Graphs in Economics – a sampler

Simon Angus
Dept. Economics, Monash University

16 Dec 2013
Dept. of Mathematics, Monash University

1 The Helicopter view: 'Graphs' & 'Networks' in Economics

Menu

1. * The Helicopter View: ~~'Graphs'~~ 'Networks' in Economics
2. Small n networks: sample – communication network formation;
3. Large n networks: sample – power-laws & small-worlds;
4. * Some examples from my own work;
5. * Concluding: final comments on synergies for Graph-Theorists.

Publishing 'Graphs' in Economics: EconLit Survey

Search: `article.title contains 'Graph Theory'`

- 16 titles mention 'Graph Theory'
- ... 69 mention 'Graph Theory' in any field:
  - A* (Econ only): Journal of Economic Theory, Games and Economic Behavior, International Economic Review.
  - A (Econ only): Journal Of Mathematical Economics, Public Choice, Energy Journal, Economic Modelling, Journal Of Post Keynesian Economics, International Journal Of Production Economics.

Search: `article.title contains 'Network'`

- 5,608 hits, year range: $[1948, \ldots, 2013]$
- Top 50 Journals (by hit) accounts for 2,313 hits (41%)
- Well ranked (ERA/ABDC A* or A):
  - A* (Econ + Field): American Economic Review (E, Top5), Rand Journal of Economics (E), Games and Economic Behavior (E), Journal of Economic Theory (E), Journal of the American Statistical Association (F), Research Policy (F), Operations Research (F), International Journal of Urban and Regional Research (F).

Search: `article.title contains 'Network'` **(cont.)**

- Well ranked (ERA/ABDC A* or A):
  - A (Econ + Field): Economics Letters (E), Journal Of Economic Geography (E), Journal Of Transport Economics And Policy (E), International Journal Of Production Economics (E), Journal Of Economic Behavior And Organization (E), Journal Of Evolutionary Economics (E), Regional Studies (F), International Journal Of Industrial Organization (F), Public Administration Review (F), International Journal Of Forecasting (F), Journal Of Regional Science (F), Papers In Regional Science (F), Business History (F), World Development (F),
  - Other Journals of note: *Econometrica* (A*, Top5), *Journal of Public Economics* (A*, Top5), *The Review of Net-*

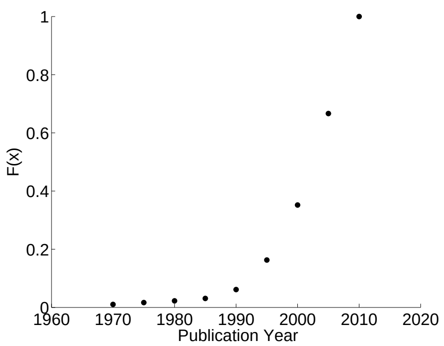
work Economics (ERA C, fr. 2010, Berkeley)

The Most Common Subjects

- transactional relationships; contracts and reputation; networks [1151]
- network formation and analysis: theory [798]
- economic sociology; economic anthropology; social and economic stratification [710]
- transportation: demand, supply, and congestion; safety and accidents; transportation noise [493]
- telecommunications [419]
- neural networks and related topics [307]
- technological change: choices and consequences; diffusion processes [288]
- multinational firms; international business [286]
- industry studies: utilities and transportation: government policy [244]
- production management [237]

Welcome to the age of Networks in Economics!

Data Source: EconLit, articles titles containing 'network'.



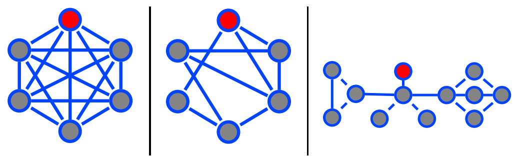
Source: EconLit, articles titles containing 'network'.

Research: Economics & Networks?

What are the key network questions of Economists?

(my interpretation) It's about selection:

Does with *whom* I interact with (to play games, get a job, trade) or become informed about (to make decisions, change behaviours, learn from) matter for economic paths & outcomes?



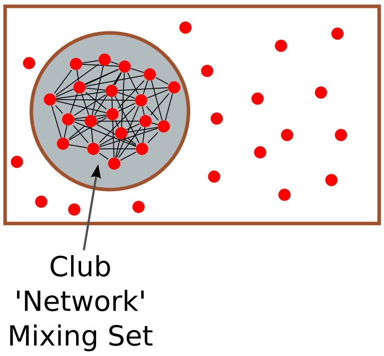
So how do diverse economic networks come about?

- By design (e.g. school classes, within-class activity groups, experimental grouping/treatment, physical/spatial location)
- By history (e.g. village level grouping, cultural/religious/ethnic/socio-economic grouping)
- By choice (e.g. self-selected associations for group work, friendship/professional contacts, ethnic/religious/cultural preferences)

Side-note: not all 'networks' are networks

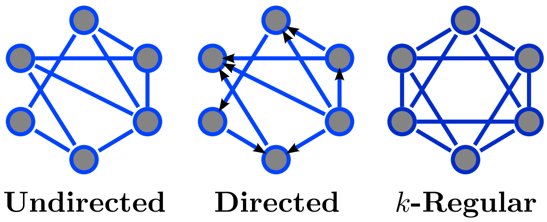
'Networks'

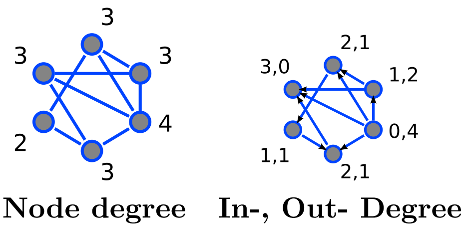
- Selection 'sets': A (1,4,5,7,9) vs. B (2,3,6,8,10)
  - Access to information;
  - Type differentiation;
  - Lower transaction costs;
  - Entry criteria? (cost?)
- Actually: 'Network' = Complete Graph



Note: 'Network' as selection sets *See an example of this for the 'old-boy' network in [23].*

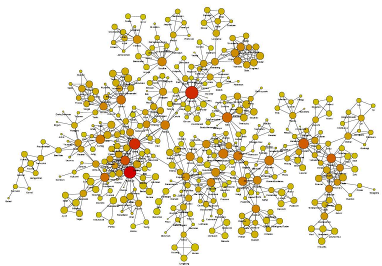
Graph-theoretic Networks . . .



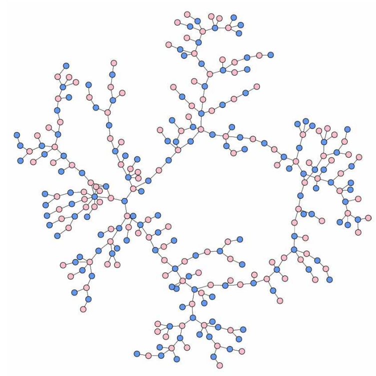


And 'Complex' Networks . . .

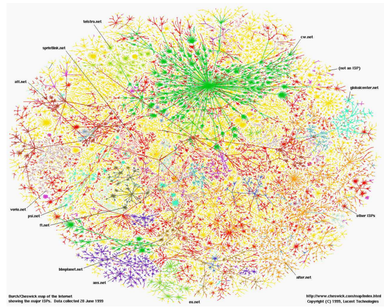
Co-Author Network (Network research) (source: Mark Newman)



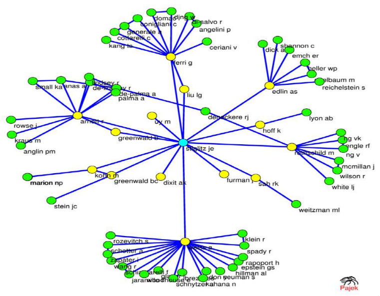
School Dating Network (source: Bearman, Moody, and Stovel)



(Physical) Internet Service Providers (source: Source Bill Cheswick/Lumeta)



Co-Author Network (Stiglitz, Economics) (source: Goyal et al.)



How complex is complex?

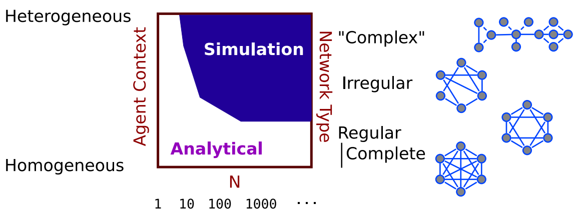
Complexity of network analysis splits research field

- Scale of <u>distinct</u> networks enormous
  - $O(n) = 2^{(n(n-1)/2)}$ (undirected graphs, n vertices)
  - Example: $O(6) = 2^{15} = 32,768$!!
- Analytical solutions impossible (so far) beyond approx. $n > 10$
- Implications:

- $n < 10 \longrightarrow$ Analytical
- $n > 10 \longrightarrow$ Computational/Numerical simulation

- But ...
  - Complexity of the graph affects this .. e.g. k-Regular graphs yield to analysis (sometimes)

Carving up 'Selection'/'Context'/'Network' Space ...



2 $n < 10$ Networks

Small n networks: Agenda

Agenda: The case of Communication Network formation

1. Theory: communication network formation as an endogenous selection activity
2. Practice: what do subjects actually do?
3. What have we learnt about endogenous network/selection issues?

Communication network formation

Example 1 (Bala & Goyal (Econometrica, 2000)[3] 'A Model of Noncooperative Network Formation). **Model**

1. Agents play **network formation game**, where a **strategy** is a vector of **link sponsorship decisions**:

$$g_i = (g_{i,1}, \dots, g_{i,i-1}, g_{i,i+1}, \dots, g_{i,n})$$

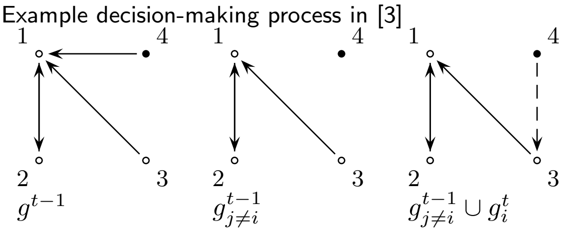
2. Sponsoring incurs a link cost, but gains access to information which is of value;
3. Information (value) **flows** in the resulting network;

Solution Method

- Assume **myopic best-response updating**: each agent simultaneously revises her strategy of $t-1$ to **best-respond** to the graph of $g^{t-1}_{j \neq i}$;
- Assume that at least one agent suffers **strategic inertia** (doesn't update) to close model.

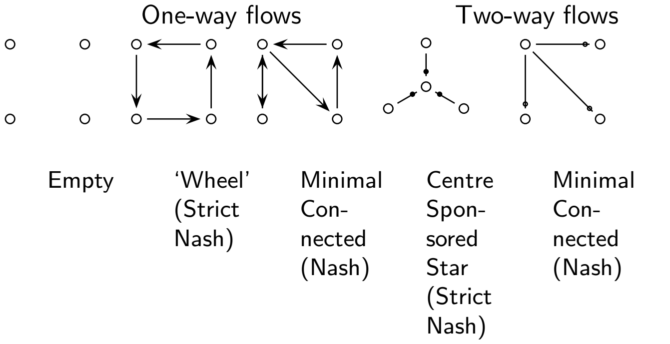
Example BG decision-making process

Example decision-making process in [3]



BG Predictions

Theoretical Predictions in [3]



BG in the Lab

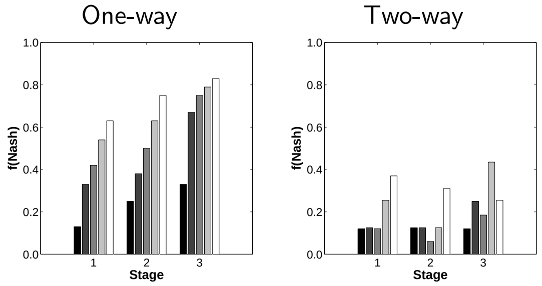
Example 2 (Falk & Kosfeld (Inst. Study Lab. 2003)[19] 'It's all about connections: evidence on network formation'). **Findings**

Flow	Edge Costs	Structure					$\langle d_i \rangle$
		m1c	('wheel')	empty	m2c	(cs-star)	
One-way	Low	0.48	(0.41)				1.19
	High	0.59	(0.49)	0.10			0.76
Two-way	Low				0.31	(0.00)	0.91
	High			nr	0.09		0.75

1. **One-way flows**: 'Strict' Nash refinement good predictor;
2. **Two-way flows**: **No** centre-sponsored star networks;
3. Strong evidence of improvement (learning?) during rounds
4. Explanations for deviations:
   - Symmetry of strategies (not convincing)
   - Inequity aversion (evidence from Probit regressions on liklihood of updating strategy)

Improvements between matching periods

Evidence of learning in the one-way network formation trials of [19]



Example 3 (*Callander & Plott (J. Pub. Econ., 2005)[8] 'Principles of network development and evolution: an experimental study'*). **Questions of Interest**

1. *Do networks converge to steady state outcomes? (What are the properties of the state?)*
2. *What principles drive the evolution of networks?*
3. *How is the process influenced by the institutional environment?*

Possible Principles of action:

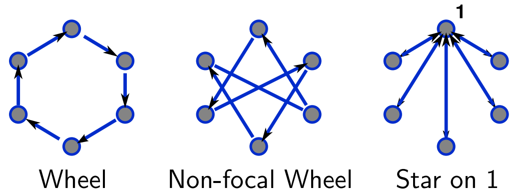
1. **Nash Equilibrium**: Nash and Strict Nash (in the sense of BG's model) considerations;
2. **Efficiency**: Information gains relative to network formation costs ('bang for your buck');
3. **Focalness**: In the spirit of Schelling ('The stragety of Conflict', 1960) – position in room, data on blackboard/screens etc.

Methodology

- $n = 6$;
- One-way links only;
- Position in lab important.

Trmnt	Link cost	Info Value	NE (Strict)	Efficient	Focal
1	\$0.15	\$0.20	wheel	wheel	cc/c wheel
2	\$0.15	\$0.25	wheel	wheel	cc/c wheel
3	\$0.30(→n), \$0.15(→o)	\$0.25	wheel	non-focal wheel	cc/c wheel
4	\$0.00(↔1), \$0.15(→o)	\$0.25	wheel/star	star on 1	cc/c wheel

Expected graph outcomes for treaments in [8]



Key results

1. **"Networks happen"** (and are economic in nature)
   - No empty graph (they happen)
   - Nor complete graph (externalities taken into account)
   - *Conclusion* 'neworks can arise by economic forces'
2. Networks **converge** to stationary configurations
   - 8/12 nets converge to stationary configurations
   - (Recall, despite massive network space)
3. Nash Eq **necessary** condition for stationarity
   - All 8 convergent nets are NE of one-shot game
4. Convergence aided by **continuous adjustment** institutions
   - **Discrete updating**: 2/5 converge
   - **Continuous updating**: 6/7 converge
5. **Reject** as necessary conditions for stationarity: **focalness**, **efficiency** and **strict NE**
   - (By counter example)
6. Nash (or Strict Nash) **not sufficient** for stationarity
   - Examples of NE, and SNE configurations that were unstable
   - Why?! – Boredom, confusion, mistakes? .. not likely
   - Common knowledge of maximizing behaviour (to drive NE results) not supported?
7. Nonconvergent networks **do not** exhibit increaseing efficiency
   - Nonconvergent experiments (10, 15, 15 and 17 rounds' worth) show no increasing trend in efficiency (info/link)
   - ... Network formation more like **public economics** (requirement of coordinating mechanism to achieve efficiency), rather than markets (where efficiency normally approached)

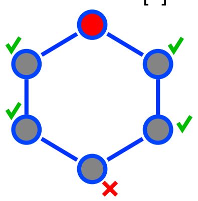
What are the subjects doing?

1. **They are following best response behaviour**
   - **No**: not even best-responding with errors
2. **They are using some kind of heuristic**
   - **Probably**: authors define **Simple Strategic Decision Rule**
   - **Rule**: 'form counter-clockwise wheel' (!)

Example 4 (*Berninghaus et al. (J. Evol. Econ., 2007)[7] 'Evolution of networks – an experimental analysis'*). **Approach**

- BG noncooperative network formation game;
- **Two-way** information flow ... stars!
- Key study: **discrete** vs. **continuous** strategy updating
- Key difference to Falk & Kosfeld – 'neighbours' only 2 hops away *if sponsoring a link to 1st neighbour*

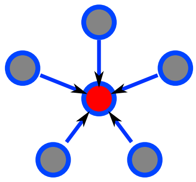
Neighbourhood definition as in [7]



Theoretical Predictions

- Should expect to see **periphery sponsored star** (PSS) (instead of centre-sponsored star as for no information decay)

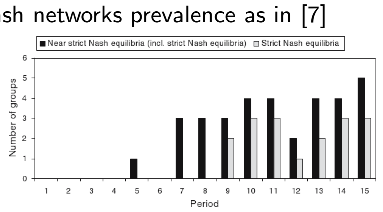
Periphery sponsored star as expected in [7]



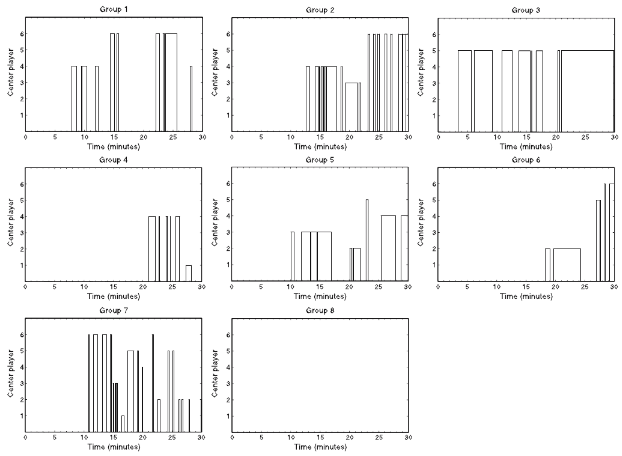
Results

- **Strict Nash Play observed**
  - Discrete: 50% of all groups reach (or get very close to) Strict Nash Network (PSS)
  - Continuous: 7/8 groups reach SNN (25% of time spent in PSS)

Strict Nash networks prevalence as in [7]



Observed stars and their centre player as reported in [7]



Other Findings

1. Solving the complex network formation problem easier in **continuous time** than in discrete time
   - Higher proportion of PSS played in second (continuous) treatment
   - Similar resuls to pure coordination games (e.g. Berninghaus and Ehrhart 1998)
2. **Inequity aversion** explains at least some of the behaviour of subjects (as per Falk & Kosfeld)
   - Sharing of centre-player
   - Willingness to undertake PSS due to smaller payoff differences than in FS
3. **Myopic best reply dynamics** not a good (only) model for individual behaviour
   - Players deviate from strict nash networks (despite incurring high losses)
   - Sometimes, several times in the one setting

Communication Network formation and Selection

How do agents build (communication) networks?

- The analytical machinery of **Game Theory** (e.g. Nash, Strict Nash) a good predictor of convergent networks (when they occur);
  - In one-way flows certainly
  - In two-way flows under continuous updating
- But .. actual **behavioural decisions** at the individual level, more likely a combination of heuristics, focal structures and best-reply considerations (and **not** best-reply only);

- **Inequity aversion** can't be rejected as a motivating factor in network formation
- Evidence for a kind of **social learning/teaching/signalling** in this complex decision-making environment (not always successful)

Return of the 'as if' assumption?

> "The systemic behavior is predicted by the model for Bala and Goyal, but the evidence at the individual level conflicts with the microbehavior postulated by their model. This combination leads to a paradox frequently observed in economic experiments that the models work well when applied at the systemic level, but the exact behavior of the agents is at odds with the behavioral principles at the foundation of the model." (Callander, p.1487)

> "These results indicate that most agents engage in strategic signaling and coordination efforts through their link selection. ... Further, the results imply that agents behave, in some sense, strategically and with foresight in network environments. Many agents employ a strategy that seems to be an attempt to teach, signal, and coordinate all agents within a network and in doing so facilitate movements towards Pareto opitmality." (Callander, p.1489)

Coda: Endogenous Networks and Behaviour

How do non-uniform interaction structures affect behaviour?

- Uniform, exogenous interactions: Kandori et al. 1993
  - Long-run: convergence on risk-dominant (rather than efficient) outcomes
- Non-uniform, exogenous interactions: Ellison (1993)
  - As above
- Non-uniform, endogenous interactions: Ely (2002), Jackson & Watts (2002)
  - (Ely) Efficient (even if not risk-dominant) eq. achievable
  - (Jackson & Watts) Possible to have stochastically stable states that are **neither** risk-dominant, nor efficient

Note: More on endogenous networks and behaviour *The references covered in the Coda include [24; 16; 17; 21], the latter of which follows an alternative network formation model by Jackson and Wolinsky (1996) [22] which introduces the concept of* **pairwise stability** *– where links are formed and maintained only if the two agents involved find it utility enhancing to do so.*

Note: Other related literature *The reader is also referred (without comment) to several other notable contributions on network formation [14; 10; 25; 13] and how networks can affect behavioural outcomes [6; 2]*

3 $n > 10$ Networks

Large n networks: Agenda

Three 'statistical' networks ...

1. What are the main types of (large) networks?
   - (Erdös and Rènyi (random) graphs);[18]
   - Power-law degree distributions;
   - 'Small-world' networks;

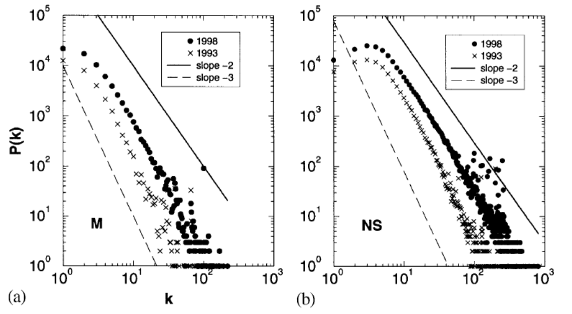
Power-laws in real data

In Networks, most popular is preferrential attachment model

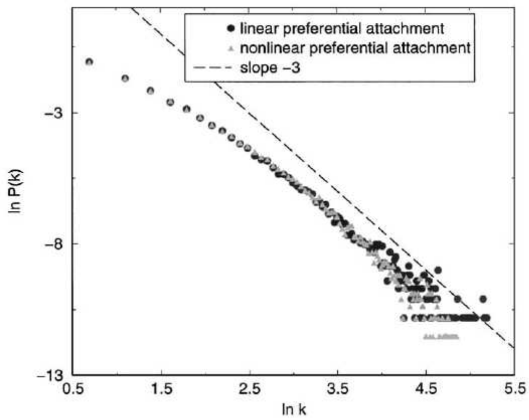
Example 5 (*Barabasi et al., (Phys. A, 2002)[4] Evolution of the social network of scientific collaborators)*. Consider two very large collaboration networks due to Mathematics (M) (70,975 authors) and neuroscience (NS) (209,293) during 1991-1998.

- Find power-law degree distributions;
- Find that 'old' authors more likely to be selected by 'new' authors;
- Find that authors who both have high degree form co-authorships more often.
- Conduct numerical simulation study of a resultant preferrential attachment model.

Field Data



Numerical simulation



(Adjusted) Power-laws in real data

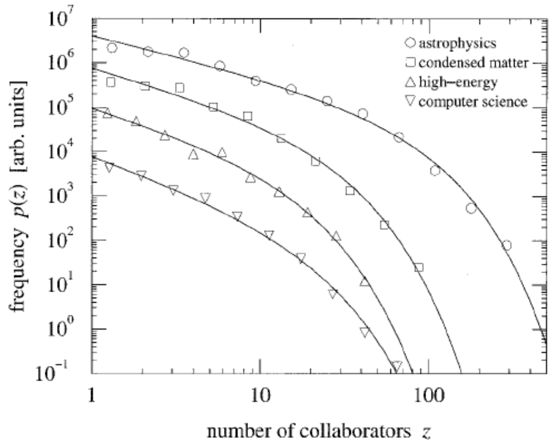
Example 6 (*Mark Newman, (PNAS, 2001)[27; 28] The structure of scientific collaboration networks).* Another very large collaboration network: MEDLINE (biomed, 2,163,923 papers), Los Alamos e-Print Archive (theo. phys., 98,502), SPIRES (high-energy physics, 66,652), NCSTRL (comp. sci., 13,169).

- Find adjusted power-law degree distributions fit very well;
- Adjustment? .. Power-law with 'exponential' cutoff:

$$P(k) \sim k^{-\tau} e^{-k/k_c}$$

- Why the cut-off?
  - Finite sampling – authorship lifetime!

Power-law cutoff in collaborator distribution



Note: On fitting power-laws to emperical data
An excellent instructional work on this non-trivial problem is to be found in [11].

The 'Small-World' Problem

The 'Small-world' effect

- Key studies: *Study of a large sociogram* (Rapoport and Horvath, 1961) and 'The Small-world Problem' (Milgram, 1967)[29; 26]
- Milgram's study:
  - Send 160 parcels across USA with name and rudimentary details of a single target person on them;
  - Intermediates to send parcel to **someone they believed to be 'closer'** (distance, socially etc.) to the person to speed the parcel.
  - Results: **Only 26% made the target**, **median 'hops': 5.5** .. a.k.a. **'Six degrees of separation'**
- Subsequent small-worlds:
  - **Acting networks**, **power-grids**, **neural network of** *C. Elegans* (Watts & Strogatz, 1998);[32]
  - **World-wide-web** (sites, not ISPs) (e.g. Adamic and Adar, 2003);[1]
  - **Firm-based collaboration networks** (Baum et al., 2003);[5]
  - **Internet dating communities** (Holme et al., 2004).[20]

Note: Follow-up Small-World Study *Further seminal studies on the Small-world effect were subsequently carried out by Milgram. For instance, [30] repeated the initial 1967 study of Milgram and found that of 217 folders that actually were sent by starting individuals, only 64, or 29% made it to their targets – very similar to the initial study. Similarly, the mean of the 'chain-length' between starters and targets was 5.2 links.*

Models of Small-worlds

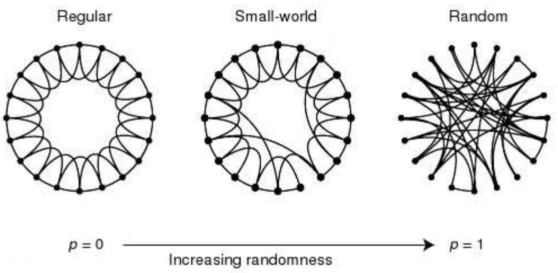
Example 7 (*(Watts & Strogatz (Nature, 1998)[32] Collective dynamics of 'small-world' networks).* The algorithm: Start with a **k-regular graph** and then progressively re-wiring randomly selected edges. Test statistics:

1. **Characteristic Path Length (L)** Average distance (in hops) from one node to every other node in the graph;
2. **Clustering coefficient (C)** Average proportion of triangles;

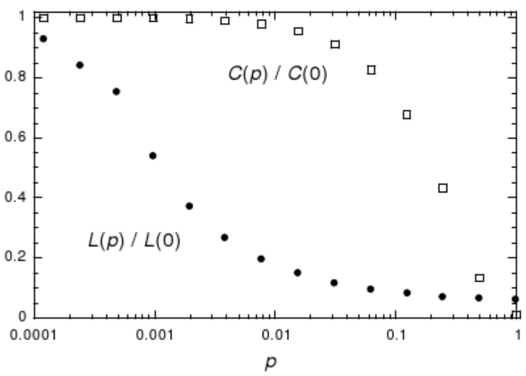
Key claims about 'small-worlds' relate to **equivalent random graphs**:

1. Equivalent (i.e. small) average path-length: $L(SW) \sim L(R) \sim \frac{\ln(n)}{\ln(\langle k \rangle)}$
2. Much higher clustering: $C(SW) \gg C(R) \sim \frac{\langle k \rangle}{n}$

Rewiring algorithm, with probability p



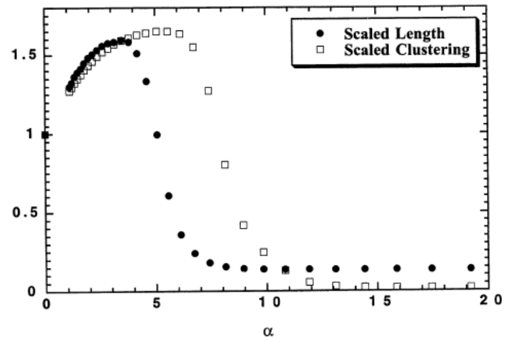
Comparision of C and L under rewiring with regular graph



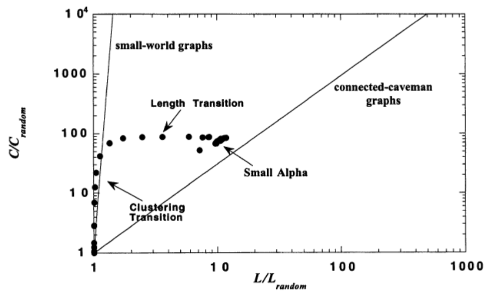
Example 8 (*Watts (1999)[31]Networks, Dynamics, and the Small-World Phenomenon).* **Proposal** Any new link between two individuals $i \leftrightarrow j$ depends on the number of **adjacent** edges at i and j, average degree of graph, and a 'tuning' parameter, α.

$$R_{i,j} = \begin{cases} 1 & m_{i,j} \geq k \\ \left[\frac{m_{i,j}}{k}\right]^{\alpha}(1-p)+p & k > m_{i,j} > 0, \\ p & m_{i,j} = 0 \end{cases}$$

Tunable parameterisation of Small-world graphs



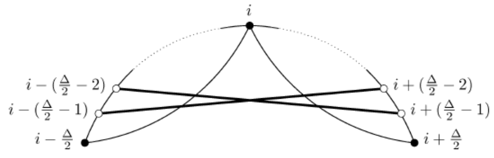
The transition away from Small-World characteristics



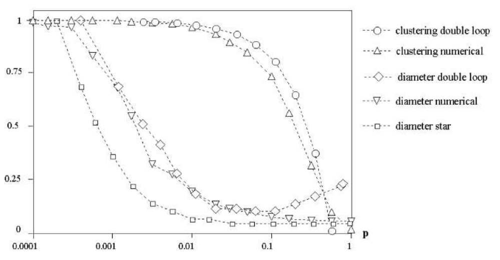
Example 9 (*Comellas et al. (Inf. Proc. Lett., 2000)[12] Deterministic small-world communication networks).* **Proposal** Unlike Watt's model, where SW occur through random linking in an uncontrolled way – form SW through specific algorithm that **preserves the regularity of the graph**.

- Main advantage is **exact results** for graphs generated;
- Useful for further analytical work.
- NB: Keeps **diameter** constant (rather than average path-distance (L).

Regularity preserving re-connections



Comparison of the Comellas and Watts & Strogatz models



Note: On Small-worlds formation hypotheses

Other authors have proposed small-world formation mechanisms, though with less formalism. For instance, [15] suggest that small worlds are formed in a bank/board-director contact network through the recruitment of directors with extensive experience (i.e. 'good' connections). Also an emperical study by [5] who finds that a combination of chance and insurgent partnering (strategic behaiour of previously peripheral nodes) explained their networks.

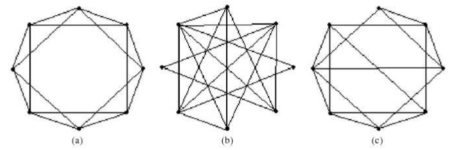
4 Linking large & small networks

Small-worlds in the Laboratory

Example 10 (*Cassar (Games & Econ. Beh., 2007)[9] Coordination and cooperation in local, random and small world networks: Experimental evidence*). **Setup**

- $n = 6$
- Study three graph types:
  1. Regular (a.k.a. 'local')
  2. Random
  3. Small-world
- .. And two games:
  1. Coordination
  2. Prisoner's Dilemma
- n fixed, $\langle k \rangle$ fixed

Regular, random and small-world graphs ($n = 6$) as in [9]



Hypotheses

- **Coordination game**

  H1 Higher C ... payoff-dominant action (closer to 'small group' action favouring efficient eq.)

  H2 Shorter L ... faster convergence (faster transmission of any action)

- **Prisoner's Dilemma**

  H3 Higher C ... cooperation (re-enforcement of gains to mutual cooperation)

  H4 Longer L ... cooperation (longer time for cooperation to be established)

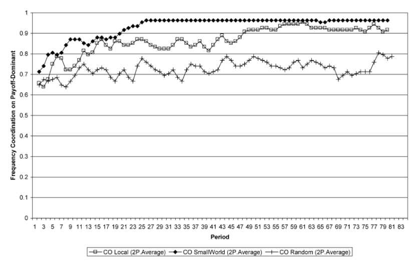
Results – Coordination

Frequency of coordination table under different network structures as in [9]

Table 3
CO frequency of coordination on payoff-dominant behavioral eq. (%)

		Period				
		1–20	21–40	41–60	61–end	1–end
Percentage	Small network	86.7	96.0	97.2	97.2	94.3
Payoff-dom.	Local network	78.4	84.6	90.8	92.7	86.8
Decisions on	Random network	69.5	71.9	75.3	73.2	72.5
	P(small = local = random)	0	0	0	0	0

Frequency of coordination on payoff-dominant play as in [9]



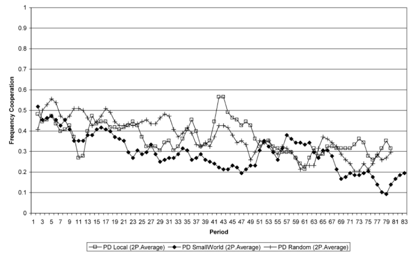
Results – Cooperation

Frequency of cooperation table under different network structures as in [9]

Table 5
PD frequency of cooperation (%)

		Period				
		1–20	21–40	41–60	61–end	1–end
Percentage	Small network	41.2	28.2	27.7	20.8	29.1
of coop.	Local network	49.9	40.7	37.1	31.7	39.7
Decisions on	Random network	50.7	41.5	34.9	28.0	38.6
	P(small = local = random)	0	0	0	0	0

Frequency of cooperative play as in [9]



Findings

- Coordination: Small-world networks facilitated higher speeds of convergence (H1)
- Coordination: SW nets facilitated higher incidence of payoff-dominant equilibria
- PD: All nets see decline in cooperation
- PD: SW nets statistically lower cooperation levels than random or regular graphs

> "Players on the small-world network achieve a defection equilibrium more often, reach this threshold more quickly, and, once attained, stay there longer than when they are on the other two networks." (Cassar, p.223)

> "This suggests that high clustering and short length may actually **encourage defection**, which we observe highest in small world networks. Interestingly, **those characteristics that helped coordination on the better outcome are here weakening cooperation**." (Cassar, p.228)

> " It is important to note that these results are exploratory, because **a theory linking network characteristics to individual behavior is yet not available**. It is hoped that these empirical ndings stimulate such theoretical development." " These results have, however, important practical implications. Many human networks (e.g., the society in which we live or the World Wide Web) tend to have small-world characteristics. What this study suggests is that this "natural" pattern of links is fertile ground for achieving coordination on Pareto superior outcomes, but might not be the best ground for cooperation to thrive." (Cassar, p.228)

5 Some Examples from my own Work

Some questions from my work

A Pair of problems

1. Coalitions: counting & finding coalitions;
2. Google Trends: community detection.

Coalitional play in networks

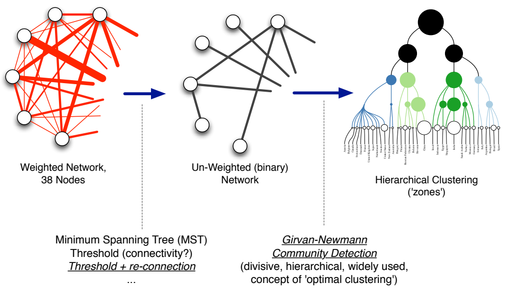
Context: *how does the inclusion of coalitional play (coordinated strategy updating by a connected sub-graph of agents) influence the speed of convergence to a better coordination equilibrium?*

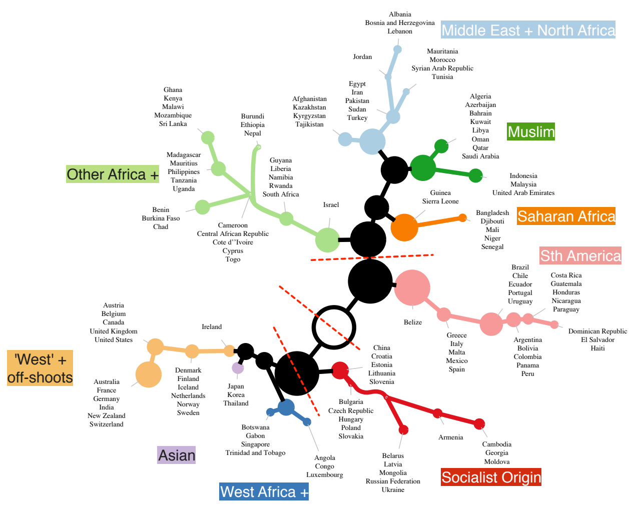
Example 11 (The Problem(s)). 1. Given the undirected graph $g = (V, E)$, in how many ways can a connected, k vertex, sub- graph $g' \subset g$ be formed?

2. Does an algorithm exist to efficiently identify all possible subgraphs for given g and k?

Google Trends: community detection

Context: *What defines a 'community' in a weighted graph?*





(my ideal) Community Detection Algorithm ...

- (ideally) Works on *weighted, undirected graphs;*

- Returns *hierarchical clustering* (major / minor divisions);
- Has a notion of *optimality* (where to stop?);
- Is not strongly *path-dependent*;
- (Is either divisive or aglomorative).

6 Concluding

Looking ahead: synergies

Problems supplied! (solutions demanded)

- *Theorists* most likely to be in need of Graph-theoretic assistance;
- (But ... data-guys also need help with *increasingly 'linked' data* (e.g. Google Trends));
- The major (highly ranked) journals in Economics tend to be more open to esoteric mathematics (i.e. open to good graph-theory) [so possibility for strong publication is relatively good];
- 'Networks' in economics is a *major area* of action at present ... though only a small fraction of this would be identifiably 'graph theory' based;

References

[1] Adamic, L. A., and Adar, E. Friends and neighbors on the web. Social Networks 25 (2003), 211–230.

[2] Bala, V., and Goyal, S. Learning from neighbours. The Review of Economic Studies 65, 3 (July 1998), 595–621.

[3] Bala, V., and Goyal, S. A noncooperative model of network formation. Econometrica 68, 5 (Sep 2000), 1181–1229.

[4] Barabasi, A. L., Jeong, H., Neda, Z., Ravasz, E., Schubert, A., and Vicsek, T. Evolution of the social network of scientific collaborations. Physica A: Statistical Mechanics and its Applications 311 (2002), 590–614.

[5] Baum, J. A. C., Shipilov, A. V., and Rowley, T. J. Where do small worlds come from? Industrial and Corporate Change 12, 4 (2003), 687–725.

[6] Berninghaus, S. K., Ehrhart, K.-M., and Keser, C. Conventions and local interaction structures: Experimental evidence. Games and Economic Behavior 39, 2 (May 2002), 177–205.

[7] Berninghaus, S. K., Ehrhart, K.-M., Ott, M., and Vogt, B. Evolution of networks – an experimental analysis. Journal of Evolutionary Economics 17, 3 (June 2007), 317—347.

[8] Callander, S., and Plott, C. R. Principles of network development and evolution: an experimental study. Journal of Public Economics 89, 8 (Aug. 2005), 1469–1495.

[9] Cassar, A. Coordination and cooperation in local, random and small world networks: Experimental evidence. Games and Economic Behavior 58, 2 (Feb. 2007), 209–230.

[10] Charness, G., Corominas-Bosch, M., and Frechette, G. R. Bargaining and network structure: An experiment. Journal of Economic Theory 136, 1 (Sept. 2007), 28–65.

[11] Clauset, A., R, S. C., and Newman, M. E. J. Power-law distributions in emperical data. arXiv pre-print, 2007.

[12] Comellas, F., Ozon, J., and Peters, J. G. Deterministic small-world communication networks. Information Processing Letters (2000), 83–90.

[13] Corbae, D., and Duffy, J. Experiments with network formation. Games and Economic Behavior In Press, Corrected Proof (2008).

[14] Corominas-Bosch, M. Bargaining in a network of buyers and sellers. Journal of Economic Theory 115, 1 (Mar. 2004), 35–77.

[15] Davis, G. F., Yoo, M., and Baker, W. E. The small world of the american corporate elite, 1982-2001. Strategic Organization 1, 3 (Aug. 2003), 301–326.

[16] Ellison, G. Learning, local interaction, and coordination. Econometrica 61, 5 (Sep 1993), 1047–1071.

[17] Ely, J. C. Local conventions. Advances in Theoretical Economics 2, 1 (2002), Article 1.

[18] Erdös, P., and Rènyi, A. On random graphs i. Publicationes Mathematicae, Debrecen 6 (1959), 290–297.

[19] Falk, A., and Kosfeld, M. It's all about connections: Evidence on network formation. Discussion Paper 777, Institute for the Study of Labor (IZA), 2003. available at http://ideas.repec.org/p/iza/izadps/dp777.html.

[20] Holme, P., Edling, C. R., and Liljeros, F. Structure and time evolution of an internet dating community. Social Networks (2004). In press.

[21] Jackson, M. O., and Watts, A. On the formation of interaction networks in social coordination games. Games and Economic Behavior 41, 2 (2002), 265–291.

[22] Jackson, M. O., and Wolinsky, A. A strategic model of social and economic networks. Journal of Economic Theory 71 (1996), 44–74.

[23] Kali, R. Endogenous business networks. The Journal of Law, Economics, & Organization 15 (3 1999), 615–636.

[24] Kandori, M., Mailath, G. J., and Rob, R. Learning, mutation, and long run equilibria in games. Econometrica 61, 1 (January 1993), 29–56.

[25] Kranton, R. E., and Minehart, D. F. A theory of buyer-seller networks. American Economic Review 91, 3 (June 2001), 485–508.

[26] Milgram, S. Small-world problem. Psychology Today 1, 1 (1967), 61–67.

[27] Newman, M. E. J. The structure of scientific collaboration networks. Proc. Nat. Acad. Sci. 98 (2001), 404–409.

[28] Newman, M. E. J. Coauthorship networks and patterns of scientific collaboration. Proceedings of the National Academy of Sciences (USA) 101, suppl. 1 (Apr 2004), 5200–5205.

[29] Rapoport, A., and Horvath, W. J. A study of large sociogram. Behavioral Science 6 (1961), 279–291.

[30] Travers, J., and Milgram, S. An experimental study of the small world problem. Sociometry 32, 4 (dec 1969), 425–443.

[31] Watts, D. J. Networks, dynamics, and the small-world phenomenon. The American Journal of Sociology 105, 2 (Sept. 1999), 493–527.

[32] Watts, D. J., and Strogatz, S. H. Collective dynamics of 'small-world' networks. Nature 393 (June 1998), 440–442.