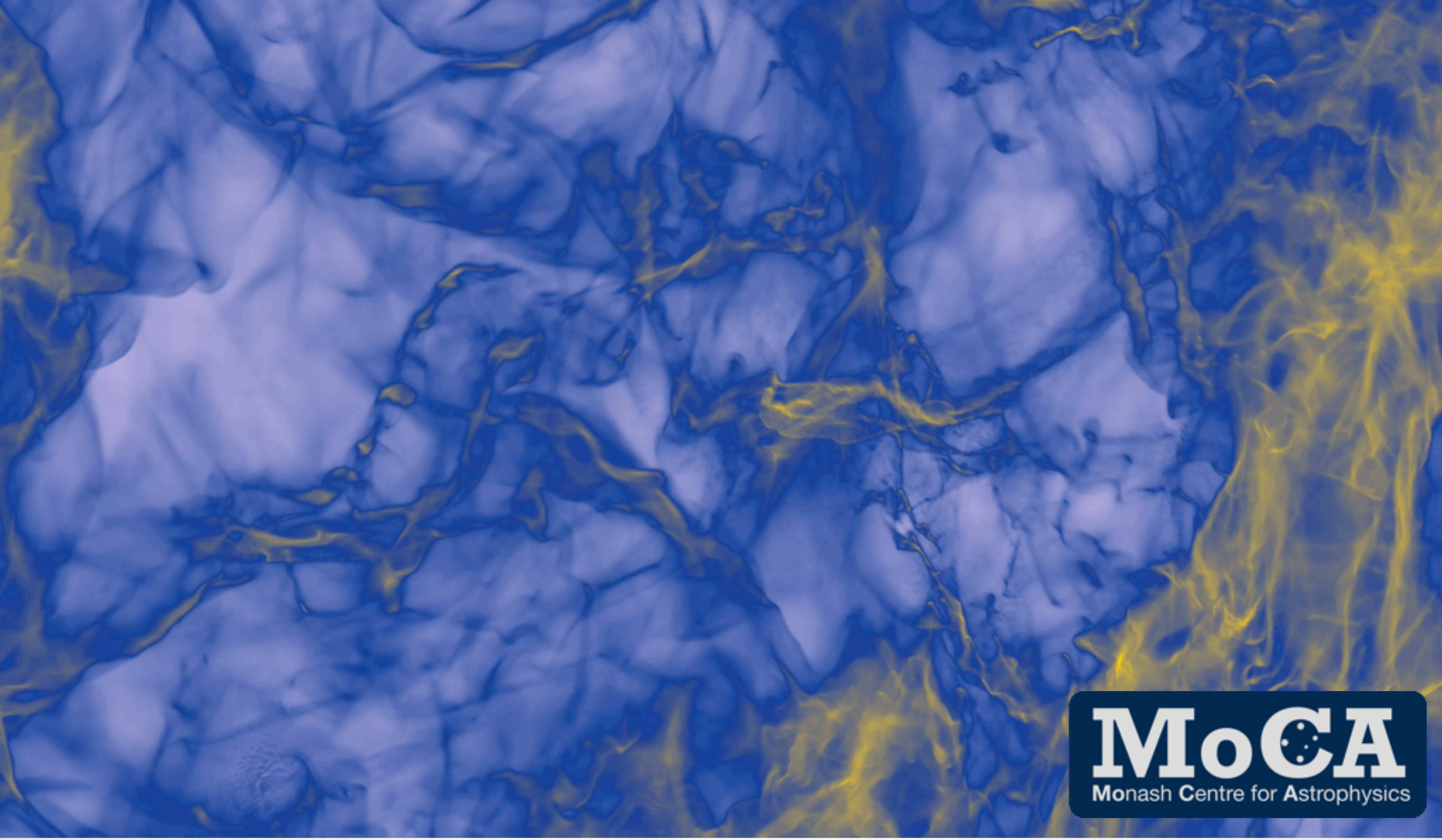


SIMULATING TURBULENCE

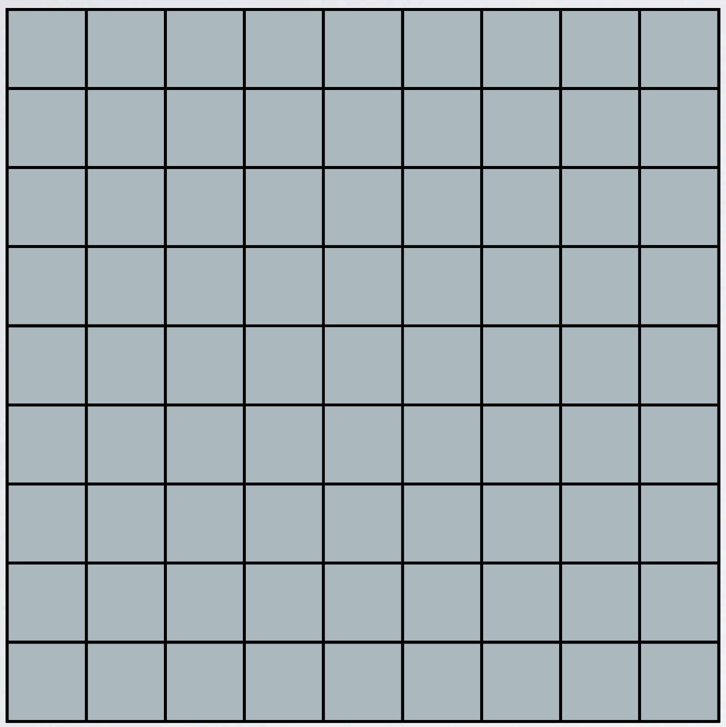
How (not) to do it

Daniel Price

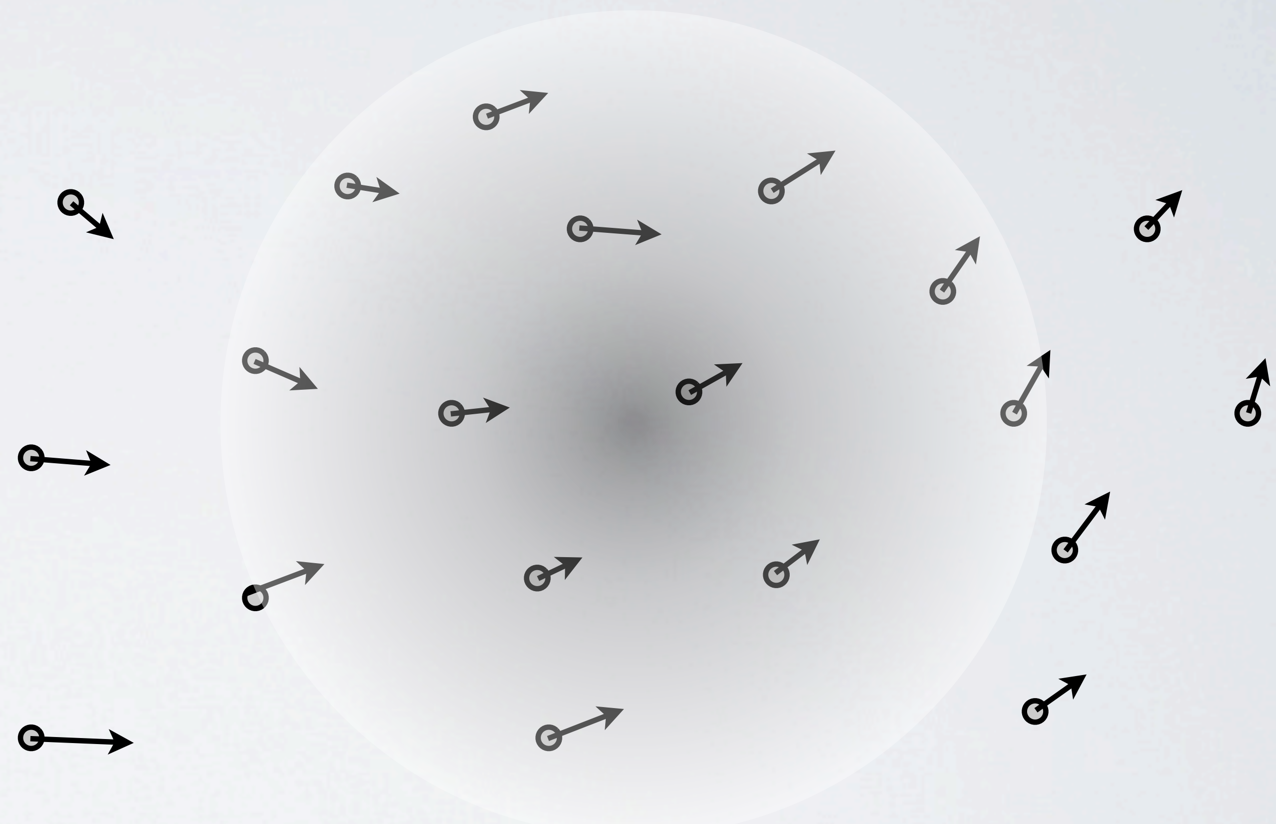
Talk given at "Turbulence in Cosmic Structure Formation", Tempe, Arizona, USA, 4th-8th March 2012

NUMERICAL METHODS

Grid



SPH



Lucy (1977), Gingold & Monaghan (1977),
Monaghan (1992, 2005), Springel (2010), Price (2012)

GRID VS. SPH

Padoan et al. (2007), commenting on Ballesteros-Paredes et al. (2006):

THE MASS SPECTRA OF CORES IN TURBULENT MOLECULAR CLOUDS AND IMPLICATIONS FOR THE INITIAL MASS FUNCTION

Javier Ballesteros-Paredes,[1] Adriana Gazol,[1] Jongsoo Kim,[2] Ralf S. Klessen,[3] Anne-Katharina Jappsen,[3] and Epimenio Tejero[1]

Received 2005 January 27; accepted 2005 September 20

"The complete absence of an inertial range with a reasonable slope, or with a reasonable dependence of the slope on the Mach number, makes their SPH simulations totally inadequate for testing the turbulent fragmentation model..."

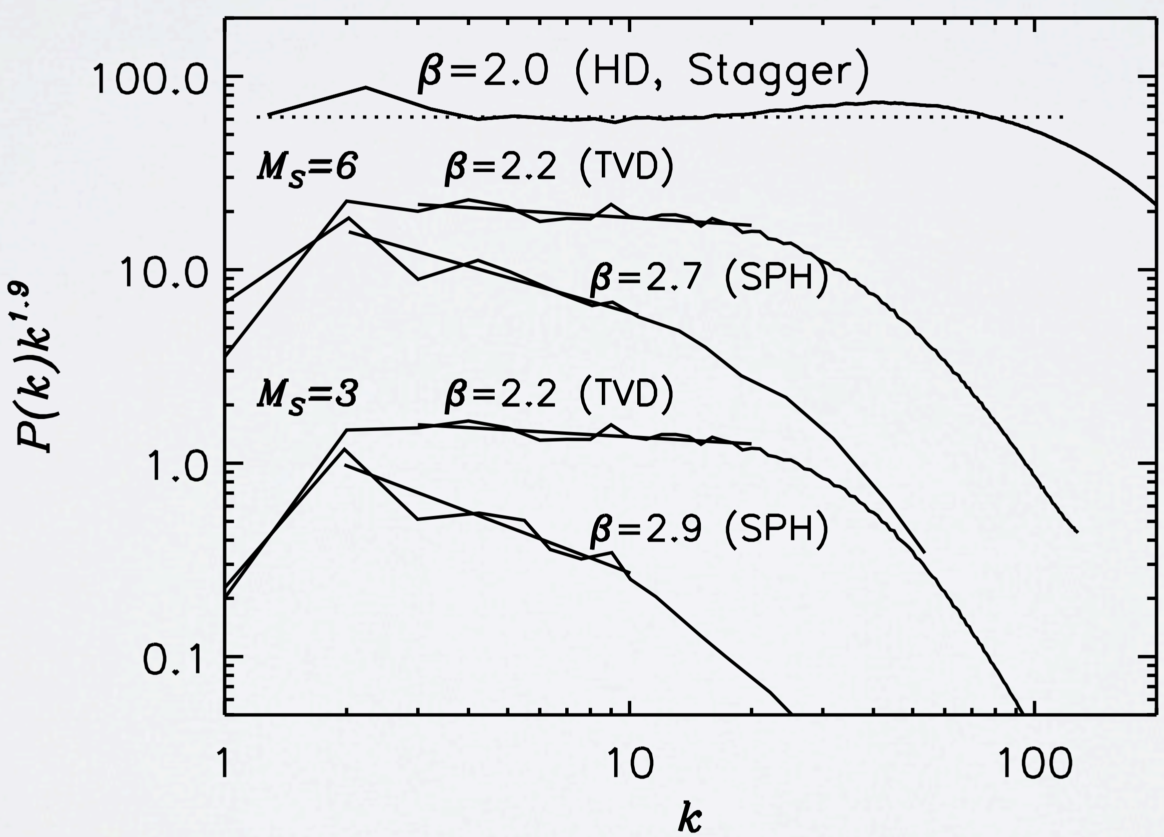
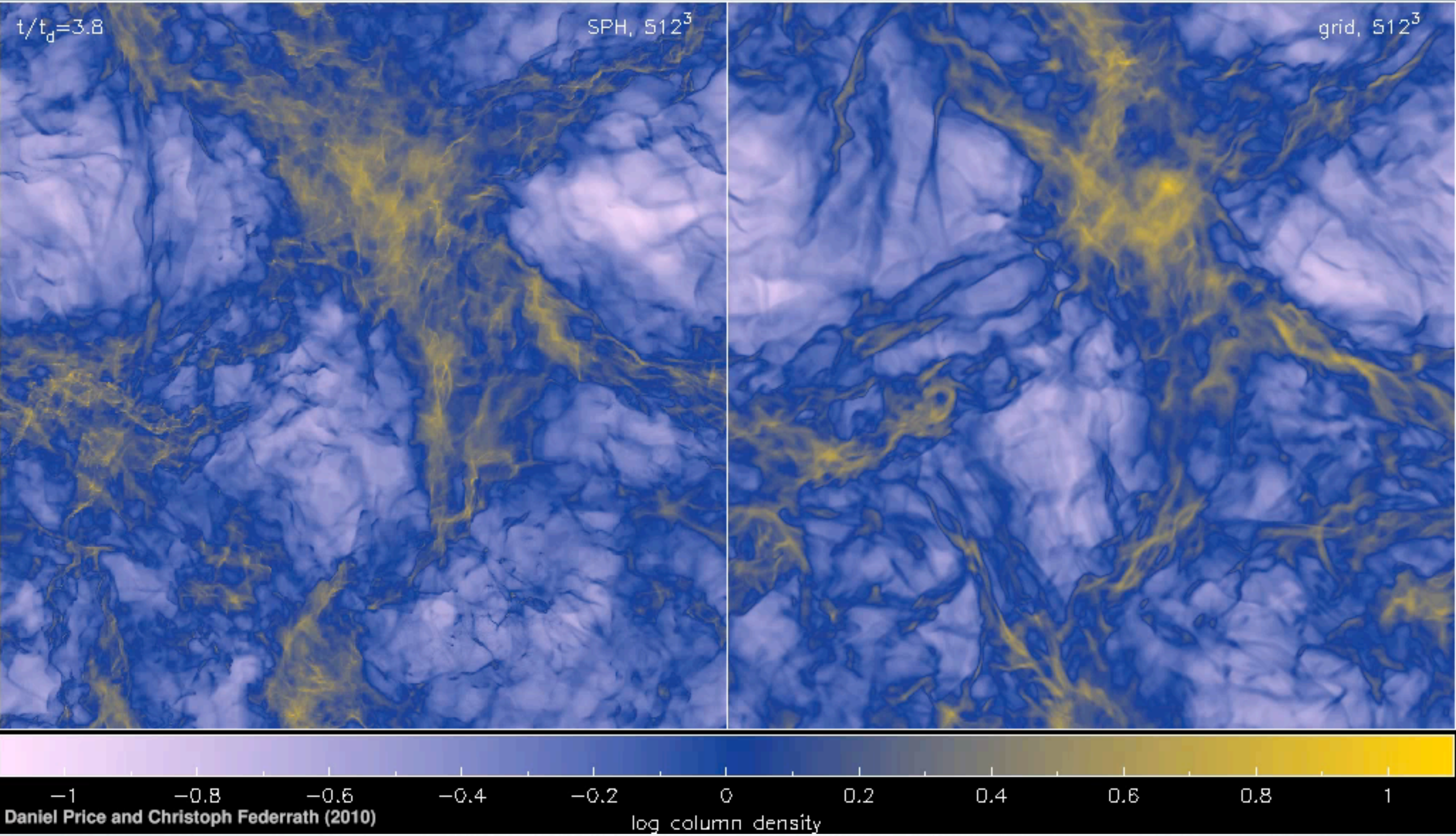


Fig. 8.—Power spectra compensated for the slope of the Stagger code HD run, $\beta = 1.9$. The TVD and SPH power spectra are the same as in Fig. 2 of Ballesteros-Paredes et al. (2006) for the Mach numbers 3 and 6.

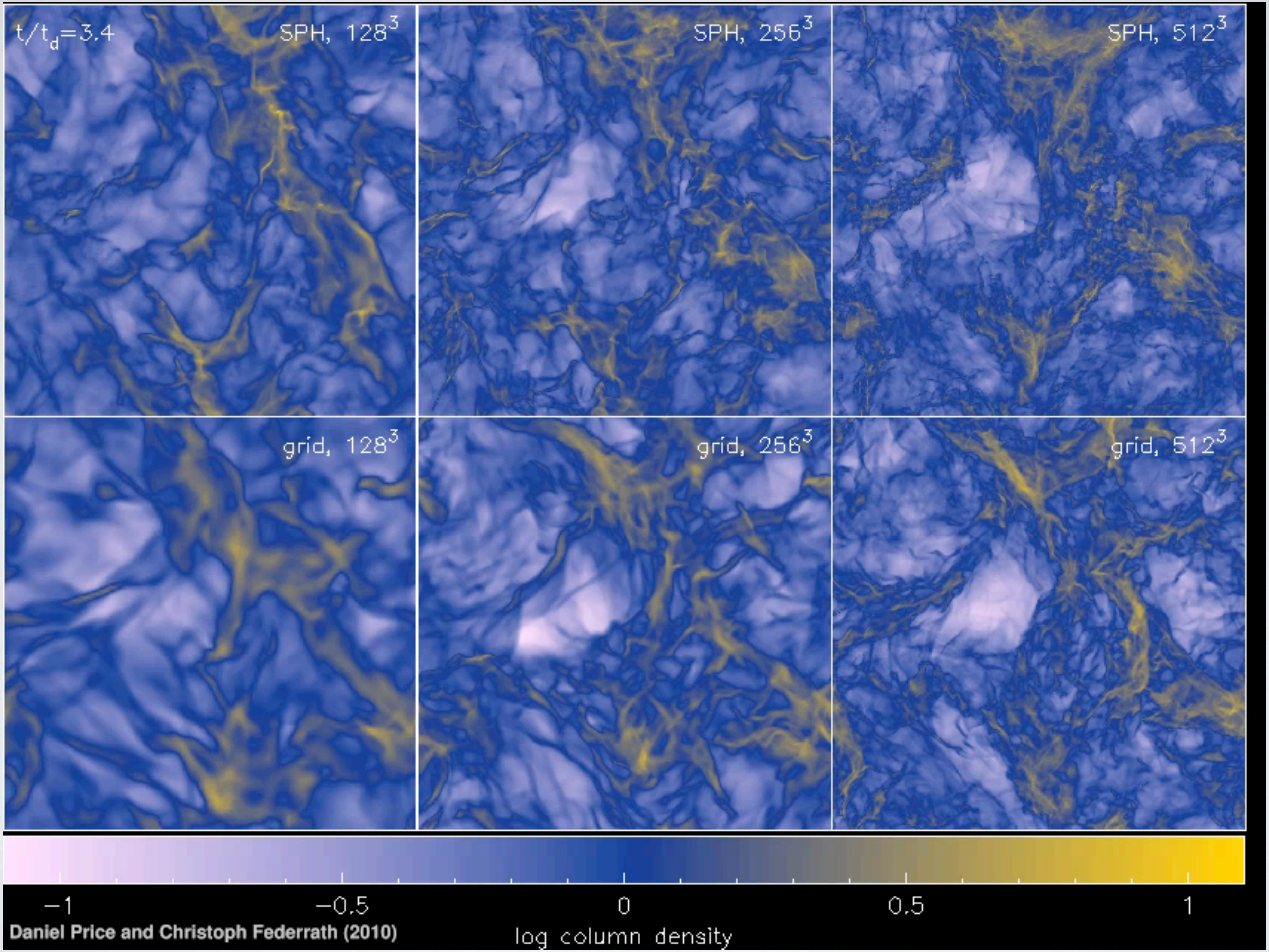
...but low resolution SPH (58^3)

Price & Federrath (2010): Comparison of driven, Mach 10 turbulence



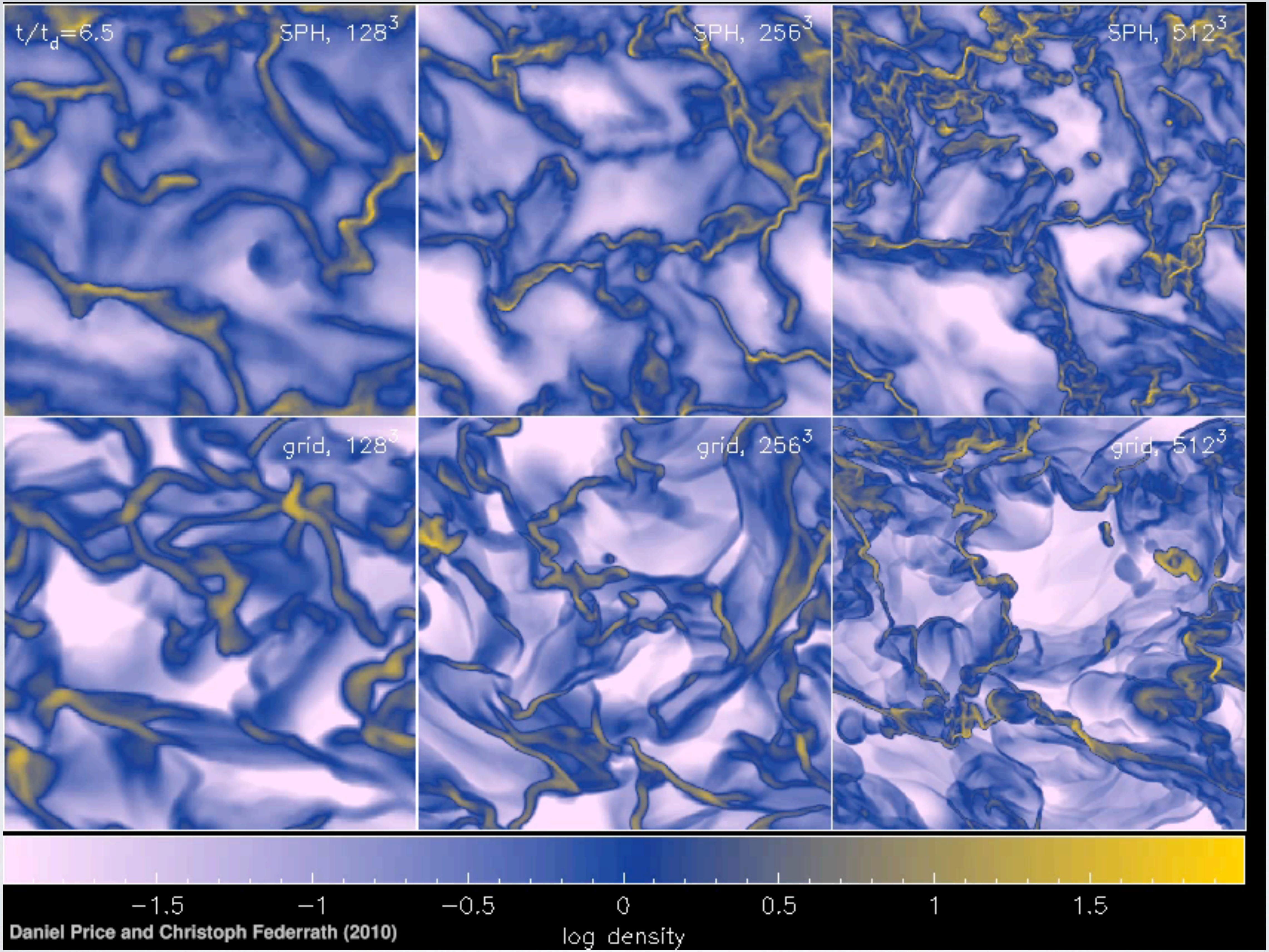
PHANTOM

FLASH



t/t_d=3.4
SPH, 128^3
SPH, 256^3
SPH, 512^3
grid, 128^3
grid, 256^3
grid, 512^3
−1
−0.5
0
0.5
1
log column density
Daniel Price and Christoph Federrath (2010)

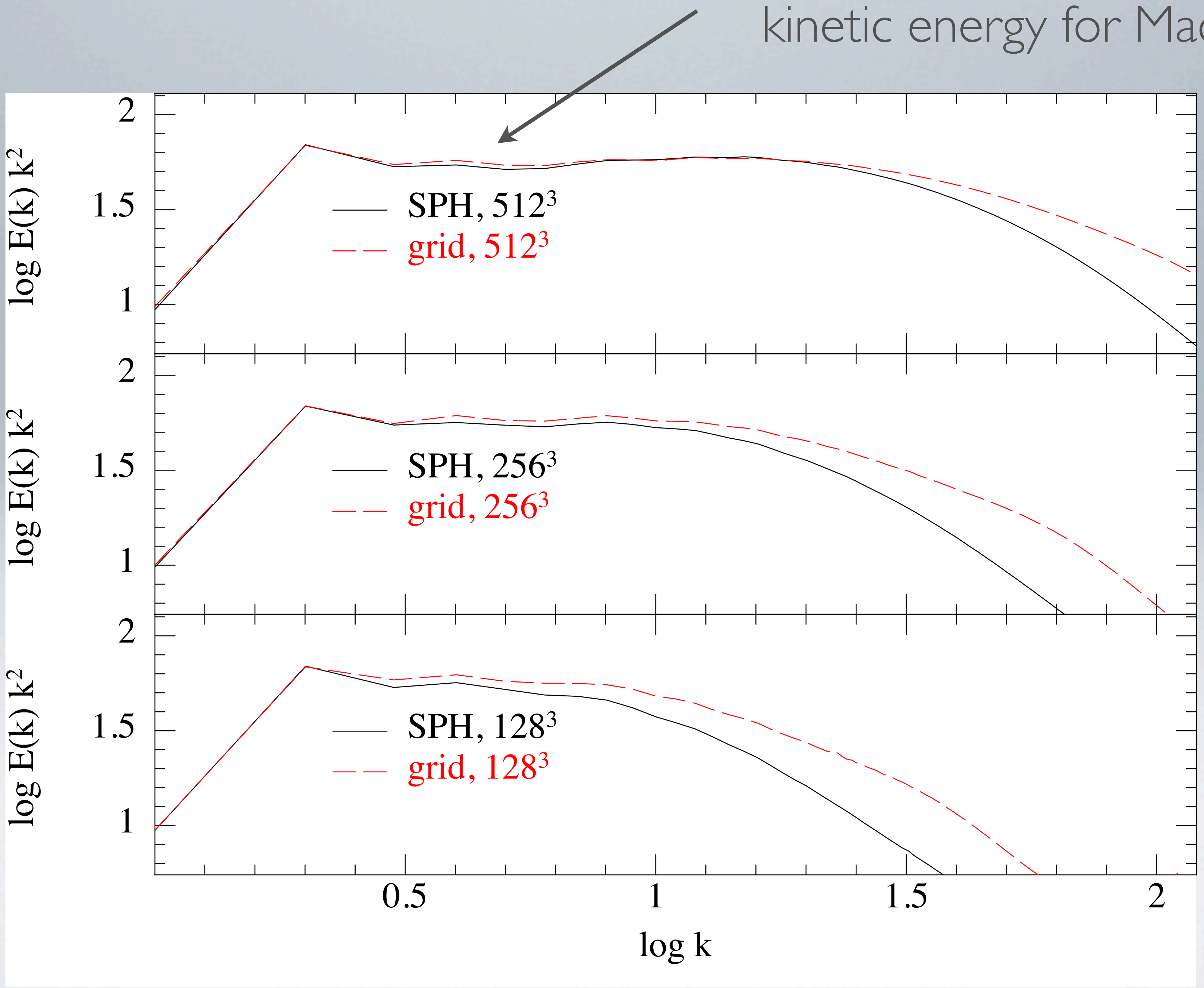
Slice:



t/t_d=6.5
SPH, 128^3
SPH, 256^3
SPH, 512^3
grid, 128^3
grid, 256^3
grid, 512^3
−1.5
−1
−0.5
0
0.5
1
1.5
log density
Daniel Price and Christoph Federrath (2010)

Kinetic energy spectra (time averaged)

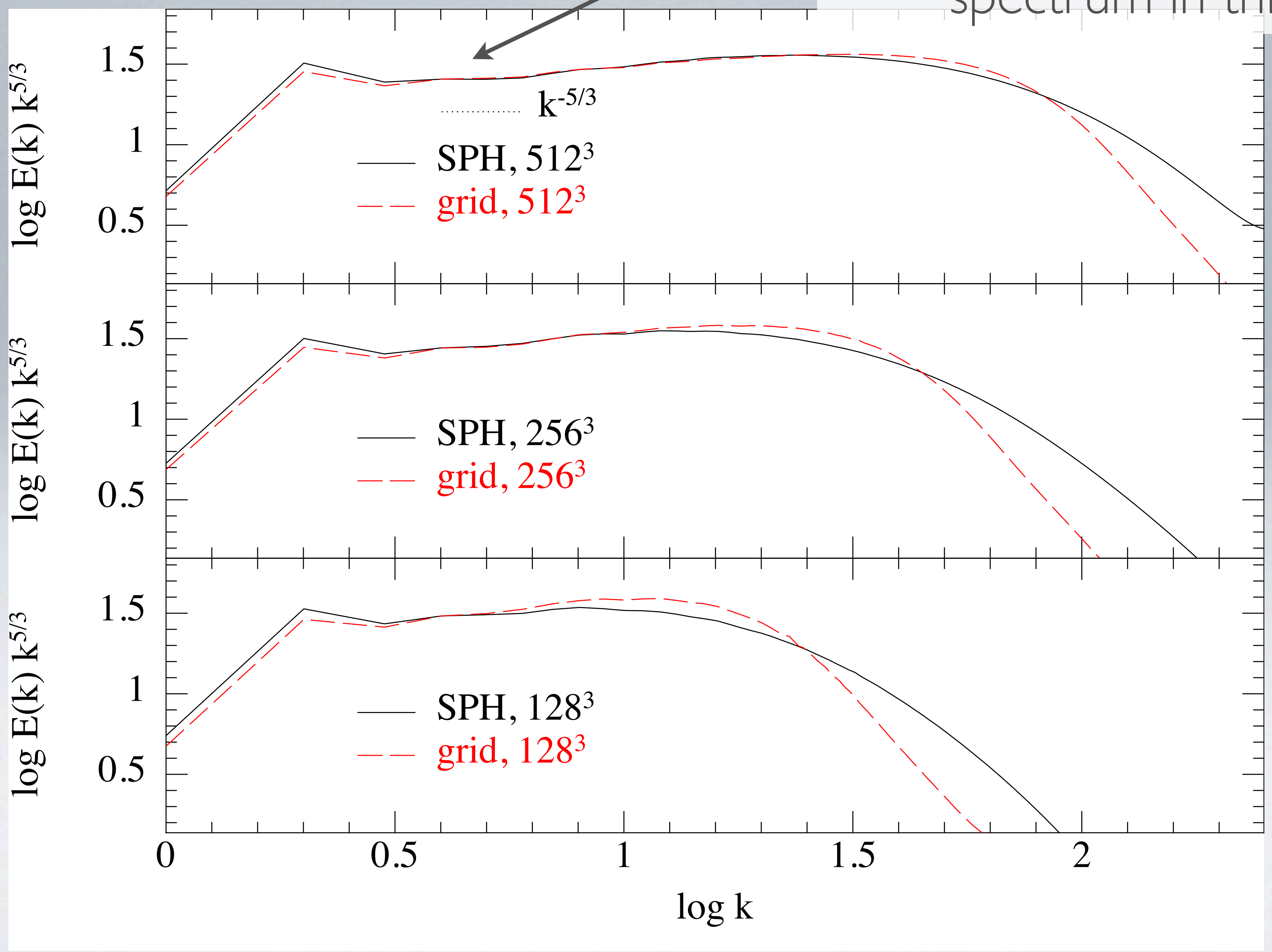
Burgers-like k^{-2} spectrum in the kinetic energy for Mach 10 hydro



Price & Federrath (2010)

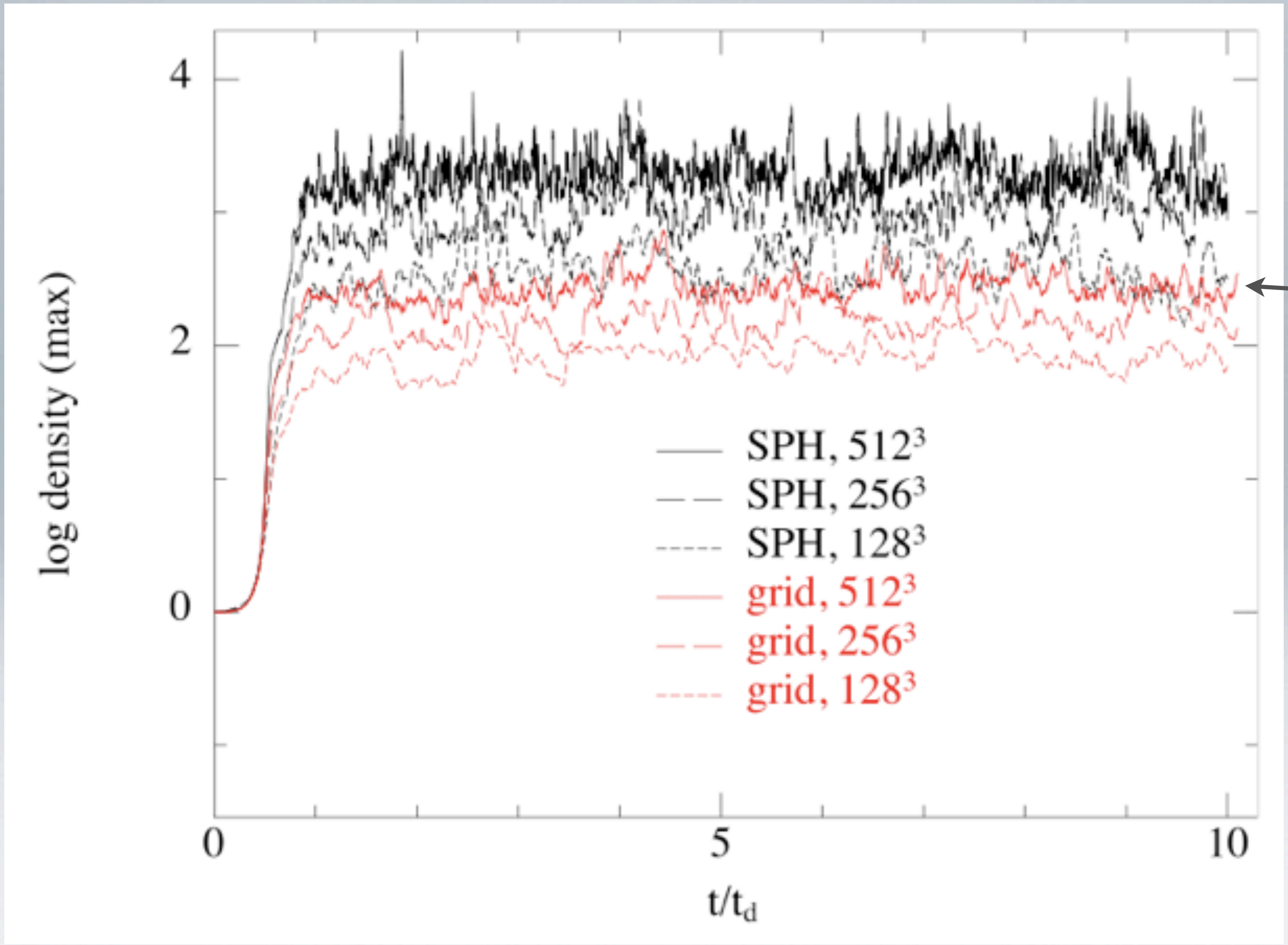
Density-weighted energy spectra $(\rho^{1/3}\mathbf{v})$

Confirms Kritsuk et al. (2007) suggestion of Kolmogorov-like $k^{-5/3}$ spectrum in this variable



Price & Federrath (2010)

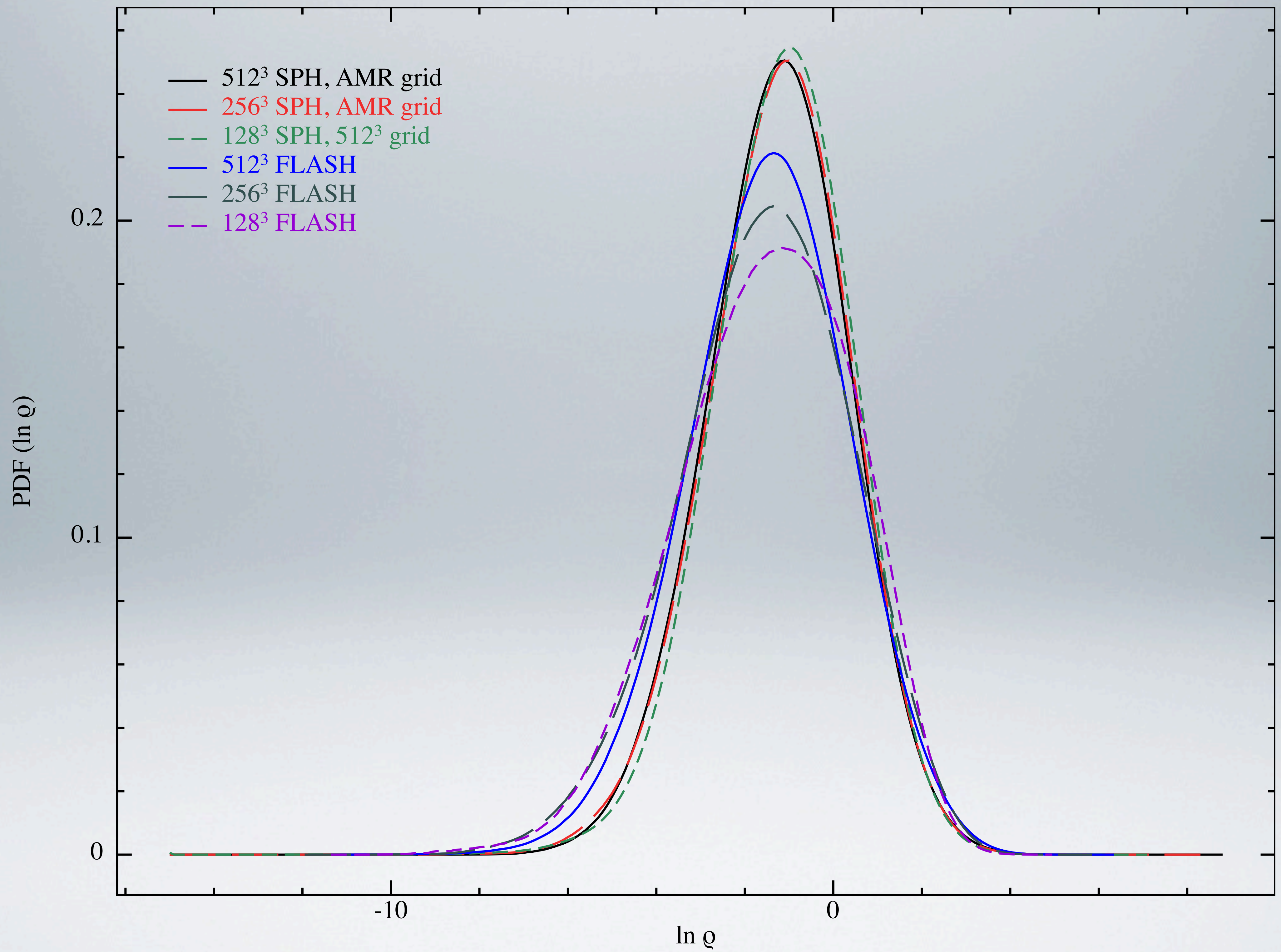
Density resolution



max density in SPH at 128^3 similar to max grid density at 512^3

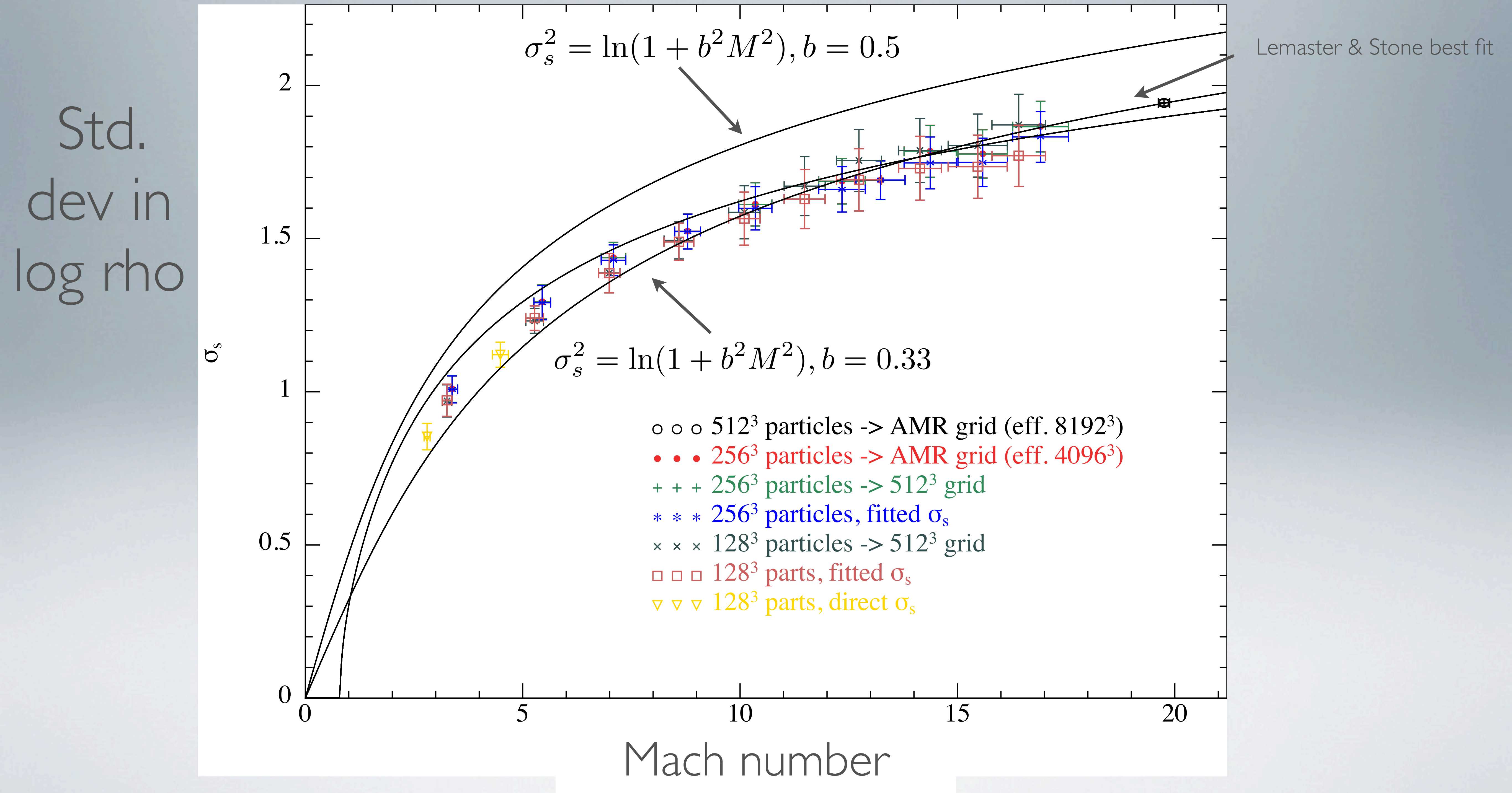
Price & Federrath (2010)

Density PDFs:

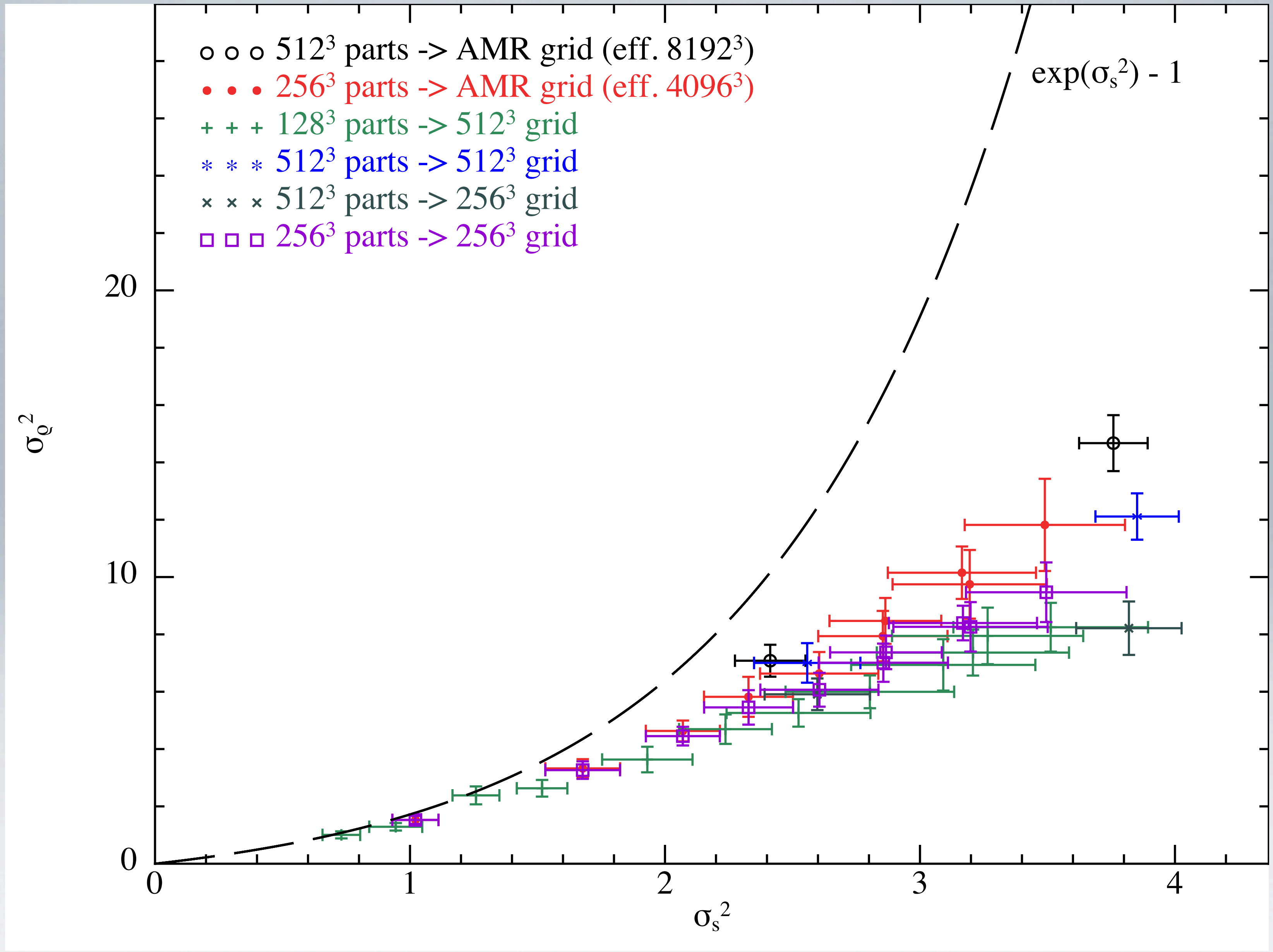


Density variance - Mach number relation in solenoidally-driven turbulence

Price, Federrath & Brunt (2011, ApJL)



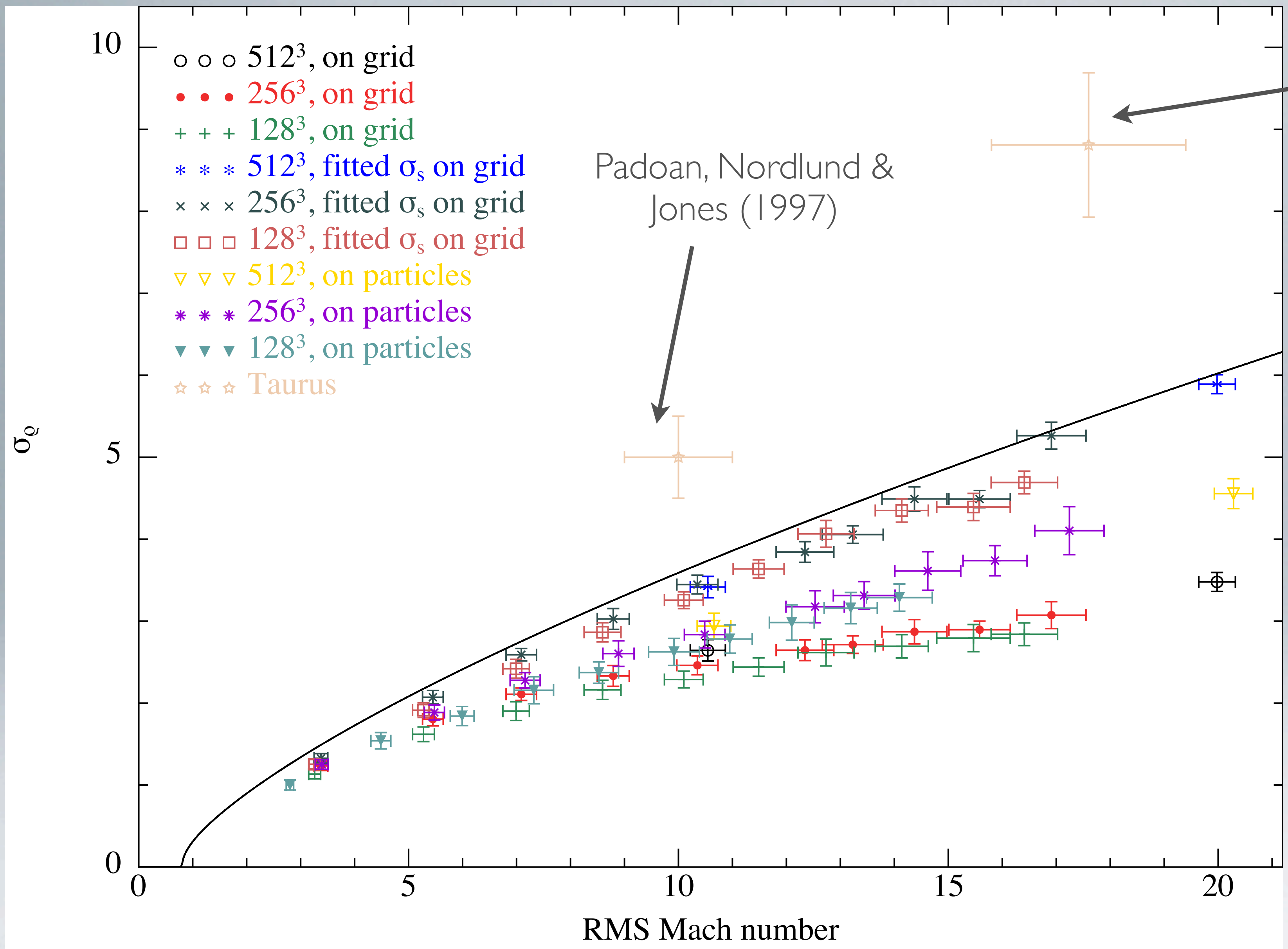
Trying to measure the (linear) density variance



If log normal, expect $$\sigma_\rho^2 = e^{\sigma_s^2} - 1$$

$$s \equiv log(\rho/\overline{\rho})$$

Comparison to observations



Brunt (2010) (based on new method for inferring 3D variance from 2D observations)

(see Brunt, Federrath and Price 2010)

need COMPRESSIVE DRIVING or GRAVITY

Price, Federrath & Brunt (2011, ApJL)

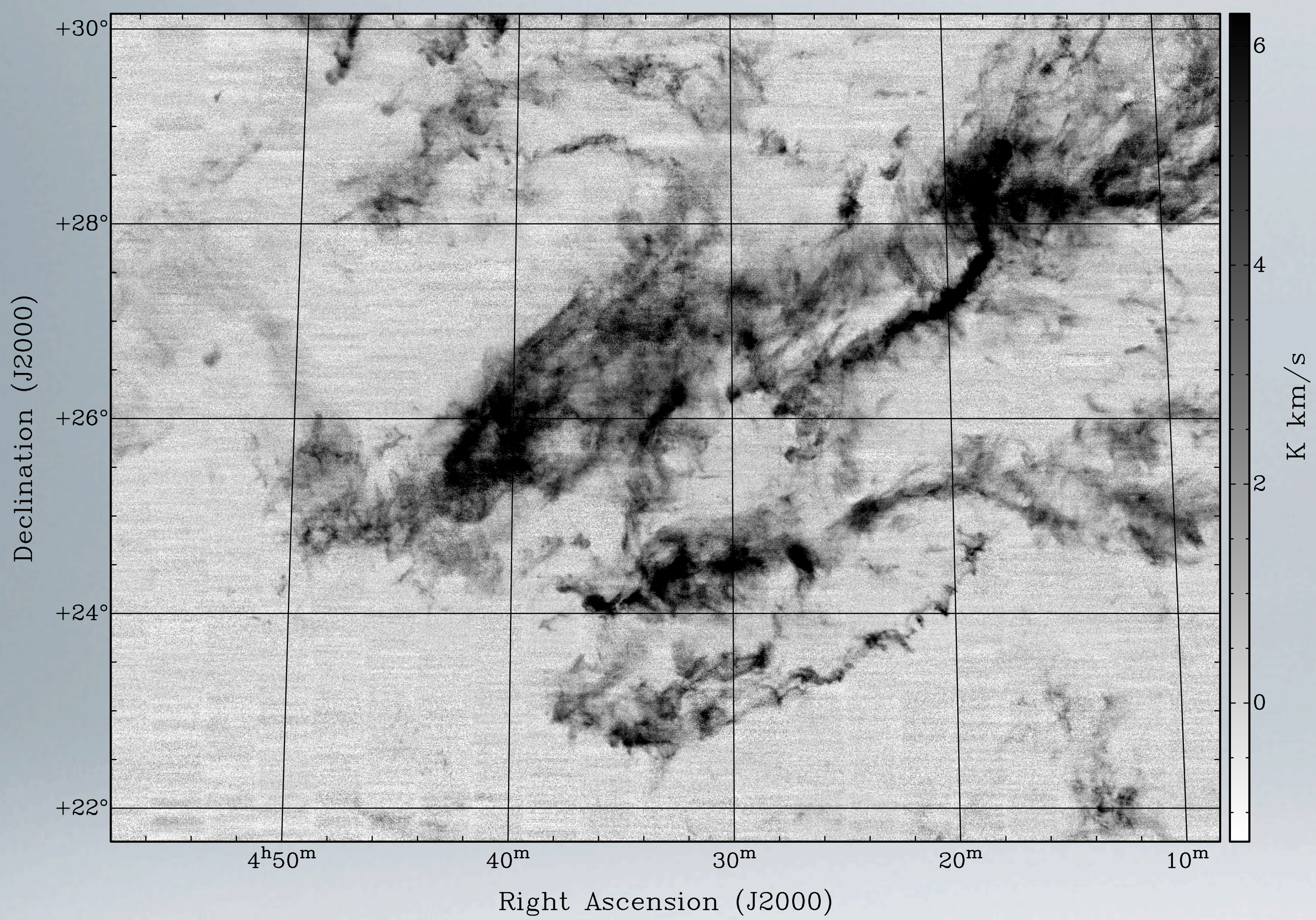
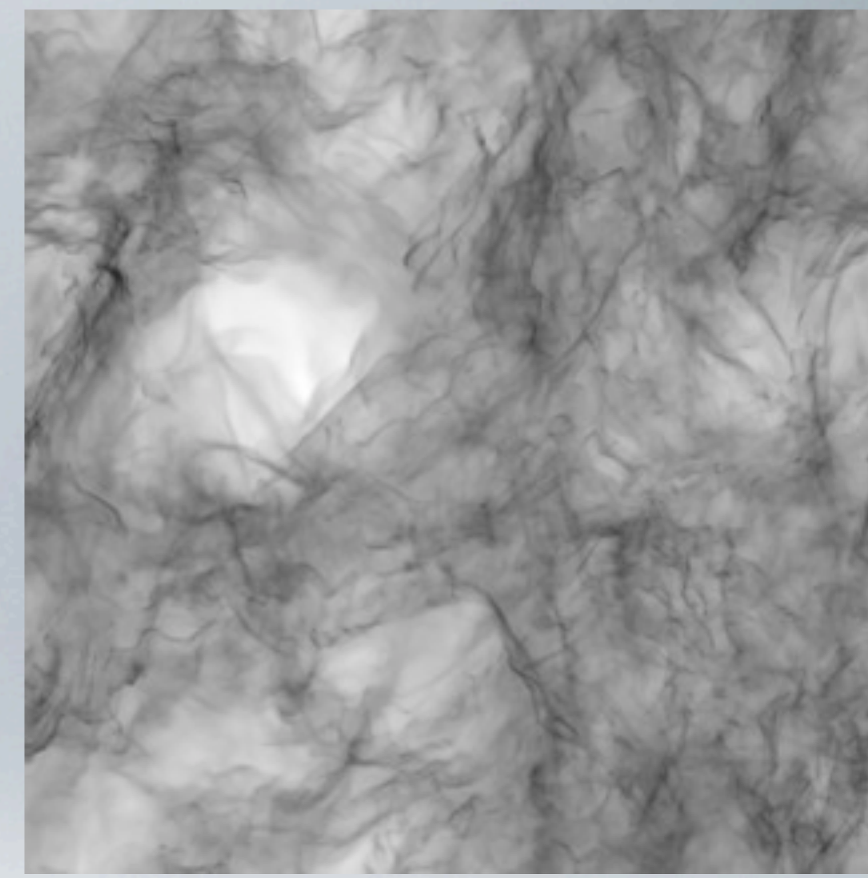
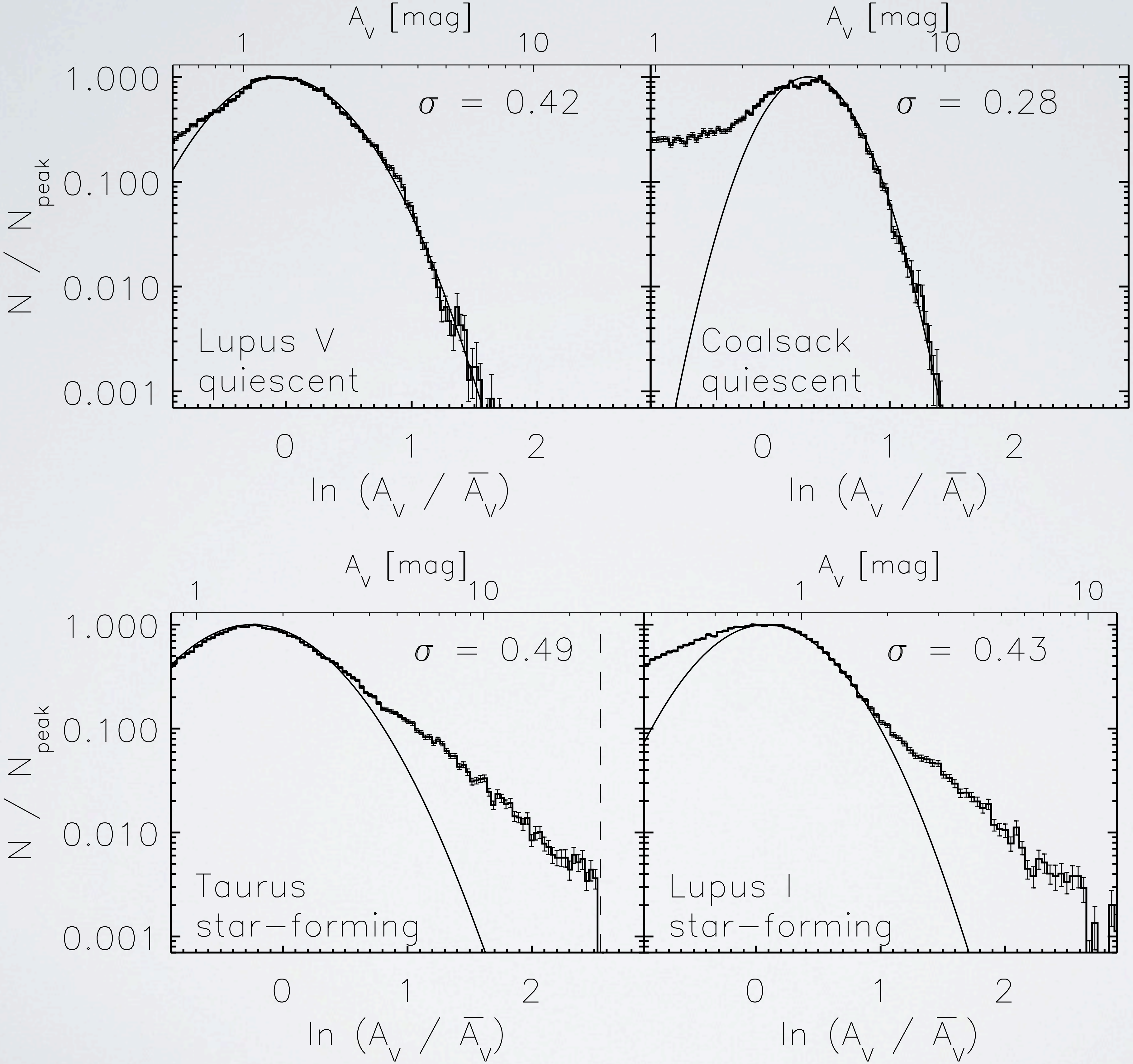


Fig. 1. Integrated intensity map of the ^{13}CO J=1–0 line over the velocity range [0, 12] km s^{-1} in the Taurus molecular cloud.

VS.



Kainulainen et al. (2009):



WHAT ABOUT LOW MACH NUMBER (ICM/IGM) TURBULENCE?

Mon. Not. R. Astron. Soc. **000**, 000–000 (0000) Printed 22 September 2011 (MN LaTeX style file v2.2)

Shocking results without shocks: Subsonic turbulence in smoothed particle hydrodynamics and moving-mesh simulations

Andreas Bauer[1⋆] and Volker Springel[1,2]

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[2]*Zentrum für Astronomie der Universität Heidelberg, Astronomisches Recheninstitut, Mönchhofstr. 12-14, 69120 Heidelberg, Germany*

22 September 2011

ABSTRACT
Highly supersonic, compressible turbulence is thought t... star formation processes in the interstellar medium (ISM)... to give rise to subsonic turbulence in the... modify the thermodynamic structur... ale mixing processes in the gas... ng the properties of astrophysi... been restricted to the super... faithfully represented by the numerica... ere we focus on comparing the accura... our new moving-mesh technique AREPO... ntact with previous results, we al... and highly supersonic t... ulence. We find that the ... SPH ... badly fails in the ... bsonic regime. Instead of building up a Kolmogorov-like tur-... de, large-scale eddies are quickly damped close to the driving scale and decay ... ale velocity noise. In contrast, our moving-mesh technique does yield power-law ... for the power spectra of velocity, vorticity and density, consistent with expecta-... turbulence. We argue that large errors in SPH's gradient ... the associated subsonic velocity noise are ultimately responsible for producing ... nphysical results in the subsonic regime. This casts doubt about the reliability ... imulations of cosmic structure formation, es ecially if turbulence in clusters of ... ndeed significant. In contrast, SPH's performa... e is much better for supersonic ... s here the flow is kinetically dominated and cha... shocks, which can be adequately captured with SPH. When... simulations of turbulence, our moving-mesh approach sho... sults, although with somewhat better resolving power at the... reduced advection errors and the automatic adaptivity of the...

Key words: hydrodynamics, shock waves, turbulence, met...

1 INTRODUCTION

Astrophysical gas dynamics in the interstellar and intergalactic medium is typically characterized by very high Reynolds numbers, thanks to the comparatively low gas densities encountered in these environments, which imply a very low physical viscosity for the involved gas. We may hence expect that turbulent cascades over large dynamic ranges are rather prevalent, provided effective driving processes exist. Such turbulence can then be an important feature of gas dynamics, for example providing an additional effective pressure contribution, or leadi... chemical elements in the gas.

In fact, it is believed that ... (ISM) plays a key role in the ... lations, determining in part the initial mass function of stars, the lifetime of molecular clouds, and the overall efficiency of star formation (e.g. Klessen et al. 2000; Mac Low & Klessen 2004). Here the turbulence is highly supersonic, and presumably driven primarily by supernova explosions. In addition, the strong radiative cool-

...we find that the widely employed standard formulation of SPH quite badly fails in the subsonic regime...

In contrast, our moving mesh technique does yield power-law scaling laws... consistent with expectations...

Instead of building up a Kolmogorov-like turbulent cascade, large-scale eddies are quickly damped close to the driving scale...

We argue that large errors in SPH's gradient estimated and associated subsonic velocity noise are ultimately responsible...

This casts doubt about the reliability of SPH for simulations of cosmic structure formation...

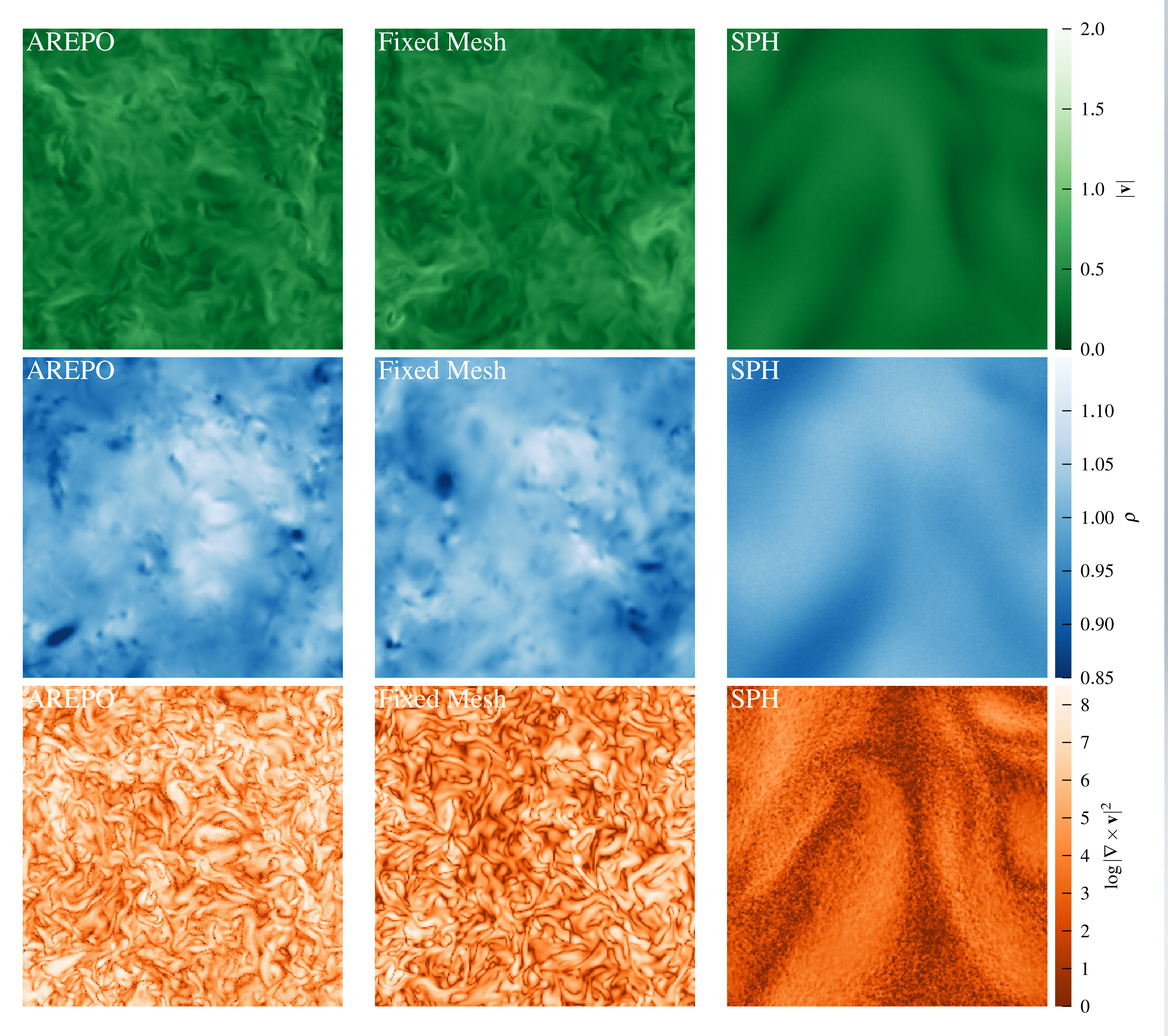


Figure 3. Visual comparison of the turbulent velocity field (top row), the density field (middle row) and the enstrophy $|\nabla \times \boldsymbol{v}|^2$ (bottom row) in quasi-stationary turbulence with $\mathcal{M} \sim 0.3$, simulated with different numerical techniques. Shown are thin slices through the middle of the perdiodic simulation box. From left to right, we show our moving grid result, an equivalent calculation on a static mesh, and an SPH calculation, as labeled.

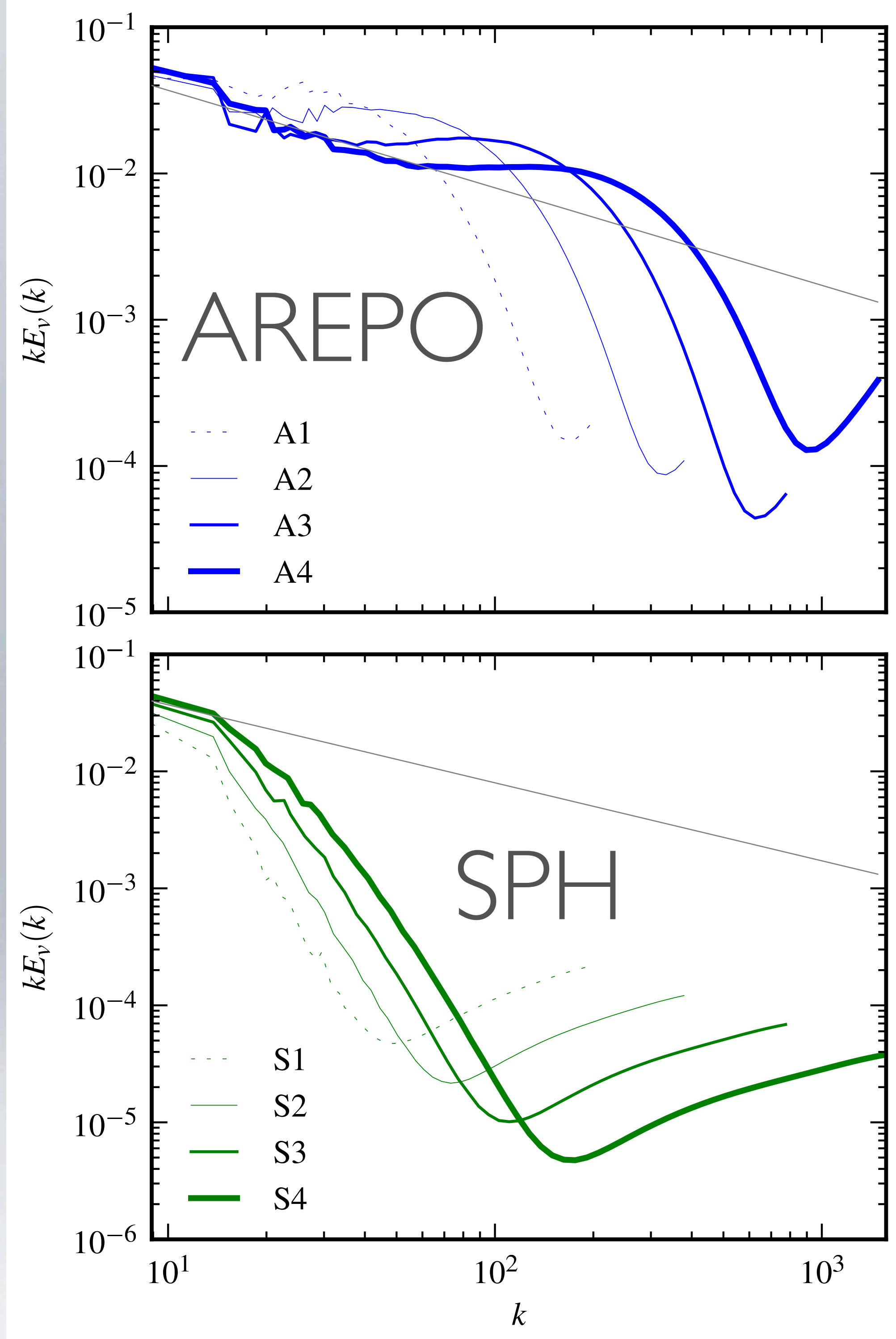


Figure 5. Convergence study for the velocity power spectrum of $\mathcal{M} \sim 0.3$ subsonic turbulence. The panel on top shows results for AREPO, from a resolution of 64^3 to 512^3 cells. The panel on the bottom gives the corresponding results for SPH. However, even at a high resolution as high 512^3 particles, no extended inertial range of turbulence can be identified in SPH. The thin grey lines show the power-law expected for Kolmogorov's theory.

What's going on?

BS explanation:

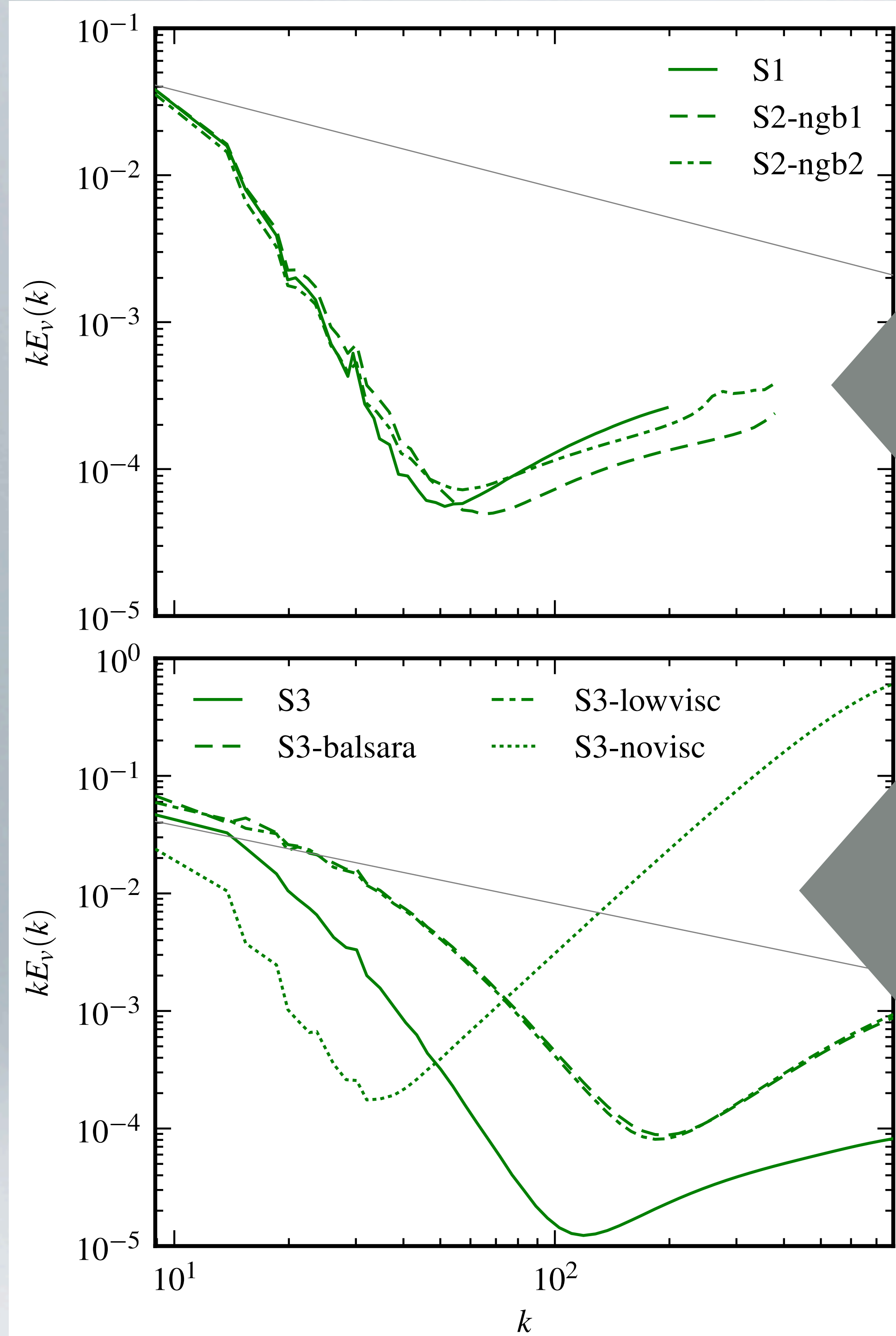
We argue that the origin of this noise lies in errors of SPH's gradient estimate. Numerous studies have pointed out that the standard approach followed in SPH for constructing derivatives of smoothed fluid quantities involves rather large error terms, especially for the comparatively irregular particle distributions in multi-dimensional simulations. The problem lies in a lack of consistency of the ordinary density estimates (which do not conserve volume, i.e. the sum of m_i/ρ_i is not guaranteed to add up to the total volume) and in a low order of the gradient estimate itself (e.g. Quinlan et al. 2006; Gaburov & Nitadori 2011; Amicarelli et al. 2011). In practice, this means that there can be pressure forces on particles even though all individual pressure values of the particles are equal,

A clue:

port. To suppress the artificial viscosity in regions of strong shear, Balsara (1995) proposed a simple viscosity limiter in the form of an additional multiplicative factor $(f_i + f_j)/2$ for the viscous tensor, defined as

$$f_i = \frac{|\nabla \cdot \boldsymbol{v}|_i}{|\nabla \cdot \boldsymbol{v}|_i + |\nabla \times \boldsymbol{v}|_i}. \tag{3}$$

This limiter is often used in cosmological SPH simulations and also available in the GADGET code. In our default simulations, we have refrained from enabling it, but we have also run comparison simulations where it is used, as discussed in our results section.



effect of "gradient error" (neighbour number)

effect of viscosity

Figure 7. Dependence of SPH turbulence results on numerical nuisance parameters. The panel on top gives results for the velocity power spectrum when the number of SPH smoothing neighbours is increased, from our default of 64 to 180, and finally to 512. Formally, the later run with 128^3 particles has the same mass and spatial resolution as our S1 run with 64^3 particles, hence the latter is included as a dashed line. The bottom panel illustrates the effect of changing the SPH viscosity parameterization. For lower α, the velocity power on large scales goes up, but the shape of the power spectrum does not improve. Note however that this also increases the

BUT WHAT IS THE REYNOLDS NUMBER?

$$\mathcal{R}_{\mathrm{e}} \equiv \frac{VL}{\nu}$$

Stokes (1851), Reynolds (1883)

DISSIPATION IN SPH

There is none (it is a Hamiltonian system)

...except what you explicitly add.

AV terms give:

$$\nu \approx \frac{1}{10}\alpha v_{\rm sig} h; \qquad \zeta \approx \frac{1}{6}\alpha v_{\rm sig} h$$

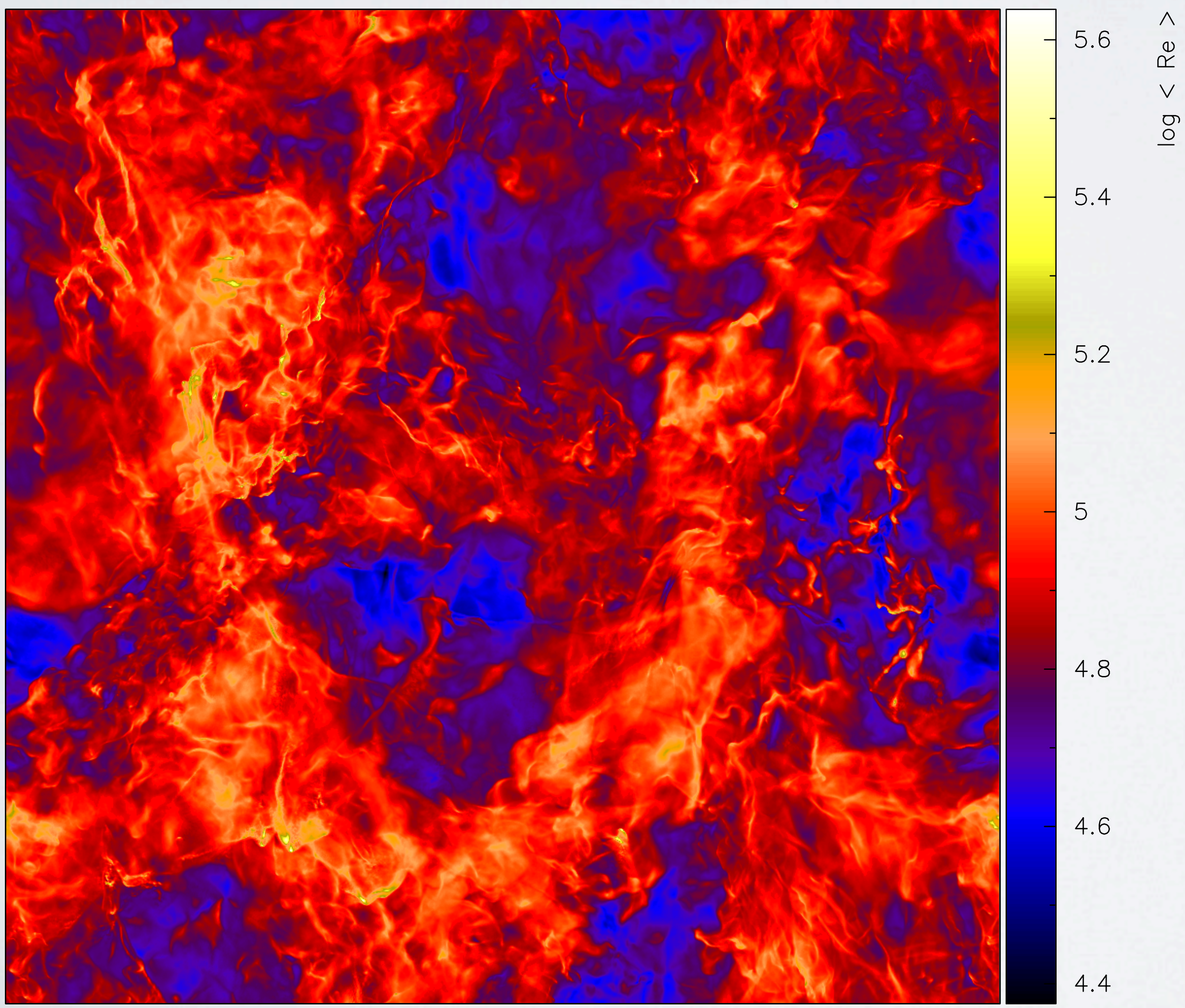
Monaghan & Lattanzio (1985): $\alpha = 1$

Morris & Monaghan (1997): $\alpha(x, t) \in [0.1, 1]$

REYNOLDS NUMBERS IN SPH

$$\mathcal{R}_{\rm e} = \frac{10}{\alpha}\mathcal{M}\frac{L}{h},$$

Price & Federrath (2010), Mach 10:



c.f. Elmegreen & Scalo (2004): R_e~10^5-10^7 in ISM

REYNOLDS NUMBERS IN SPH

$$\mathcal{R}_{\rm e} = \frac{10}{\alpha}\mathcal{M}\frac{L}{h},$$

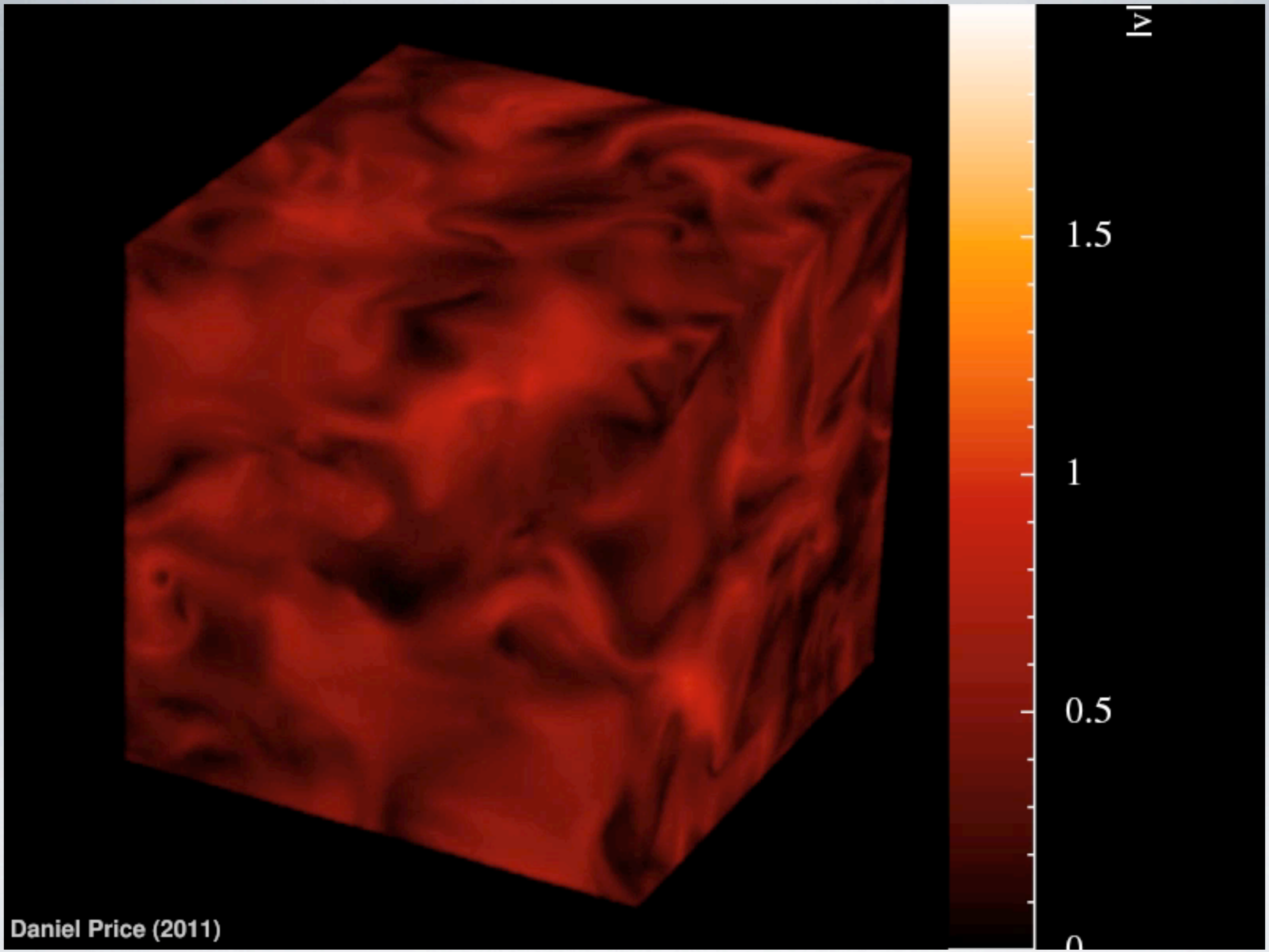
Linear dependence on Mach number

$$\mathcal{R}_{\rm e} = 2.4 n^{1/3} \left(\frac{\mathcal{M}}{0.3}\right) \left(\frac{\alpha}{1.0}\right)^{-1} \left(\frac{N_{\rm ngb}}{64}\right)^{-1/3},$$

In BS calculations:

$n = 64^3$	$n = 128^3$	$n = 256^3$
$\mathcal{R}_{\rm e} = 154$	$\mathcal{R}_{\rm e} = 307$	$\mathcal{R}_{\rm e} = 614$

Using standard (15yo) viscosity switches:



see also Dolag et al. (2005) and Valdarnini (2011) on importance of viscosity switches for SPH simulations of ICM/IGM turbulence

Price (2012), MNRAS 420, L33

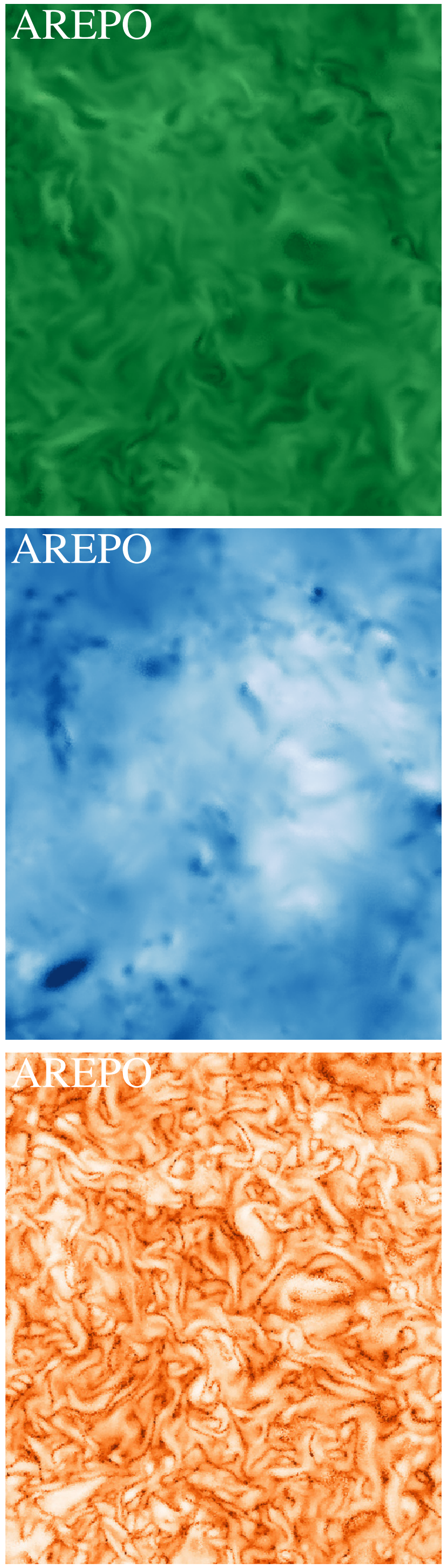


Figure 3. Visual comparison of the turbul
turbulence with $\mathcal{M} \sim 0.3$, simulated wit
to right, we show our moving grid result,

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Resolving high Reynolds numbers in smoothed particle hydrodynamics simulations of subsonic turbulence

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ABSTRACT

Accounting for the Reynolds number is critical in numerical simulations of turbulence, particularly for subsonic flow. For smoothed particle hydrodynamics (SPH) with constant artificial viscosity coefficient α, it is shown that the effective Reynolds number in the absence of explicit physical viscosity terms scales linearly with the Mach number – compared to mesh schemes, where the effective Reynolds number is largely independent of the flow velocity. As a result, SPH simulations with $\alpha = 1$ will have low Reynolds numbers in the subsonic regime compared to mesh codes, which may be insufficient to resolve turbulent flow. This explains the failure of Bauer & Springel to find agreement between the moving-mesh code AREPO and the GADGET SPH code on simulations of driven, subsonic ($v \sim 0.3c_s$) turbulence appropriate to the intergalactic/intracluster medium, where it was alleged that SPH is somehow fundamentally incapable of producing a Kolmogorov-like turbulent cascade. We show that turbulent flow with a Kolmogorov spectrum can be easily recovered by employing standard methods for reducing α away from shocks.

Key words: hydrodynamics – turbulence – methods: numerical – galaxies: clusters: intracluster medium – intergalactic medium.

1 INTRODUCTION

Turbulence in astrophysics is of key importance for the interstellar medium (ISM), intracluster medium (ICM) and intergalactic medium (IGM). Compressible, hydrodynamic turbulence is characterized by two dimensionless parameters, the Mach number $\mathcal{M} \equiv V/c_s$ and the Reynolds number (Stokes 1851; Reynolds 1883)

$$\mathcal{R}_e \equiv \frac{VL}{\nu}, \tag{1}$$

where V is the flow velocity, L is a typical length-scale, ν is the viscosity of the fluid and c_s is the sound speed. Physically, these parameters estimate the relative importance of each of the terms in the Navier–Stokes equations – the Mach number specifies the ratio of the inertial term, $(\boldsymbol{v} \cdot \nabla)\boldsymbol{v}$, to the pressure term, $\nabla P/\rho$, while the Reynolds number specifies the ratio of the inertial term to the viscous dissipation term, $\nu\nabla^2\boldsymbol{v}$. Mathematically, these two parameters – along with the boundary conditions and driving – entirely characterize the flow.

Given the importance of turbulence in theoretical models, it is crucial that agreement can be found between codes used for simulations of the ISM and ICM/IGM. Several comparison projects have been published recently comparing simulations of both decaying (Kitsionas et al. 2009) and driven (Price & Federrath 2010a) supersonic turbulence relevant to molecular clouds. However, fewer calculations appropriate to the ICM or IGM have been performed. In a recent preprint, Bauer & Springel (2011) have set out to extend the high Mach number comparisons to the mildly compressible, driven, subsonic turbulence thought to be appropriate to the ICM and IGM. In this case, the motions are comparable to or smaller than the sound speed, turbulent motions are dissipated by viscosity, and the flow is mainly characterized by the Reynolds number, similar to turbulence in the laboratory. In particular, it is well known from laboratory studies that the transition from laminar flow to fully developed turbulence only occurs once a critical Reynolds number is reached – for example, for Poiseuille flow (water flowing in a pipe) this is observed for $\mathcal{R}_e \gtrsim 2000$ (e.g. Reynolds 1895).

Since at low Mach number the Reynolds number controls not only the transition to turbulence, but also the character of such turbulence (e.g. the extent of the inertial range), it is critical to specify, or at least estimate, the Reynolds number employed in numerical simulations of turbulence in order to compare with the physical Reynolds numbers in the problems of interest. For the ISM, the physical Reynolds numbers are high [e.g. Elmegreen & Scalo (2004) estimate $\mathcal{R}_e \sim 10^5$–10^7 for the cold ISM] so the approach adopted has been to fix the Mach number and try to reach high numerical Reynolds numbers by minimizing numerical dissipation away from shocks. Estimates for $\mathcal{R}_e$ in the ICM/IGM are more difficult. Brunetti & Lazarian (2007) estimate $\mathcal{R}_e \sim 52$, but

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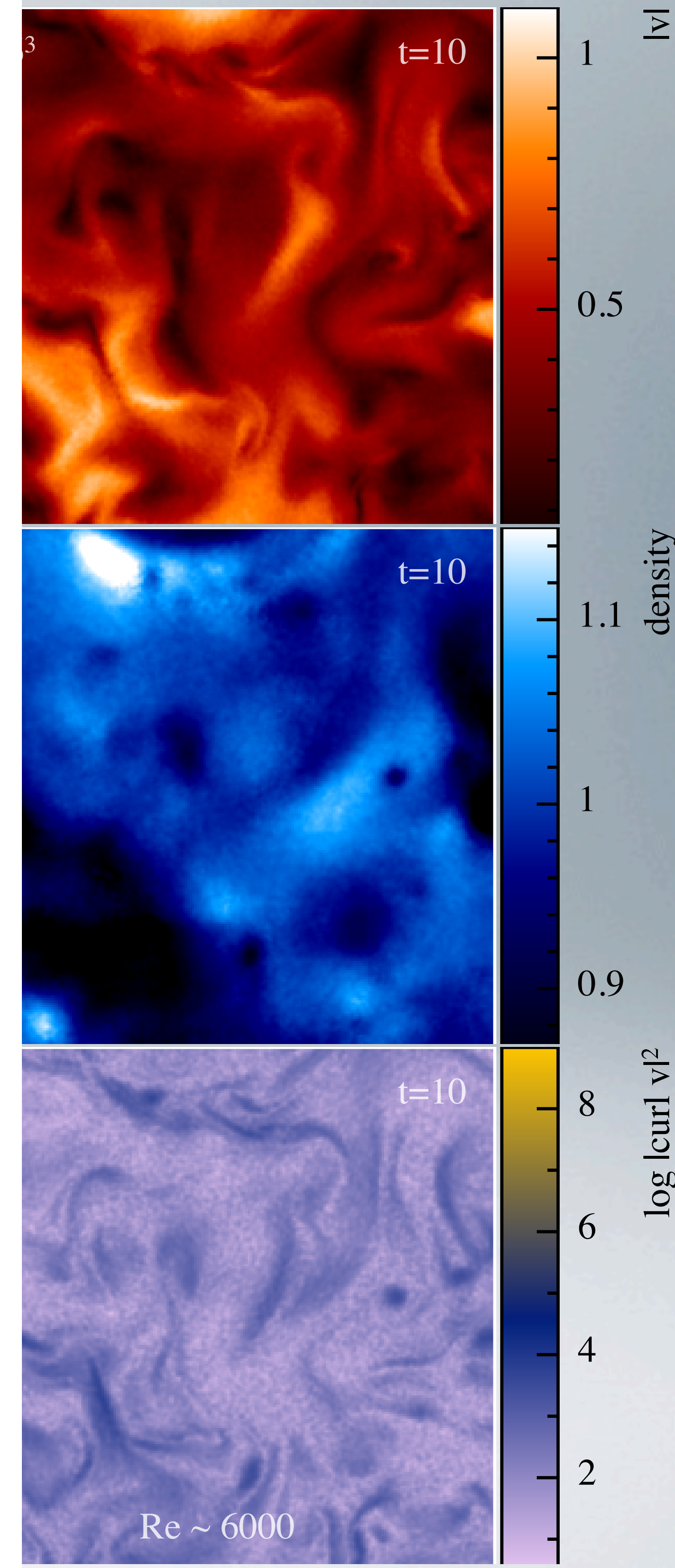


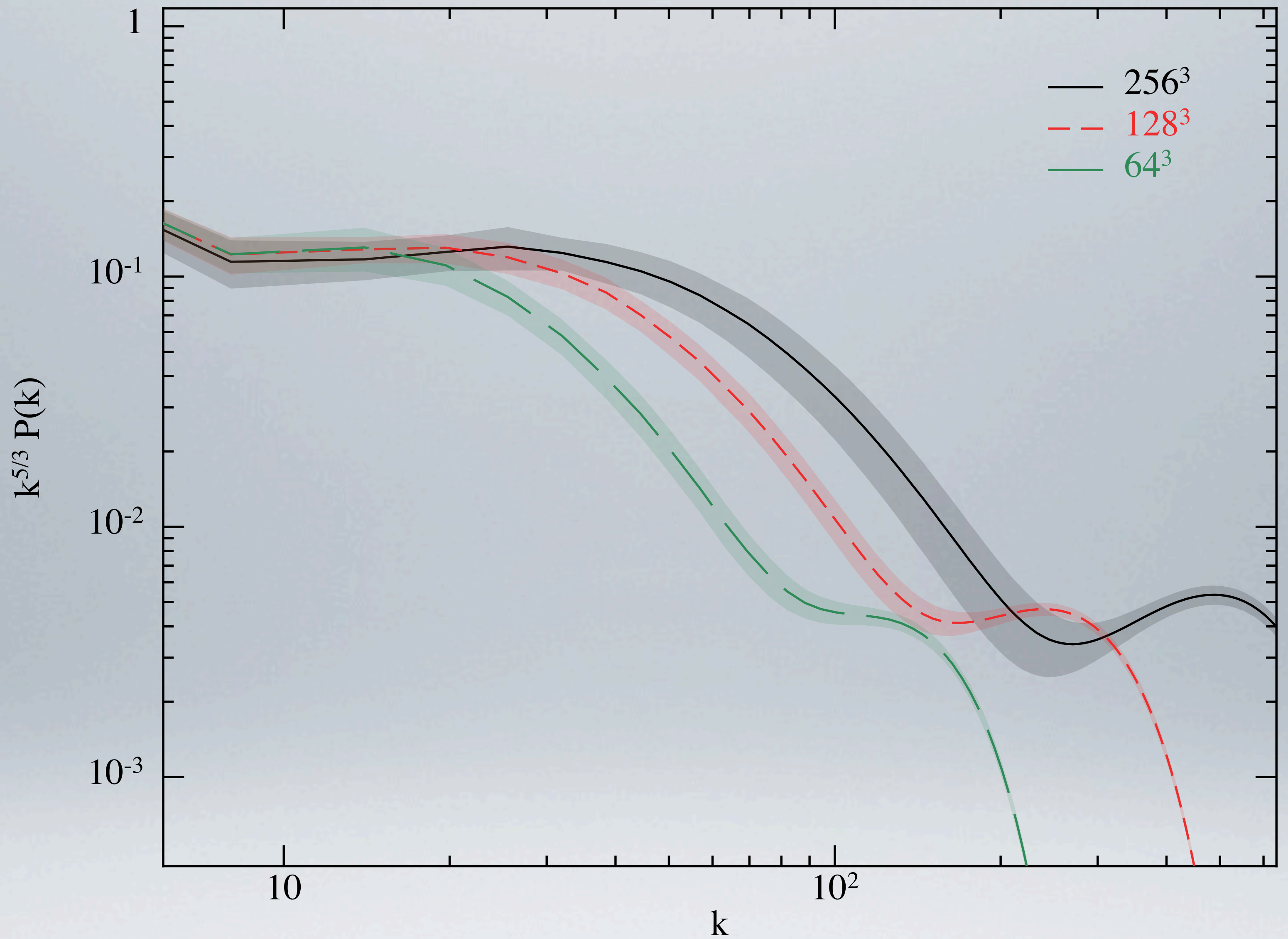
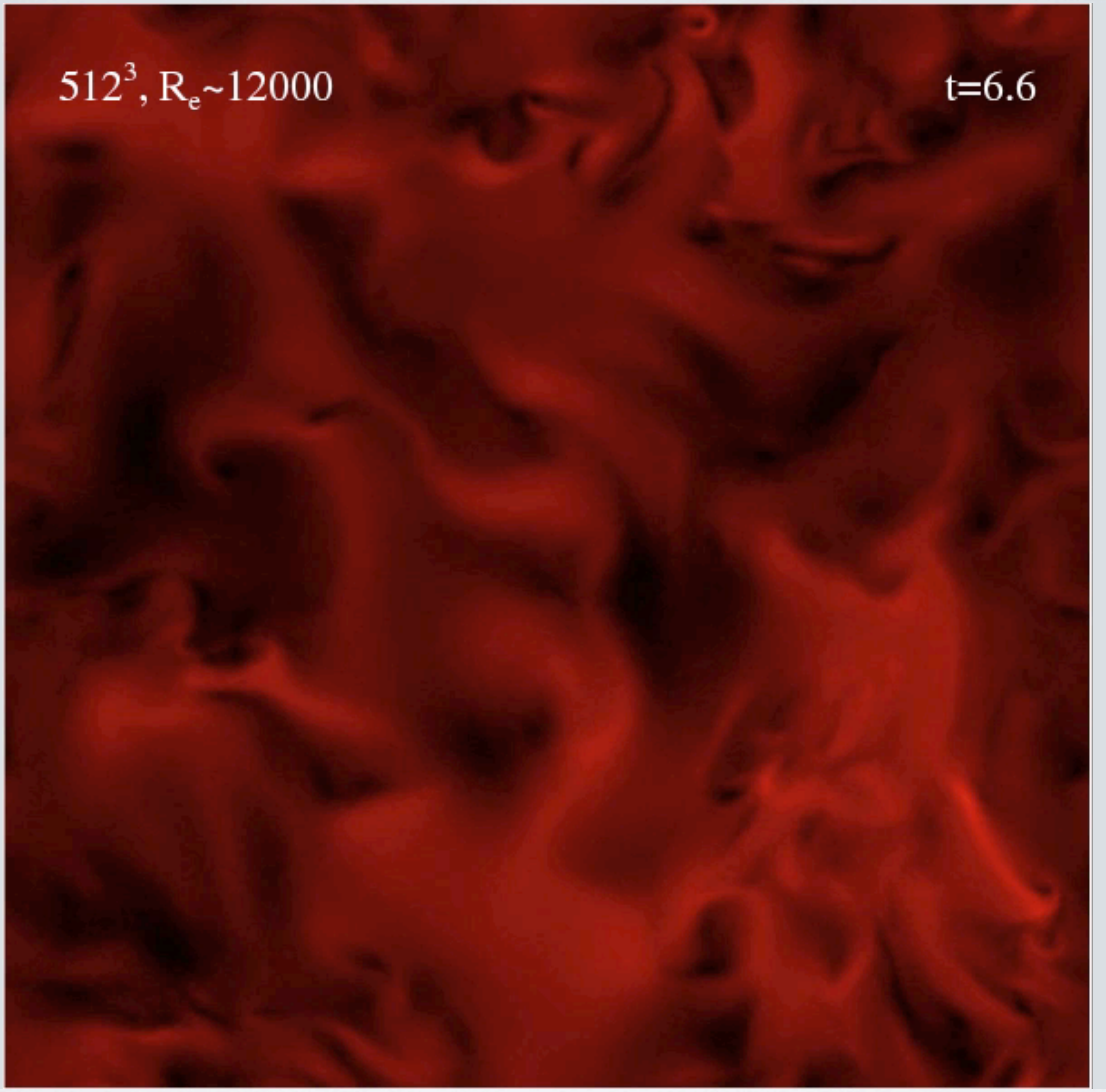


Figure 2. Time-averaged $k^{5/3}$-compensated power spectra from subsonic SPH turbulence calculations using the Morris & Monaghan (1997) viscosity switch at a resolution of 64^3, 128^3 and 256^3 particles, as indicated, for which the corresponding Reynolds numbers are $\sim$ 1500, 3000 and 6000, respectively. The shaded regions show the 1σ errors from the time-averaging. At the highest Reynolds numbers a Kolmogorov-like $k^{-5/3}$ slope is evident at large scales.



Also, much better viscosity switches now available
(e.g. Cullen & Dehnen 2010)

CONCLUSIONS

- Don't believe everything you read on astro-ph
- SPH gives comparable results to grid methods for turbulence studies, but more efficient only if one is interested in the density field / gravity is involved
- Know your Reynolds number - it defines the flow!
- Viscosity switches are the key to high Reynolds numbers in SPH at low Mach number - also easier to achieve high Re at high Mach number