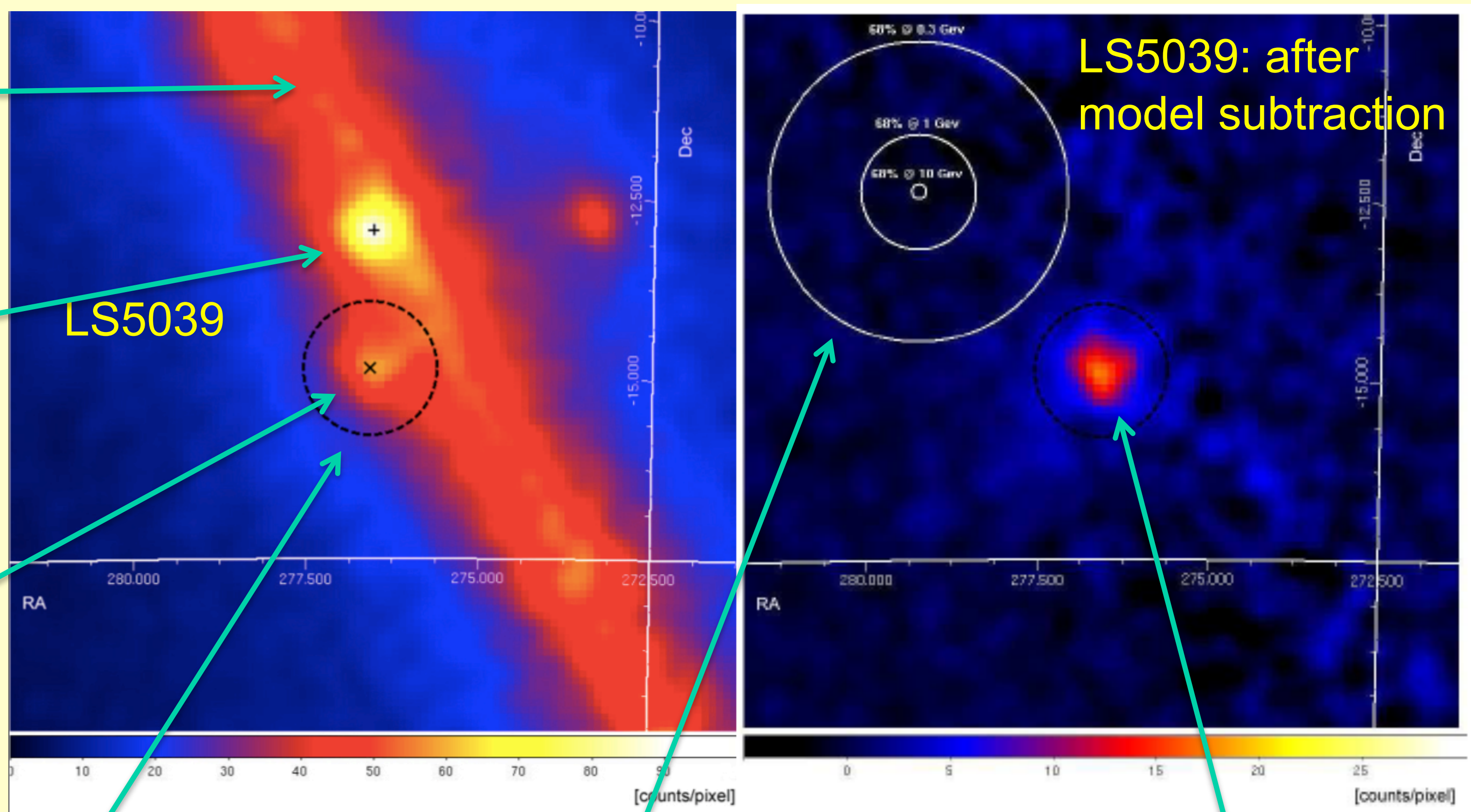
Issues in a Nutshell

Strong diffuse background: 100x source in 10^0 region

Source confusion: nearby pulsar

Source enters and exits FoV: in view 30 mins every 3 hours



Lots of systematics and sanity checks to make: energy cuts, Aeff, diffuse response etc.

Large PSF at low energies

Low stats: 4k photons in 1 yr

Why Likelihood?

• There aren't many photons. In an interesting part of the sky we will collect thousands, but the instrument response has many dimensions: time, angles, energy, and instrument-specific quantities. With sensible binning, most bins won't contain enough photons for χ^2 analysis to be valid.

• With the LAT's broad PSF, many sources will overlap. Care is required to distinguish nearby pairs.

• Direct image deconvolution is dangerous. Poisson noise is amplified to swamp the result. Some sort of regularization is needed, either by making assumptions about the statistical properties of the image (MaxEntropy) or by assuming a simple physical model with adjustable parameters. We choose the latter.

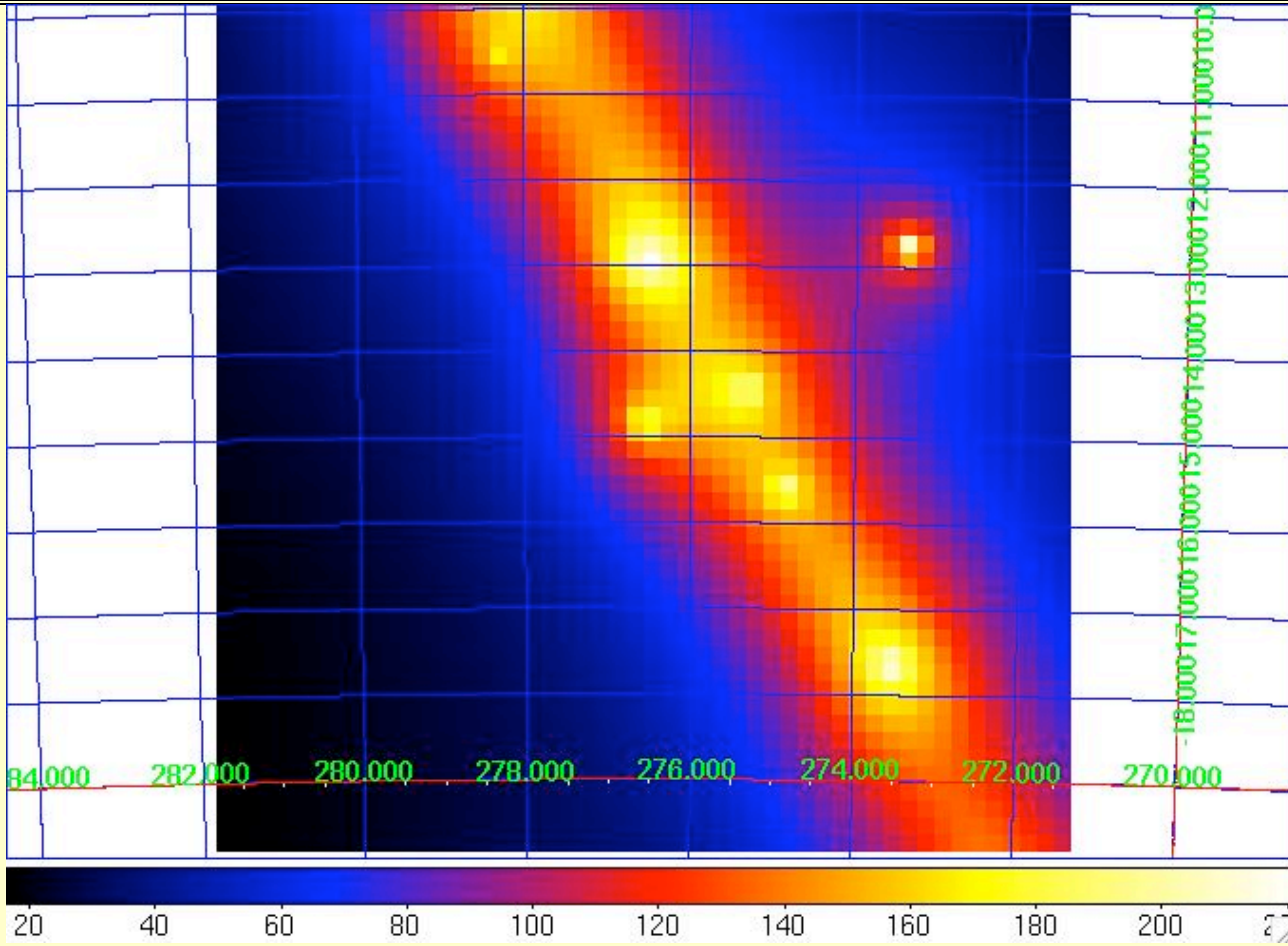
• Pro: Gives quantitative results.
• Con: We won't discover anything we aren't looking for.

What is Likelihood?

The likelihood, L, of a set of data is the probability of observing that data, given our belief about the physical processes that produced the photons that were detected. When that belief takes the form of a model with adjustable parameters, the likelihood can be expressed as a function of the parameters. The parameter values which produce the maximum value of L are useful estimators of the “true” values. Under fairly mild conditions, this process is unbiased and efficient.

The maximum likelihood value does not provide a test of “goodness of fit.” The statistical significance of a point source, for instance, can be determined by the ratio of maximum likelihood values for models with and without the source.

Example of A Model: LS5039



Source Model and Instrument Response

Source model is the sum of point and diffuse sources:

$$S(\varepsilon,\hat{p}) = \sum_i S_i(\varepsilon,\hat{p})$$

energy

position

$$S_i(\varepsilon,\hat{p}) = s_i(\varepsilon)\delta(\hat{p}-\hat{p}_i)$$ for a point source

s_i is typically power law, exponential cutoff etc

Parameterise the instrument response:

$$R(\varepsilon',\hat{p}';\varepsilon,\hat{p},t) = A(\varepsilon,\hat{p},t)P(\hat{p}';\varepsilon,\hat{p},t)D(\varepsilon';\varepsilon,\hat{p},t)$$

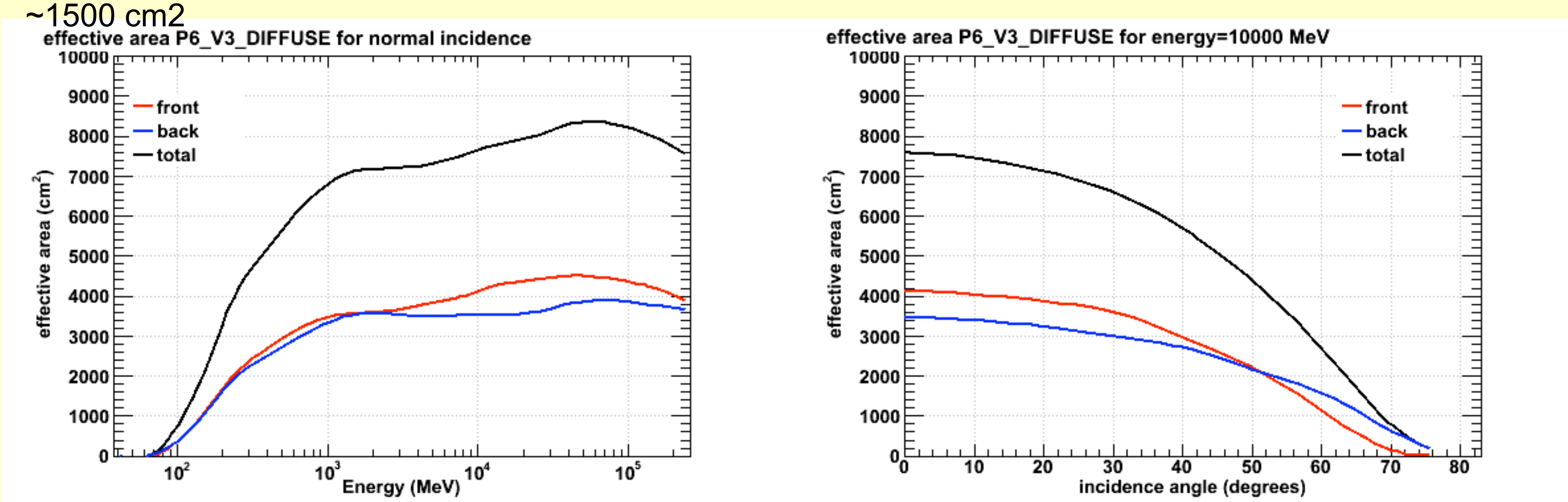
Effective area

PSF

Energy dispersion (ignored)

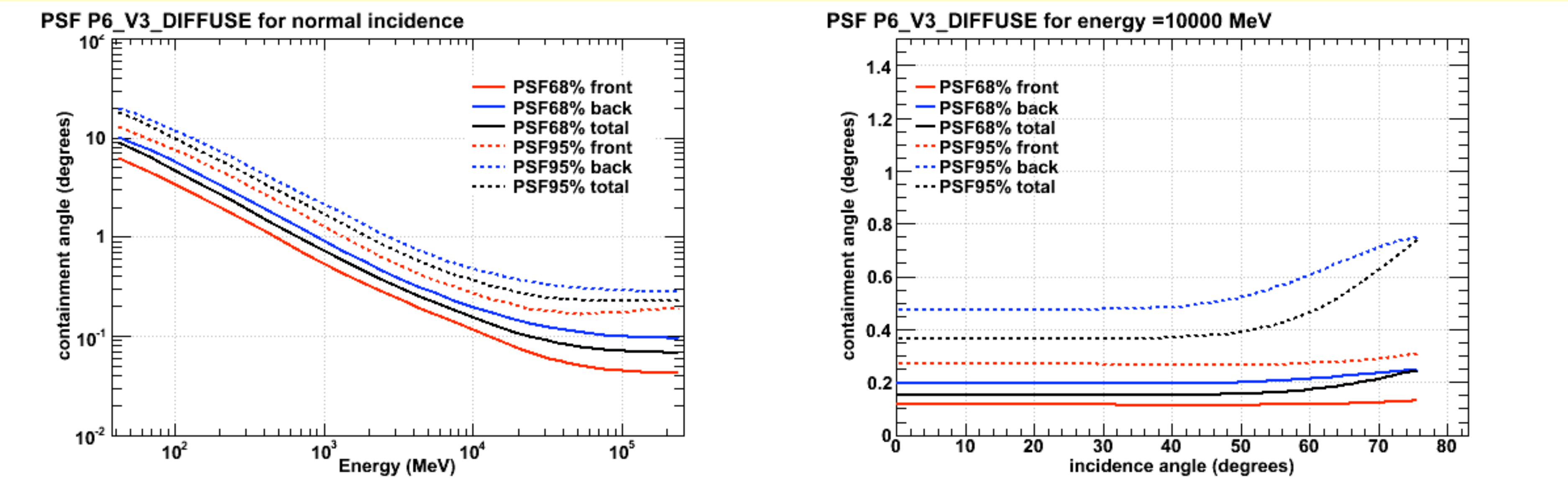
LAT Performance Aeff

c.f. EGRET
~1500 cm2



- Effective area rises rapidly up to 1 GeV.
- Useful data collected out to 65-70 deg from the LAT boresight.

LAT Performance: Angular Resolution



- Angular resolution rapidly improves with increasing energy.
- Improved sensitivity (less background); greatly improved source locations, reduced source confusion - particularly for hard spectrum sources.
- Source localizations 5-10's arcmin typically - can follow up with MW observations.
  - Everything is better when we know where to look!

The Math of Likelihood

We use Extended Maximum Likelihood (EML). This is the proper form to use when the number of photons is not determined before the observation. The quantity to be maximized is

$$\ln(\mathcal{L}) = \sum_i \ln M(x_i) - N_{pred} \quad , \quad N_{pred} = \int M(x)dx$$

M(x) is the model rate of photon detection

$$\sum_j \log\left(\sum_i \int d\varepsilon\, s_i(\varepsilon) A(\varepsilon,\hat{p}_i,t_j) P(\hat{p}'_j;\varepsilon,\hat{p}_i,t_j) D(\varepsilon'_j;\varepsilon,\hat{p}_i,t_j)\right)$$

j is events; i is sources. D is taken to be a delta function in energy. For diffuse sources one must integrate over all positions: [ ] is precomputed for each event

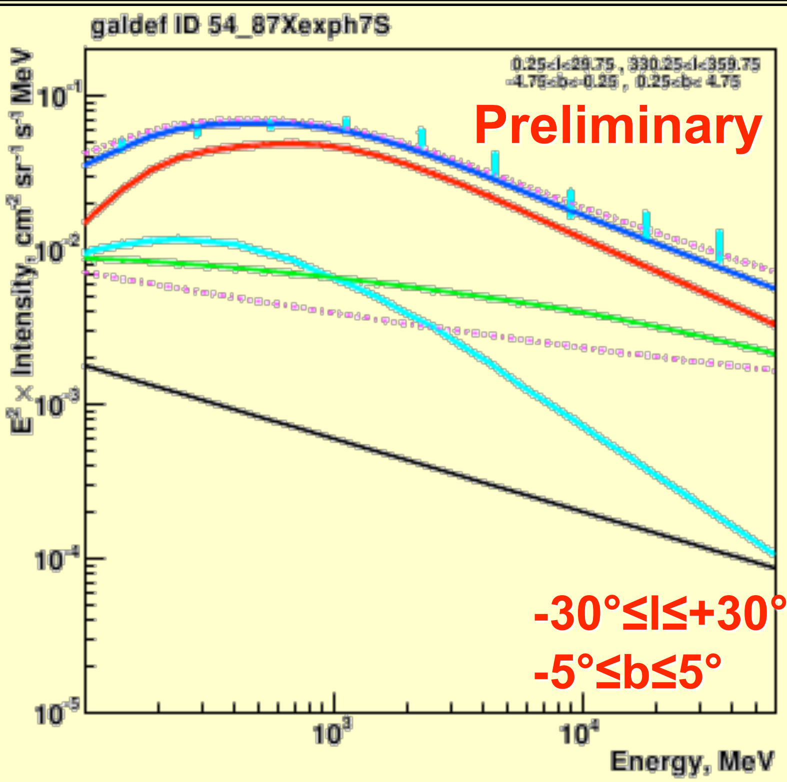
$$\sum_j \log\left(\sum_i s_i(\varepsilon'_j)\left[\int d\hat{p}\, \tilde{S}_i(\hat{p}) A(\varepsilon'_j,\hat{p},t_j) P(\hat{p}'_j;\varepsilon'_j,\hat{p},t_j)\right]\right)$$

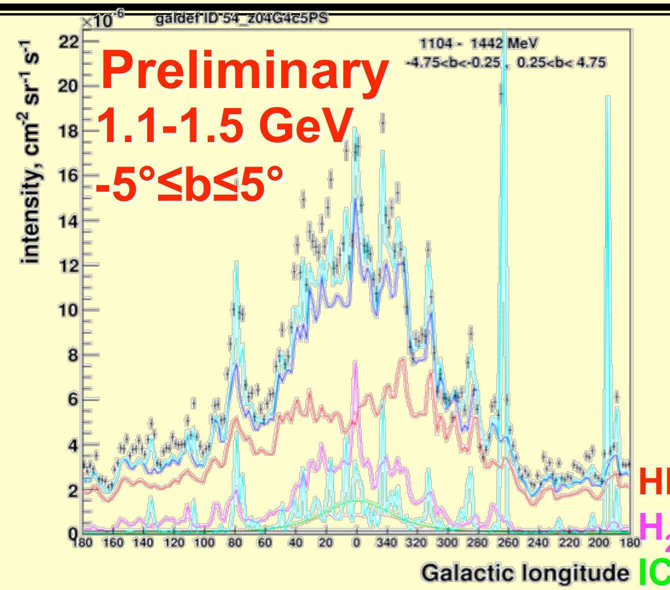
Diffuse Galactic Emission: Spectrum, Longitude, and Latitude Profile

IC:
Total
OPT
IR
CMB
π^0-decay
Brem

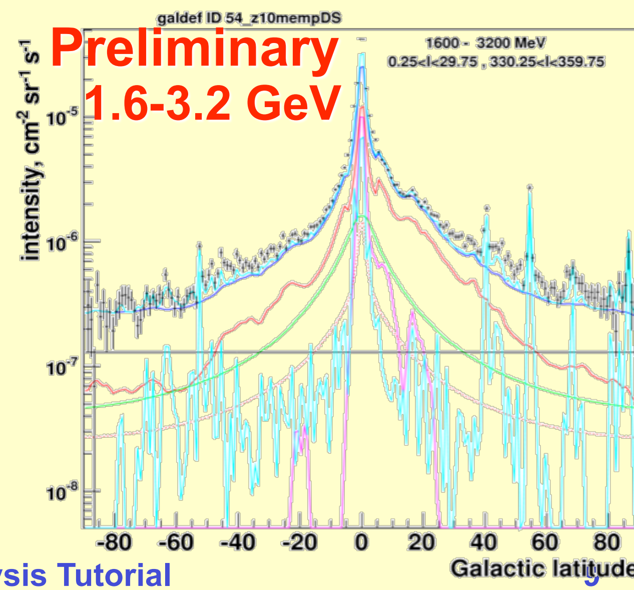
Catalogue sources

Model total





HI
H_2
IC
Model
Model + Sources



Model describes large-scale diffuse emission over whole sky within 10%

From Troy Porter

Likelihood vs. χ^2

If the data are governed by Poisson (counting) statistics and the number of counts is large, then the familiar minimum-χ^2 estimator is an approximation to the exact likelihood. Up to an irrelevant additive constant,

$$\chi^2 \approx -2\ln(L)$$

Or when comparing two models,

$$\Delta\chi^2 \approx -2\ln(L_2 / L_1)$$

Wilks's Theorem guarantees that this likelihood ratio will be drawn from a χ^2 distribution if the total number of photons is large, even if they can't be put into bins with large numbers.

TS

A basic tool of the EGRET analysis is the “Test Statistic”, or TS.

When two models are compared, $TS \equiv -2\ln(L_2 / L_1)$
Where L_1 and L_2 are the maximum likelihood values for the two models.
As we have seen, TS has an asymptotically χ^2 distribution.

TS is applied when the difference between the two models is the presence of an extra point source in model #2. The statistical significance of the new source can be determined by treating TS as a χ^2 value with one degree of freedom, or sqrt(TS) as the gaussian “sigmas” of detection.

A TS map can be made by placing the putative source in many places, calculating TS in each place. This is a means of searching for unknown point sources.

Exposure

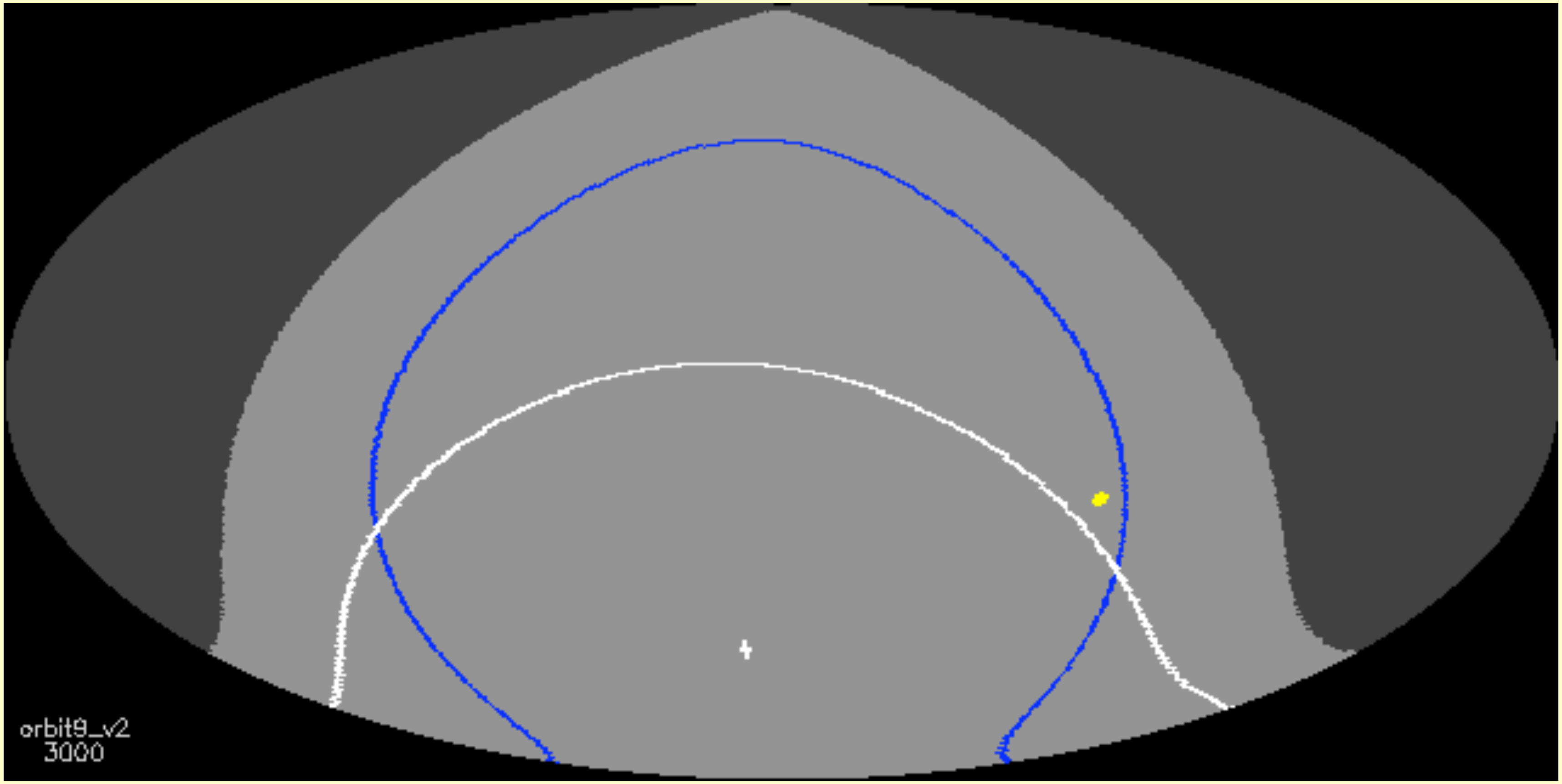
The calculation can be simplified by the use of "exposure". If the source spectrum is the only set of parameters that can be adjusted, most of the integral needs to be calculated only once. Exposure has dimensions area × time. It describes how deeply a spot on the sky has been examined. For diffuse sources, exposure must be calculated at many points.

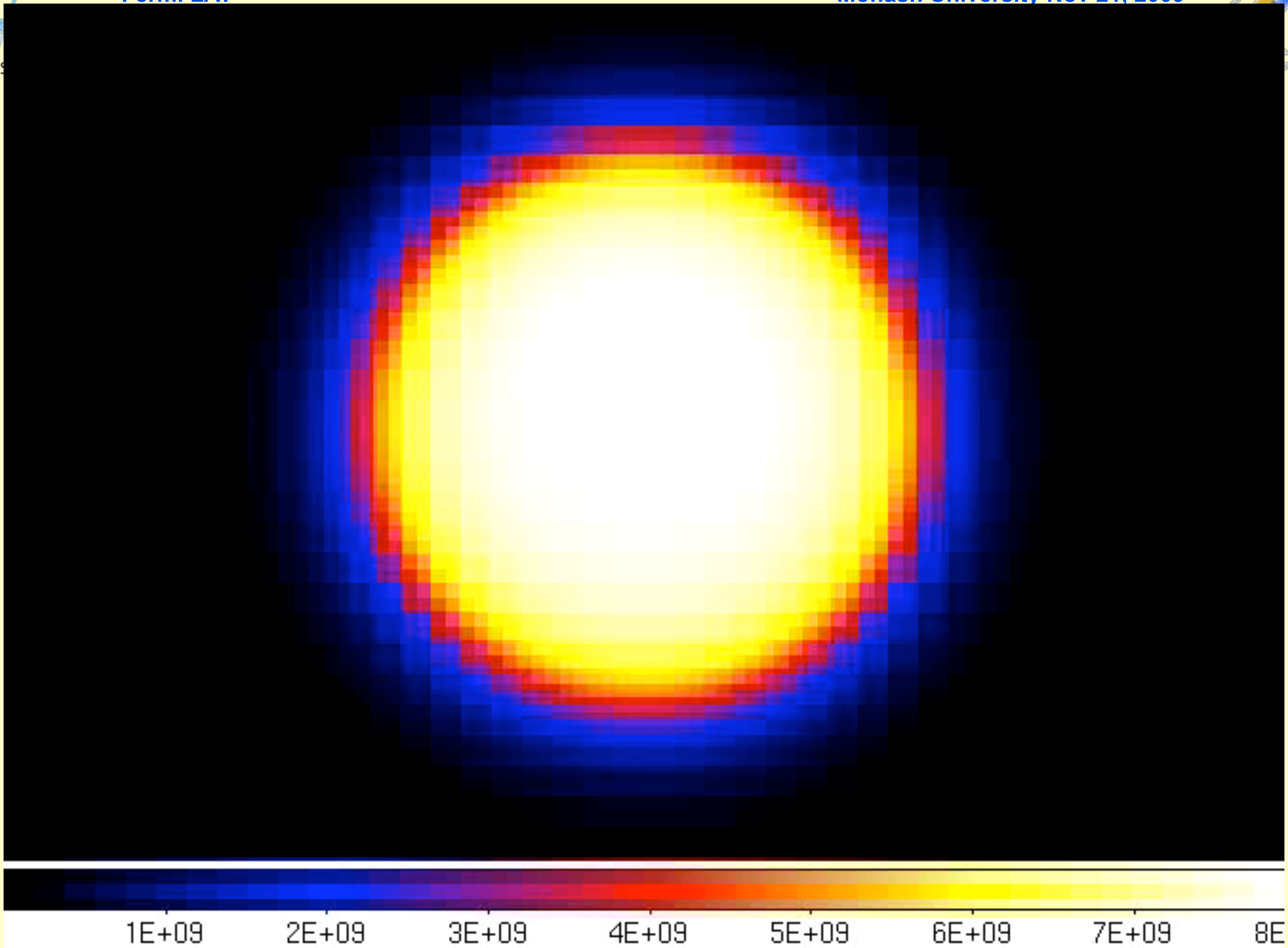
The exposure calculation isn't simple. It looks like this for a particular energy E_m:

$$\mathcal{E}_m(\mathbf{p}) = \int dE' \int d\mathbf{p}' \int dt\, h(t) \sum_k H(\mathbf{u}(t)\cdot\mathbf{p}' - \mu_k(E')) \times A_k(E_m, \theta(\mathbf{p},t), \phi(\mathbf{p},t))\, P_k(\mathbf{p}'; E_m, \theta(\mathbf{p},t), \phi(\mathbf{p},t))\, D_k(E'; E_m, \theta(\mathbf{p},t), \phi(\mathbf{p},t)) \quad (1)$$

The d**p**' integral covers the ROI, but this must be evaluated for directions **p** within the larger source region.

Survey Orbit Animation




1E+09
2E+09
3E+09
4E+09
5E+09
6E+09
7E+09
8E

Approximations

To save CPU time, approximations can be made.

•The energy dispersion can be treated as a delta function.

•If a point source is far from the edge of the Source Region, its PSF integrates to 1.

•The effects of the zenith cuts on N_{pred} can be ignored.

•Etc., etc.