incidence angle for vv polarisation. These results would indicate that information on the near-surface soil moisture profile may be maximised by using a low frequency sensor with vv polarisation and a large incidence angle.

The comparisons of backscattering simulations using the Modified IEM with a frequency dependent transition rate factor against EMSL data show good agreement. It may therefore be concluded that a variable transition rate factor is feasible, and that by using the Modified IEM, information may be obtained on the soil moisture profile down to the observation depth, which can be as deep as 5 cm for *vv* polarisation at low frequencies. However, replacement of the modified reflection coefficients with an approximate solution to the Riccati equation should ease some of the restrictive assumptions of the current Modified IEM.