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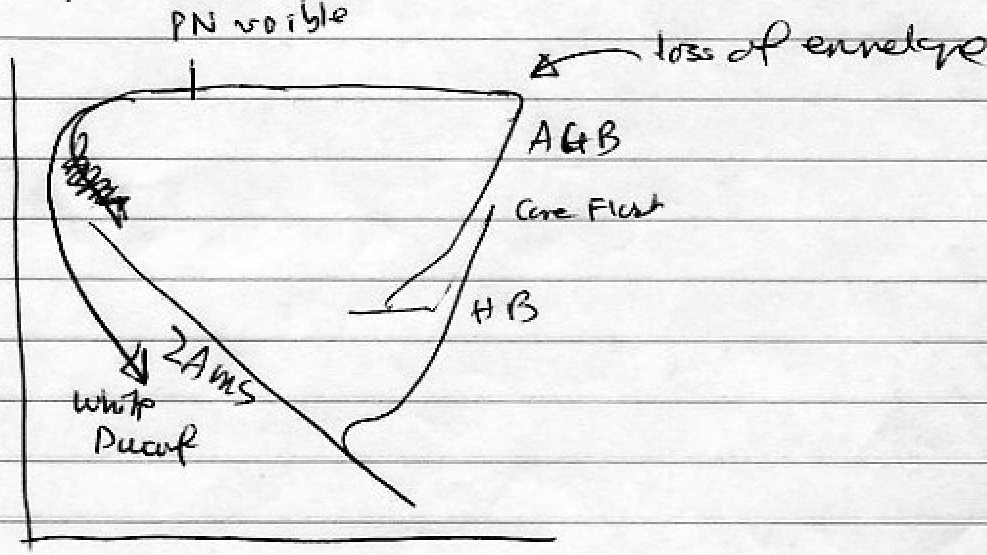
Death of the Sun

As the Sun evolves up the AGB it loses mass from its envelope. The evolution proceeds until there is no envelope left.....



The star then leaves the AGB, and a hot core is revealed. It gets hotter as it is exposed, eventually reaching 50 000 K or so. At this point the UV photons are absorbed by the surrounding gas lost earlier in the AGB evolution... it is then visible as a planetary nebula!!

Increase of Core-Mass

Suppose all energy comes from H shell...

$$L = \frac{\text{energy}}{\text{time}} = \text{rate of H burning} \times \text{energy released}$$

$$= \frac{d}{dt}(X M_c) \times E_H$$

H mass fraction

6.4×10^{18} erg/gm

$$L\left(\frac{\text{erg}}{\text{sec}}\right) = X \cdot E_H \frac{dM_c}{dt} \,(\text{g/s})$$

$X \simeq 0.7$

$$= 4.4 \times 10^{18} \frac{\text{erg}}{\text{gm}} \frac{dM_c}{dt} \,(\text{gm/sec})$$

$$\frac{L}{L_\odot} = \frac{4.4 \times 10^{18}}{3.86 \times 10^{33}} \frac{\cancel{\text{erg}}\ \text{sec}}{\cancel{\text{erg}}\ \text{gm}} \frac{dM_c}{dt} \,(\text{g/s})$$

$$= \frac{4.4 \times 10^{18}}{3.86 \times 10^{33}} \frac{dM_c}{dt} \frac{2 \times 10^{33}}{3.1 \times 10^{7}} \,(M_\odot/y)$$

$$= \frac{8.8 \times 10^{11}}{3.1 \times 3.86} \frac{dM_c}{dt} \,(M_\odot/yr)$$

$$\frac{dM_c}{dt} = 1.4 \times 10^{-11} \left(\frac{L}{L_\odot}\right)$$

in $M_\odot/y$

Typically $L \simeq 5000\, L_\odot$ so $\frac{dM_c}{dt} \simeq 7 \times 10^{-8} M_\odot/yr$ = small.

But mass-loss also changes M:

Let $q = \frac{M_c}{M}$

Then $\dot{q} = \frac{dq}{dt} = \frac{M\dot{M}_c - M_c\dot{M}}{M^2}$

$$= \frac{\dot{M}_c}{M} - \frac{M_c}{M}\frac{\dot{M}}{M}$$

So $\dot{q}/q = \frac{\frac{\dot{M}_c}{M} - \frac{M_c}{M}\frac{\dot{M}}{M}}{M_c/M}$

$$= \frac{\dot{M}_c}{M_c} - \frac{\dot{M}}{M}$$

Thus $\frac{1}{\tau} = \frac{\dot{q}}{q} = \frac{1}{\tau_c} - \frac{1}{\tau_e}$

A rough fit to mass-loss on the AGB is the "Reimers Law"

$$\dot{M} \simeq -3.2 \times 10^{-13} \left(\frac{L}{L_\odot}\right)^{1.66} \left(\frac{M}{M_\odot}\right)^{-1} M_\odot/\mathrm{yr}$$

Core Mass - Luminosity Relation

Models show that

$$\frac{L}{L_\odot} \simeq 60{,}000 \left(\frac{M_c}{M_\odot} - 0.5 \right)$$

eg1 Sun: $M_c \simeq 0.6 \Rightarrow L \simeq 6000$ $(M_{bol} \simeq -4.7)$

eg2 Max mass white-dwarf: $M_c = 1.4$ $L = 54000\,L_\odot$

$(M_{bol} = -7.1)$

Relative Timescales

eg1 Sun $M_c = 0.6$ $L = 6000\,L_\odot$ $M = 0.8$ typically

$$\dot{M} = -7.5\times10^{-7}\ M_\odot/yr$$
$$\dot{M}_c = 8.4\times10^{-8}\ M_\odot/yr$$

$$\therefore \tau_c = M_c/\dot{M}_c = \frac{0.6}{8.4\times10^{-8}}\ yr \simeq 7\times10^6 y$$

$$\tau_e = M_e/\dot{M}_e = \frac{0.8}{7.5\times10^{-7}}\ yr \simeq 10^6 y$$

ie $\underline{\tau_c \sim 7\,\tau_e}$

Mass-loss dominates!

eg2 Max mass WD:

$$\dot{m} = -1.6 \times 10^{-4}\, M_\odot / yr$$
$$\dot{m}_c = 7.6 \times 10^{-7}\, M_\odot / yr \ll \dot{M} \; !$$

$$\tau_c = \frac{1.4}{7.6 \times 10^{-7}}\, yr \simeq 1.8 \times 10^{6}\, y$$

$$\tau_e = \frac{1.4}{1.6 \times 10^{-4}}\, yr \simeq 9000\, yr \; !!$$

$$\tau_e \ll \tau_c \; !!$$

Time on AGB

$$\frac{dM_c}{dt} = 1.4 \times 10^{-11} \frac{L}{L_\odot}$$

$$\& \quad \frac{L}{L_\odot} = 60000 \left(\frac{M_c}{M_\odot} - 0.5 \right)$$

$$\text{So} \quad \frac{dM_c}{dt} = 1.4 \times 10^{-11} \left[60000 \left(\frac{M_c}{M_\odot} - 0.5 \right) \right]$$

$$= a \left(b M_c - c \right)$$

M_c is in $M_\odot$
$a = 1.44 \times 10^{-11}$
$b = 60000$
$c = 0.5$

$$\int \frac{dM_c}{M_c - \frac{c}{b}} = \int ab\,dt$$

$$\ln\left(M_c - \frac{c}{b}\right) = abt + k$$

$$M_c = \frac{c}{b} + Ae^{abt}$$

When $t=0$ $\quad M_c = \frac{c}{b} + A \longleftarrow$ depends on M & star's composition!

For Sun: $\quad t=0$ (on AGB) has $M_c \simeq 0.55$.

$$M_c \simeq 0.55 = \cancel{\frac{0.5}{60000}}^{\simeq 0} + A$$

$$\Rightarrow A \simeq 0.55$$

$$\therefore \boxed{M_c \simeq 0.55\, e^{8.6\times10^{-7}t}} \qquad t \text{ in years}$$

Given $M_c(\text{max})$ for Sun $\simeq 0.6 = 0.55\, e^{8.6\times10^{-7} t_{max}}$

$$0.6/0.55 = 1.1 = e^{8.6\times10^{-7} t_{max}}$$

$$\ln(1.1) = 0.095 = 8.6\times10^{-7} t_{max}$$

$$t_{max} \simeq \frac{0.095}{8.95\times10^{-7}} \simeq 110{,}000\,y$$

re AGB lifetime $\simeq 10^5 y$

Since $\Delta t_{pulse} \simeq 10^4 \Rightarrow \simeq 10$ pulses (as we found before).