

# STELLAR STRUCTURE AND EVOLUTION NOTES

## PART VI: Stellar Evolution

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| = for reference, not examinable

## I. The Zero-Age Main-Sequence

The gravitational contraction results in temperatures high enough to sustain nuclear reactions: the star has reached the main sequence, called the “zero-age” main sequence, or ZAMS, because one usually counts the age from this point. Figure 3 shows a ZAMS for  $X = 0.685, Y = 0.294$  (and thus  $Z = 0.021$ ). Figure 4 shows the  $R(M)$  relation for these models, together with the observed  $R(M)$  relation for binary systems (the only ones for which  $R$  and  $M$  can be determined observationally). Typically we have

$$R \sim M^\xi$$

where  $\xi \simeq 0.8$  for  $M \lesssim M_\odot$  and  $\xi \simeq 0.6$  for  $M \gtrsim M_\odot$ . Figure 5 is analogous to figure 4, but it shows the luminosity variation. Again we find simple scaling behaviour, with a change of the gradient at  $M \sim M_\odot$ :

$$L \sim M^\nu$$

where  $\nu \simeq 3$  for  $M \lesssim M_\odot$  and  $\nu \simeq 4$  for  $M \gtrsim M_\odot$ .

To contrast the interior structure of ZAMS stars of different masses, figure 6 shows many interior variables for  $M = 1$  and  $M = 10M_\odot$  ZAMS models. Note that the density is everywhere higher in the lower mass model (Figure 6a), which is also more centrally condensed (6b). The higher mass model requires higher pressures for hydrostatic equilibrium, so its temperature is higher (6c) with consequently higher energy generation  $\epsilon$  (6d). Likewise, the region of energy generation is more centrally condensed in the  $10M_\odot$  star, where energy liberation is due to CN(O) cycling ( $\epsilon \sim T^{18}$ ) rather than pp reactions ( $\epsilon \sim T^4$ ) which are responsible for the energy generation in the  $1M_\odot$  model (6e).

Figure 7 shows the locus of  $(\rho_c, T_c)$  for ZAMS models of different masses. Also marked is the division between pp and CNO cycling, and lines of constant degeneracy  $\psi$ . We see that even on the ZAMS the lowest mass stars have partially degenerate cores.

Figure 8 shows the regions of convection in ZAMS stars of different masses. Note that the lowest masses are fully convective. As mass increases a radiative core develops and grows. At the same time that the convective envelope disappears a convective core develops—both taking place at  $M \simeq M_\odot$ , depending on the composition. This change in behaviour is responsible for the different gradients of the mass-radius and mass-luminosity relations above and below  $\sim 1M_\odot$ . It is convenient to define stars as belonging to the “upper main sequence” if they have convective cores, and the “lower main sequence” if they have radiative cores. Finally, note that although the mass in a convective envelope is small, it nevertheless can cover a large fraction of the stellar radius.

In high mass stars the high temperature dependence of the CN(O) cycles results in small nuclear burning cores, as revealed in figure 8 by plots of masses interior to which 50% and 90% of the total luminosity are generated. The steep energy dependence is responsible for a steep temperature gradient, and thus  $\nabla > \nabla_{ad}$ , so that a convective core develops. In contrast, the less temperature-sensitive pp-cycles have much more extended regions of nuclear energy generation, and hence shallower  $\nabla$  and radiative cores.

Figure 9a shows the mass variation of the central temperature and density, and the total stellar radius with total mass for ZAMS stars. Figure 9b shows the contributions to the luminosity from pp and CNO cycling, again as a function of total mass. Figure 9c shows the mass in the convective core, or convective envelope, as a function of mass (much like figure 8).