Probabilistic and Extremal Combinatorics Downunder

Monash University
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Abstracts

(listed alphabetically by surname)

Structure, randomness and universality Mon 10:00

Noga Alon
Tel Aviv University

What is the minimum possible number of vertices of a graph that contains every k-vertex graph as an induced subgraph? What is the minimum possible number of edges in a graph that contains every k-vertex graph with maximum degree 3 as a subgraph? These questions and related ones have been studied by a combination of probabilistic arguments and explicit, structured constructions. I will survey the subject briefly, focusing on a recent asymptotic solution of the first question, where the answer turns out to be $(1 + o(1))2^{(k-1)/2}$. This improves earlier estimates by several authors. The proof combines combinatorial and probabilistic arguments with group theoretic tools.

Edge decompositions of graphs Tue 14:50

Ben Barber
Bristol University

An F-decomposition of a graph G is a partition of $E(G)$ into copies of F. Determining whether a graph has an F-decomposition is NP-complete, but it is much easier to find 'fractional decompositions. I'll explain the connection between these ideas and how it can be exploited to attack Nash-Williams' conjecture that every large graph on n vertices with minimum degree at least $3n/4$, $e(G)$ divisible by 3 and every degree even has a triangle decomposition.

Probabilistic lower bounds on maximal determinants of binary matrices Thu 9:45

Richard Brent and Judy-anne Osborn
ANU and University of Newcastle

Hadamard (1893) proved an upper bound $n^{n/2}$ on the determinant of a $\{\pm 1\}$-matrix of order n. The bound is tight if and only if there exists a Hadamard matrix of order n. For other ("non-Hadamard") orders various lower bounds on the maximal determinant have been proved by deterministic methods, but they differ from the Hadamard bound by a factor at least of order $n^{1/2}$. In this talk we consider applying the probabilistic method to obtain sharper lower bounds. A straightforward application of the probabilistic method is useless, as almost all $\{\pm 1\}$-matrices have determinants that are exponentially smaller than the Hadamard bound. To overcome this obstacle, we bias the distribution to which the probabilistic method is applied by considering a suitable subset of those $\{\pm 1\}$-matrices of order n obtained by bordering a Hadamard matrix of slightly smaller order $n - d$. This gives lower bounds within a factor c_d of the Hadamard bound, where c_d is positive and independent of n. The case $d = 1$ was considered previously by Erdős and Spencer.
As a corollary, if the Hadamard conjecture is true, then the best lower bounds are within a constant factor $c_3 > 0.11$ of the Hadamard upper bound.
This is joint work with Warren D. Smith; for details see arXiv:1501.06235.

Improper coloring graphs on surfaces with girth conditions Thu 16:05

Ilkyoo Choi
KAIST

A graph is $(d_1, \dots, d_r)$-colorable if its vertex set can be partitioned into r sets $V_1, \dots, V_r$ where the maximum degree of the graph induced by V_i is at most d_i for each $i \in \{1, \dots r\}$. Given r and $d_1, \dots, d_r$, determining if a (sparse) graph is $(d_1, \dots, d_r)$-colorable has attracted much interest. For example, the Four Color Theorem states that all planar graphs are 4-colorable, and therefore $(0, 0, 0, 0)$-colorable. The question is also well studied for partitioning planar graphs into three parts. For two parts, it is known that for given d_1 and d_2, there exists a planar graph that is not (d_1, d_2)-colorable. Therefore, it is natural to study the question for planar graphs with girth conditions. Namely, given g and d_1, determine the minimum $d_2 = d_2(g, d_1)$ such that planar graphs with girth g are (d_1, d_2)-colorable.
We continue the study and ask the same question for graphs that are embeddable on a fixed surface. Given integers k, γ, g we completely characterize the smallest k-tuple $(d_1, \dots, d_k)$ in lexicographical order such that each graph of girth at least $g \leq 7$ that is embeddable on a surface of Euler genus γ is $(d_1, \dots, d_k)$-colorable. All of our results are tight, up to a constant multiplicative factor for the degrees d_i depending on g. In particular, we show that a graph embeddable on a surface of Euler genus γ is $(0, 0, 0, K_1(\gamma))$-colorable and $(2, 2, K_2(\gamma))$-colorable, where $K_1(\gamma)$ and $K_2(\gamma)$ are linear functions in γ. This talk is based on results and discussions with H. Choi, F. Dross, L. Esperet, J. Jeong, M. Montassier, P. Ochem, A. Raspaud, and G. Suh.

Core forging by warning propagation Tue 10:00

Amin Coja Oghlan
Goethe University

The k-core of a graph is the largest subgraph of minimum degree k. The threshold for the emergence of a non-empty k-core for $k \geq 3$ in the random graph $G(n, m)$ was first determined by Pittel, Spencer and Wormald (JCTB 1996) via an analysis of the peeling process, an algorithm that determines the k-core. Several alternative proofs, a central limit theorem as well as extensions to other random graph or hypergraph models have been put forward since. I am going to present a novel approach to the k-core problem based on Warning Propagation, a message passing algorithm inspired by physics ideas. Apart from (yet) another derivation of the k-core threshold, this approach yields a short proof of a local limit theorem for the number of vertices and edges in the k-core and other assorted parameters. The talk is based on joint work with Oliver Cooley, Mihyun Kang and Kathrin Skubch.

Finite reflection groups and graph norms

Mon 11:30

David Conlon
Oxford University

For any given graph H, we may define a natural corresponding functional $\|.\|_H$. We then say that H is norming if $\|.\|_H$ is a semi-norm. A similar notion $\|.\|_{r(H)}$ is defined by $\|f\|_{r(H)} := \||f|\|_H$ and H is said to be weakly norming if $\|.\|_{r(H)}$ is a norm. Classical results show that weakly norming graphs are necessarily bipartite. In the other direction, Hatami showed that even cycles, complete bipartite graphs, and hypercubes are all weakly norming. Using results from the theory of finite reflection groups, we identify a much larger class of weakly norming graphs. This result includes all previous examples of weakly norming graphs and adds many more. We also discuss several applications of our results. In particular, we define and compare a number of generalisations of Gowers' octahedral norms and we prove some new instances of Sidorenko's conjecture. Joint work with Joonkyung Lee.

Subgraph counting in series-parallel graphs and infinite dimensional systems of functional equations

Tue 15:45

Michael Drmota
Technische Universität Wien

In this talk we consider series-parallel graphs (and more generally so-called subcritical graph classes) and show that the number of occurrences of a given subgraph follows a normal limiting distribution with linear expectation and variance. The main ingredient in our proof is the analytic framework developed by Drmota, Gittenberger and Morgenbesser to deal with infinite systems of functional equations. This is joint work with Lander Ramos and Juanjo Rue.

A local limit theorem for QuickSort key comparisons via multi-round smoothing

Tue 11:10

Jim Fill
Johns Hopkins University

It is a well-known result, due independently to Régnier (1989) and Rösler (1991), that the number of key comparisons required by the randomized sorting algorithm QuickSort to sort a list of n distinct items (keys) satisfies a global distributional limit theorem. We resolve an open problem of Fill and Janson from 2002 by using a multi-round smoothing technique to establish the corresponding local limit theorem. This is joint work with Béla Bollobás and Oliver Riordan.

Maximum density of copies of a subgraph in the n-cube

Thu 12:20

John Goldwasser
West Virginia University

The n-cube, denoted Q_n, is the graph whose vertices are the set of binary n-tuples, with two vertices adjacent if and only if they differ in precisely one coordinate. Let H be a set of vertices, which we call a configuration, in Q_d, for some fixed d. The d-cube-density of H, denoted $\pi(H, d)$, is the limit as n goes to infinity of the maximum fraction, over all subsets S of the vertices of Q_n, of sub-d-cubes whose intersection with S is an exact copy of H. For which configuration H in Q_4 is $\pi(H, 4)$ smallest? By using a "blow-up" of H, it is not hard to show that $\pi(H, 4) \geq 3/32$ for every configuration H in Q_4. Let C_{2d} denote a "perfect" $2d$-cycle a cycle where each pair of opposite vertices is Hamming distance d apart. We show that $\pi(C8, 4) = 3/32$, and flag algebra evidence says it may be the only one of the 238 non-isomorphic configurations in Q_4 which has this minimum possible 4-cube-density. So it is the subgraph of Q_4 most difficult to create lots of copies of in a subset of the vertices of a large n-cube. We conjecture that the same

is true of C_{2d} among all configurations in Q_d for any $d > 3$. To obtain our results we found an equivalent problem counting the number of sequences of length d of an n-set with a certain property. To solve the sequence problem we needed to determine which bipartite graph with n vertices induces the most copies of the graph on four vertices with two disjoint edges.
Joint work with Ryan Hansen.

The average number of spanning trees in sparse graphs with given degrees

Mon 16:25

Catherine Greenhill
University of New South Wales

Consider a random graph with a given degree sequence. How many spanning trees do you expect it to contain? We calculate an asymptotic formula for the expected number of spanning trees in the sparse case. Specifically, our result holds when the number of vertices n and the number edges m satisfy $m \geq n + d_{\max}^4/2$, where $d_{\max}$ denotes the maximum entry in the degree sequence. Our proof uses a known asymptotic enumeration formula for the number of graphs with given degrees containing a specified subgraph, and a martingale concentration argument that makes use of the Prüfer sequence of a tree. Joint work with Mikhail Isaev, Matthew Kwan and Brendan McKay.

Matchings and covers in 3-uniform hypergraphs

Thu 9:00

Penny Haxell
University of Waterloo

A *matching* in a hypergraph H is a set of pairwise disjoint edges of H. A *cover* of H is a set C of vertices that meets all edges of H. It is easy to see that every 3-uniform hypergraph H has a cover of size at most $3\nu(H)$, where $\nu(H)$ denotes the size of a largest packing in H, and this bound is tight. However, stronger bounds are known for special classes of hypergraphs H, including tripartite hypergraphs and triangle hypergraphs. We discuss some recent results on lower bounds for $\nu(H)$, and on conjectures of Ryser and Tuza about the existence of small covers for these classes.

The extremal function for Petersen minors

Thu 15:10

Kevin Hendrey
Monash University

A graph H is a minor of a graph G if H can be obtained from a subgraph of G by contracting edges. Graph minor theory often provides a neat way of characterising and working with graphs in a particular class. A natural question at the intersection of graph minor theory and extremal graph theory is, given a graph H, what is the maximum number of edges in an n-vertex graph with no H-minor? We answer this question when H is the Petersen graph, and show how this result leads to a tight bound on the chromatic number of Petersen-minor-free graphs. No knowledge of graph minor theory is assumed.

Treeable graphings are local limits of finite graphs Fri 14:00

Lucas Hosseini
École des Hautes Études en Sciences Sociales

Let G be a graphing, that is a Borel graph defined by d measure preserving involutions. We prove that if G is treeable then it arises as the local limit of some sequence of graphs with maximum degree at most d. This extends a result by Elek (for G a treeing) and consequently extends the domain of the graphings for which the Aldous-Lyons conjecture is known to be true.

Complex martingales and combinatorial enumeration (Part 2) Mon 15:30

Mikhail Isaev
The Australian National University

In this talk we continue the study of $\mathbb{E}e^{f(\mathbf{X})}$, as it appears in asymptotic enumeration problems in combinatorics (see the abstract for Part 1).
In many important cases, the estimate $\mathbb{E}e^{f} \sim e^{\mathbb{E}f + \frac{1}{2}\mathbb{V}f}$ is limited by the magnitude of the imaginary part of f, which appears necessarily in the error term. In this talk we discuss additional conditions on $f(\boldsymbol{X})$ that allow us to bypass this obstacle. These conditions frequently hold in combinatorial problems and also permit extension of the theory to more precise estimates that employ higher-order cumulants. Joint work with Brendan McKay.

Jigsaw percolation on random hypergraphs Fri 11:25

Mihyun Kang
Graz University of Technology

Jigsaw percolation process on graphs was introduced by Brummit, Chatterjee, Dey, and Sivakoff. Whether the process percolates may be viewed as a characterisation of joint connectivity of two graphs on a common vertex set. In this talk we consider a hypergraph analogue of this process for a family of possible definitions of connectivity. We provide the asymptotic order of the critical threshold probability for percolation when both hypergraphs are chosen binomially at random, generalising a graph result of Bollobás, Riordan, Slivken, and Smith. (Joint work with Bollobás, Cooley, and Koch.)

Refining old results on disjoint and longest cycles in graphs Thu 10:50

Alexandr Kostochka
University of Illinois at Urbana-Champaign

We discuss and refine some results on cycle structure of graphs from the sixties.
Some of these results are due to Dirac and Erdős from 1963 related to the well-known Corrádi-Hajnal Theorem. Let $V_{\geq t}(G)$ (respectively, $V_{\leq t}(G)$) denote the set of vertices in G of degree at least (respectively, at most) t. With this notation, the Corrádi-Hajnal Theorem says that for every positive integer k, if $|V(G)| \geq 3k$ and $V_{\geq 2k}(G) = V(G)$, then G has k disjoint cycles. Dirac and Erdős allowed some vertices of "small" degree. They proved that if $k \geq 3$ and G is a graph with $|V_{\geq 2k}(G)| - |V_{\leq 2k-2}(G)| \geq k^2 + 2k - 4$, then G also has k disjoint cycles. For planar graphs, they weakened the restriction on $|V_{\geq 2k}(G)| - |V_{\leq 2k-2}(G)|$ to $5k - 7$ and showed a series of examples of graphs G with $|V_{\geq 2k}(G)| - |V_{\leq 2k-2}(G)| = 2k - 1$ that do not have k disjoint cycles.
We present some refinements of the Dirac–Erdős results. In particular, we show that each graph G with $|V_{\geq 2k}(G)| - |V_{\leq 2k-2}(G)| \geq 3k$ has k disjoint cycles. This is sharp if we do not impose restrictions on $|V(G)|$. This is joint work with Kierstead and McConvey.

The Turán-type theorems of Erdős and Gallai from 1959 on the most edges in n-vertex graphs that do not have paths/cycles with at least k vertices were sharpened later by Faudree and Schelp, Woodall, and Kopylov. Let $h(n,k,a) = \binom{k-a}{2} + a(n-k+a)$. The strongest result (by Kopylov) was: if $t \geq 2$, $k \in \{2t+1, 2t+2\}$, $n \geq k$, and G is an n-vertex 2-connected graph with at least $\max\{h(n,k,2), h(n,k,t)\}$ edges, then G contains a cycle of length at least k unless $G = H_{n,k,t} := K_n - E(K_{n-t})$. We prove stability versions of these results. In particular, if $n \geq k$ and the number of edges in an n-vertex 3-connected graph G with no cycle of length at least k is greater than $\max\{h(n,k,3,h(n,k,t-1)\}$, then G is a subgraph of $H_{n,k,t}$ or of $H(n,k,2)$. The lower bound on $|E(G)|$ is tight. This is joint work with Füredi, Luo and Verstraëte.

Long cycles in expanding graphs, with applications

Thu 14:00

Michael Krivelevich
Tel Aviv University

We present sufficient conditions for the existence of long cycles in graphs in terms of local expansion. We then discuss some applications of our criteria and techniques to random graphs and to positional games.

Cutoff in the Bernoulli-Laplace diffusion model

Fri 10:40

Malwina Luczak
Queen Mary, University of London

We re-examine the Bernoulli-Laplace diffusion Markov chain, for which a cut-off was first established by Diaconis and Shahshahani (1987). In this work, we provide a probabilistic proof of this result, using couplings and concentration of measure inequalities.
This is joint work with Andrew Barbour and Graham Brightwell.

Hypergraphs, randomness, and a touch of topology

Wed 10:55

Tomasz Łuczak
Adam Mickiewicz University

We shall define and discuss some properties of hypergraphs of a topological flavour and present a number of recent results in this area. This is a joint work with Yuval Peled.

Foster's network theorems on finite and infinite graphs and connections to random walks

Tue 14:00

Greg Markowsky
Monash University

Foster's First and Second Theorems give the average electric resistance on a finite graph between pairs of adjacent points and points of distance 2. These theorems can be proved using probabilistic techniques from Markov chain theory, which can then be extended to give "Foster's n-th Theorem". Also available is a method of extending the techniques to infinite graphs with a high degree of symmetry, giving a very elegant method of deducing certain resistances between points in grids and similar infinite graphs. The talk will include a fair amount of overview of the area, as well as some recent work done by myself and José Palacios on this subject.

A conjecture on shattering-extremal set systems

Tue 14:25

Tamás Mészáros
Freie Universität Berlin

We say that a set system $\mathcal{F} \subseteq 2^{[n]}$ shatters a given set $S \subseteq [n]$ if

$$\mathcal{F}|_S = \{F \cap S : F \in \mathcal{F}\} = 2^S.$$

One related notion is the VC-dimension of a set system: the size of the largest set shattered by $\mathcal{F}$. The Sauer inequality states that in general, a set system $\mathcal{F}$ shatters at least $|\mathcal{F}|$ sets. A set system is called shattering-extremal if it shatters exactly $|\mathcal{F}|$ sets. Such families have many interesting features. Here we present several approaches to study shattering-extremal set systems together with a conjecture about the eliminability of elements from extremal families.

On subgraph containment probabilities for random graphs of given degree sequence

Tue 11:55

Stephen Mildenhall
Australian National University

Many 'real-world' networks studied in recent years are characterised by a small number of vertices of high degree, but many more vertices of much lower degree. For the graphs underlying these sorts of networks, we provide an improved estimate for the probability that an arbitrary known subgraph is contained within a larger labelled graph of given degree distribution. Our approach aims to update the results achieved by McKay in 1981, using a three-way switching operation instead of the original two-way operation. We illustrate the main theorems with simple examples and asymptotics, and make comparisons with the previous results.

Clique colourings of random graphs

Tue 12:20

Colin McDiarmid
Oxford University

A clique colouring of a graph G is a colouring of the vertices so that no maximal clique is monochromatic (ignoring isolated vertices). The smallest number of colours in such a colouring is the clique chromatic number $\chi_c(G)$. For example, if G is triangle-free this is the chromatic number $\chi(G)$. We shall discuss clique chromatic number, in particular for random graphs.
For random perfect graphs, we will see that with high probability the clique chromatic number is 2. We will discuss both deterministic geometric graphs (or unit disk graphs), and random geometric graphs for a range of threshold distances r. Similarly, we will discuss binomial random graphs $G(n, p)$ for a range of edge probabilities $p = p(n)$.
This material is from recent joint work with Nikola Yolov (concerning random perfect graphs) and with Dieter Mitsche and Pawel Pralat.

Complex martingales and combinatorial enumeration (Part 1)

Mon 15:05

Brendan McKay
The Australian National University

Many enumeration problems in combinatorics, including such fundamental questions as the number of regular graphs, can be expressed as high-dimensional complex integrals. The asymptotic behaviour is found by concentrating the integral in a small region and then approximating the integrand inside that region.

Evaluation of such integrals one variable at a time produces a martingale and the sought-after quantity has the form $\mathbb{E}e^{f(\boldsymbol{X})}$, where $f(\boldsymbol{X})$ is some complex-valued function of a multidimensional measure (usually a truncated gaussian measure). We generalise the problem to arbitrary complex martingales and find conditions under which $\mathbb{E}e^{f} \sim e^{\mathbb{E}f+\frac{1}{2}\mathbb{V}f}$, where $\mathbb{V}f$ is the pseudovariance. The result appears to cover all previous combinatorial enumeration problems done by the integral method, and also applies to discrete martingales such as functions of random permutations and random subsets. Joint work with Mikhail Isaev.

New developments in hypergraph Ramsey theory Thu 11:35

Dhruv Mubayi
University of Illinois at Chicago

I will describe lower bounds (i.e. constructions) for several hypergraph Ramsey problems. These constructions settle old conjectures of Erdős-Hajnal on classical Ramsey numbers as well as more recent questions due to Conlon-Fox-Lee-Sudakov and others on generalized Ramsey numbers and the Erdős-Rogers problem. Most of this is joint work with Andrew Suk.

Clustering graphs in a local convergent sequence Fri 14:25

Patrice Ossona de Mendez
École des Hautes Études en Sciences Sociales

The cluster analysis of very large objects is an important problem, which spans several theoretical as well as applied branches of mathematics and computer science. Here we suggest a novel approach: under assumption of local convergence of a sequence of finite structures we derive an asymptotic clustering. This is achieved by a blend of analytic and geometric techniques, and particularly by a new interpretation of the authors' representation theorem for limits of local convergent sequences, which serves as a guidance for the whole process. Our study may be seen as an effort to describe connectivity structure at the limit (without having a defined explicit limit structure) and to pull this connectivity structure back to the finite structures in the sequence in a continuous way.

Variants of Hadwiger's conjecture Thu 16:30

Sang-il Oum
KAIST

Hadwiger conjectured that every graph with no K_t minor is $(t-1)$-colorable; in other words, every graph with no K_t minor admits a partition of its vertex set into $t-1$ independent sets. This conjecture is still widely open and if true, it implies the four color theorem. Gerards and Seymour made a stronger conjecture claiming that every graph with no K_t odd minor is $(t-1)$-colorable, commonly called the odd Hadwiger's conjecture.

We prove variants of Hadwiger's conjecture. First, we prove that every graph with no K_t minor admits a partition of its vertex set into $t-1$ sets, each inducing a subgraph of bounded maximum degree. Secondly, we prove that every graph with no K_t minor admits a partition of its vertex set into $3(t-1)$ sets, each inducing a subgraph having no large components. We also prove variants of the odd Hadwiger's conjecture as follows: Every graph with no odd K_t minor admits a partition of its vertex set into $7t-10$ sets, each inducing a subgraph of bounded maximum degree. As a corollary, we prove that every graph with no odd K_t minor admits a partition of its vertex set into $16t-19$ sets, each inducing a subgraph with no large components. The last result improves the result of Kawarabayashi, who showed it with $496t$ sets. This talk is a combination of three works; first with K. Edwards, D. Kang, J. Kim, and P. Seymour, second with C. Liu, and third with D. Kang.

Between probabilistic and extremal combinatorics: VC-dimension

Fri 15:20

János Pach
EPFL Lausanne

It was discovered about 20 years ago that intersection graphs of simple geometric objects in the plane obey much stronger Ramsey-type theorems than most other graphs. This result was extended to a wide range of geometrically defined graphs and hypergraphs and to other extremal and structural problems. It turned out that to prove some of these results, including the Szemerédi-Trotter theorem on incidences between points and lines, it is sufficient to explore that the underlying set systems satisfy a purely combinatorial property: their Vapnik-Chervonenkis dimension is bounded.

The Vapnik-Chervonenkis dimension of a graph is defined as the Vapnik-Chervonenkis dimension of the set system induced by the neighborhoods of its vertices. In this talk, we discuss how several classical results in extremal graph theory can be strengthened by restricting our attention to graphs with bounded VC-dimension. In particular, we show that every such n-vertex graph contains a clique or an independent set of size $e^{(\log n)^{1-o(1)}}$. This improves on the general bound of $e^{c\sqrt{\log n}}$, proved by Erdős and Hajnal for all graphs. Joint work with J. Fox and A. Suk.

The excess degree of a polytope

Mon 14:15

Guillermo Pineda-Villavicencio
Federation University Australia

This talk revolves around the excess of a d-polytope P and some of its applications. The excess $\mathbf{E}(P)$ of a d-polytope P is defined as $\mathbf{E}(P) = 2e - dv$, where v and e denote the number of vertices and edges of the polytope.

Our first important result is that the excess degree of a d-polytope does not take every natural number. Indeed, the smallest values are 0 and $d-2$; a polytope is simple iff its excess degree is 0.

In our second result, the excess theorem is applied to questions about the (Minkowski) decomposability of polytopes. In particular, we show that polytopes with small excess (i.e., $\mathbf{E} = d-2, d-1$) behave in a similar manner to simple polytopes in terms of decomposability. That is, they are decomposable or a pyramid and their duals are indecomposable. This is best possible since there are d-polytopes with excess d which are decomposable and d-polytopes which are not.

Joint work with David Yost and Julien Ugon.

Modularity of random graphs

Fri 12:10

Fiona Skerman
Bristol University

An important problem in network analysis is to identify highly connected components or 'communities'. Most popular clustering algorithms work by approximately optimising modularity. Given a graph G, the modularity of a partition of the vertex set measures the extent to which edge density is higher within parts than between parts; and the maximum modularity $q*(G)$ of G is the maximum of the modularity over all partitions of V(G) and takes a value in the interval $[0, 1)$ where one indicates a highly clustered graph. Knowledge of the maximum modularity of random graphs can help determine the significance of a division into communities/vertex partition of a real network.

We investigate the evolution of the maximum modularity of Erdős-Rényi random graphs as the edge probability increases. Three different phases of the likely maximum modularity are found. For $np = 1 + o(1)$ the maximum modularity is $1 + o(1)$ whp and for $np \to \infty$ the maximum modularity is $o(1)$ whp. For $np = c$ with $c > 1$ a constant, functions are constructed with $0 < a(c) < b(c) < 1$ and $b(c) \to 0$ as $c \to \infty$ such that whp the maximum modularity is bounded between these functions. Concentration of the maximum modularity about its expectation and structural properties of an optimal partition are also established. This is joint work with Colin McDiarmid.

Powers of cycles in random graphs and hypergraphs

Fri 9:45

Nemanja Skoric
ETH Zurich

We prove that if $p^k >= C \log^8 n/n$ for a constant k, then asymptotically almost surely the random graph G_{np} contains the k-th power of a Hamilton cycle. This determines the threshold for the appearance of the square of a Hamilton cycle up to the logarithmic factor, improving the result of Kühn and Osthus. Using similar ideas, we also give a combinatorial proof for the appearance of a tight Hamilton cycle in the random k-uniform hypergraph, for the hyperedge probability $p \geq C \log^8 n/n$. Our proofs are based on the absorber method and follow the strategy of Kühn and Osthus and Allen et al. The new ingredient is the proof of the Connecting Lemma using a hypergraph matching criteria of Haxell, which allows us to connect tuples of vertices using more complex structures. This is a joint work with Rajko Nenadov.

Local resilience for squares of almost spanning cycles in sparse random graphs

Wed 10:10

Angelika Steger
ETH Zurich

In 1962, Pósa conjectured that a graph $G = (V, E)$ contains a square of a Hamiltonian cycle if $\delta(G) \geq 2n/3$. Only more than thirty years later Komlós, Sárkőzy, and Szemerédi proved this conjecture using the so-called Blow-Up Lemma. Here we extend their result to a random graph setting. We show that for every $\epsilon > 0$ and $p = n^{-1/2+\epsilon}$ a.a.s. every subgraph of $G_{n,p}$ with minimum degree at least $(2/3+\epsilon)np$ contains the square of a cycle on $(1-o(1))n$ vertices. This is almost best possible in three ways: (1) for $p \ll n^{-1/2}$ the random graph will not contain any square of a long cycle (2) one cannot hope for a resilience version for the square of a *spanning* cycle (as deleting all edges in the neighborhood of single vertex destroys this property) and (3) for $c < 2/3$ a.a.s. $G_{n,p}$ contains a subgraph with minimum degree at least cnp which does not contain the square of a path on $(1/3+c)n$ vertices.

Independent sets in regular graphs, spectral stability and generalizations

Fri 9:00

Prasad Tetali
Georgia Institute of Technology

Results of Kahn and of Zhao show that among all d-regular graphs on n vertices, the disjoint union of $K_{d,d}$'s maximizes the number of independent sets. In this talk, we consider various related aspects, including a spectral stability result, Widom-Rowlinson model, and independent sets in linear hypergraphs.

Solutions to linear equations — Mon 12:15

Andrew Thomason
University of Cambridge

We survey some old-ish results with possibly new-ish proofs. These are concerned with solutions to linear equations of the form $Ax = b$, say over a finite field F, where A is an integer matrix. Let $\mathrm{ex}(F, A, b)$ be the largest size of a subset X of F such that contains no solution. By the container method it is often possible to compute the number of solution-free sets in terms of $\mathrm{ex}(F, A, b)$, and similarly to determine the largest solution-free subset within a randomly chosen subset of F.

The phase transition in the random d-process — Wed 9:00

Lutz Warnke
University of Cambridge

In this talk we consider the random d-process, which starts with n isolated vertices, and evolves by sequentially adding new random edges so that the maximum degree remains at most d. We shall discuss several results concerning the phase transition in the d-process (for the emergence of a linear size giant component), which solve problems of Wormald from 1997 and of Balińska and Quintas from 1990. This is joint work with Nick Wormald.

The local stability method and some applications — Thu 14:45

Liana Yepreman
McGill University

The Turán number of a graph G, denoted by $ex(n, G)$, is the maximum number of edges in an G-free graph on n vertices. The Turán density of a graph G, denoted by $\pi(G)$, is the limit as n tends to infinity of the maximum edge density of an G-free graph on n vertices. Unless $\pi(G) = 0$, it captures the asymptotic behaviour of $ex(n, G)$.
During this talk I will discuss a method, which we call local stability method, that allows one to obtain exact Turán numbers from Turán density results. This method can be thought of as an extension of the classical stability method by generically utilizing Lagrangians and symmetrization. Using it, we solved a conjecture of Frankl and Füredi from 1980's and obtained the Turán number of a hypergraph called generalized triangle, for uniformities 5 and 6. Further, our method is more generally applicable for determining Turán numbers of so-called extensions. This is joint work with Sergey Norin.

Lower bounds for general polytopes — Mon 14:40

David Yost
Federation University Australia

For a d-dimensional polytope with v vertices, $d+1 \le v \le 2d$, we calculate precisely the minimum possible number of m-dimensional faces, when $m = 1$ or $m \ge 0.62d$. This confirms a conjecture of Grünbaum, for these values of m. We also characterise the minimising polytopes. For $v = 2d+1$, we solve the same problem when $m = 1, d-1$ or $d-2$.