19th Australasian Fluid Mechanics Conference
Melbourne, Australia
8-11 December 2014

Large-Eddy Simulation of Turbulent Flow in a Pipe at Extreme Reynolds Numbers

N. Saito[1], **D. I. Pullin**[1] **and H. M. Blackburn**[2]

[1]Graduate Aerospace Laboratories
California Institute of Technology, Pasadena, CA 91125, USA

[2]Department of Mechanical and Aerospace Engineering
Monash University, Clayton, Victoria 3800, Australia

Abstract

Large-eddy simulations (LES) of turbulent flow in a smooth-wall pipe at Reynolds numbers based on the pipe Radius R and wall friction velocity in the range $Re_\tau = 2,000-2,000,000$ are discussed. Using a Reynolds number based on D and the bulk-flow velocity, then corresponding values are $Re_D = 91,800-165,000,000$. For the LES, the spectral-element code SEMTEX was used combined with the stretched-vortex subgrid-scale model and a wall model. Calculated friction factors are found to be in reasonable agreement with super-pipe measurements. An estimate of the Kármán constant κ is made using a least-square fit to the friction-factor verses Re_D results obtained from the LES, giving $\kappa = 0.43$.

Introduction

The turbulent flow of a fluid in a pipe of constant circular cross section has been recognized as a canonical, wall-bounded turbulent flow since the time of Osborne Reynolds. An empirical framework for characterizing the friction factor as a function of a roughness parameter and a bulk-flow Reynolds number for turbulent pipe flow was developed through experiments and modeling [13, 12]. Subsequently, turbulent pipe flows have attracted much attention from the experimental, theoretical, and computational fluid dynamics communities. The study of pipe-flow, fluid-dynamic turbulence received considerable impetus from the seminal "super-pipe" experiments of Zagarola & Smits[14] and from studies stimulated by this work. In the Princeton super-pipe, unprecedented Reynolds numbers were achieved by pressurization techniques producing increased fluid density, with consequent reduction in the kinematic viscosity, while retaining low Mach number.

The Reynolds numbers achieved by the super-pipe experiments, up to $Re_\tau \approx 5\times10^5$ [14], remain well out of range of present-day direct-numerical simulation capability. In contrast, large-eddy simulation methodology that includes wall-modeling capable of resolving weak Reynolds number effects over much of the outer-flow region, have been extended to extreme Reynolds numbers for both turbulent channel flows [4] and for turbulent boundary layers [6]. Presently we report on the application of these techniques to pipe flow with the aim of making contact with data from the super-pipe experiments.

LES performed

We report on the LES of smooth-wall pipe flow at extremely large Reynolds numbers. The LES were performed using the SEMTEX platform of Blackburn and Sherwin [1]. Within SEMTEX, the building blocks of the geometry are convex spectral elements that adopt standard Gauss-Lobatto-Legendre nodal basis expansions of n^{th} order. Within a three-dimensional flow domain, one co-ordinate is selected and represented by an N-term Fourier series. Each orthogonal, two-dimensional, flat plane is then covered by contiguous spectral elements. For the present pipe-flow geometry, in cylindrical (r,θ,z) co-ordinates, the Fourier co-ordinate is selected to be the mean stream-wise flow direction z with M spectral elements covering the $(r-\theta)$ plane.

The pipe surface is defined by $r = R$. The LES equations for incompressible viscous flow were implemented within SEMTEX [1]. There are two aspects of the subgrid-scale (SGS) modeling used for the LES of wall-bounded flows. First, in the main body of the fluid within the pipe, the SGS model as described in Chung and Pullin [4] was utilized. This does not contain model adjustable constants and all internal parameters are determined locally and dynamically. Second, for the large Reynolds numbers considered presently, a wall model is required. This is described in reference [4], where it was applied to channel flow, and also in [6] for the zero-pressure-gradient, flat-plate boundary layer.

The wall model itself contains two components. In the first, the local value of the wall-friction velocity $u_\tau(R,\theta,x,t)$, defined by $u_\tau^2 = \tau_w/\rho$ (τ_w is the wall shear stress), is determined by the numerical solution of an ordinary-differential equation (ODE) obtained using near-wall averaging of the stream-wise momentum equation combined with an ansatz constructed from local inner-scaling for the wall-parallel, filtered, stream-wise velocity component. The second part constructs an effective wall slip velocity at a raised wall-parallel plane, based on a log-like relation obtained from the subgrid model applied between the raised plane and the pipe wall, and with dynamic computation of an effective Kármán-like parameter. This slip velocity requires as input, the local u_τ provided by the ODE solve. In turn, this generates a wall boundary condition for the outer LES. Both the ODE and the slip boundary condition are applied independently at every wall point. The height of the raised plane was fixed as 20% of the distance to the spectral element nodal point nearest the wall. This is a parameter of the present wall model but is considered as part of the LES computational grid. The offset in the log relationship used for the slip velocity is an empirical input to the overall wall model: see [4] for a detailed discussion.

The dimensions of the present pipe-shaped domain are $(R,\theta,l_z) = (0.5,2\pi,15.7)$ and we note that l_z/R is larger than the minimum value of 8π recommended by Chin *et al.*[3] for the convergence of statistics in the outer region. The spectral-element parameters were $M = 48$, $N = 160$ with $n = 10^{\text{th}}$-order polynomials used within each element leading to approximately 1.5×10^6 total node points within the full domain. Figure 1 shows the distribution of spectral elements within the pipe cross section. Based on the pipe radius R and the pipe inner-wall-averaged mean friction velocity $\overline{u}_\tau$, the friction Reynolds number was $Re_\tau = 2\times10^3$ to $Re_\tau = 2\times10^6$. The bulk flow Reynolds number is $Re_D = u_b D/\nu$, where D is the pipe diameter and u_b the bulk velocity. Corresponding values are $Re_D = 9.18\times10^4 - 1.65\times10^8$ respectively. The spectral element method guarantees $C0$ but not $C1$ continuity at element boundaries on the $r-\theta$-plane. Hence for the present LES, the

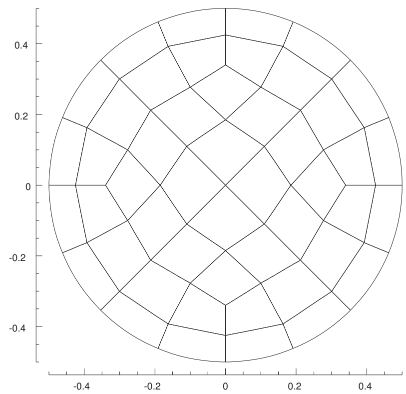

Figure 1: Distribution of spectral elements within the pipe cross section

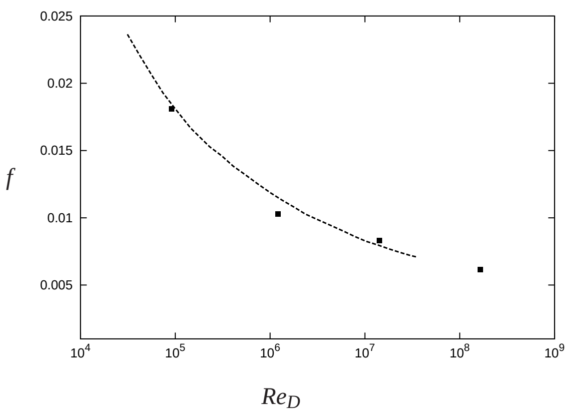

Figure 2: Friction factor f versus Re_b. Squares: LES; dashed line: experimental result of McKeon *et al.* [9].

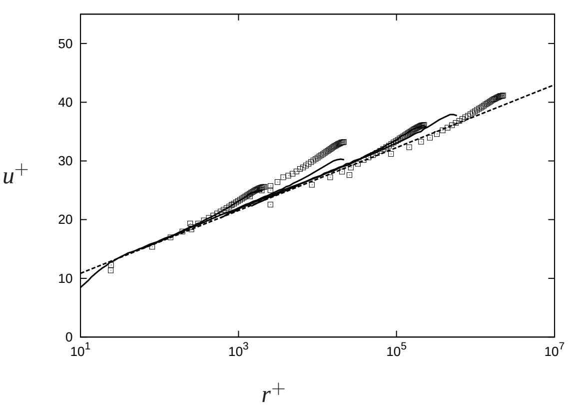

Figure 3: Pipe flow mean streamwise velocity profiles. Squares: LES; solid line: experiment McKeon *et al.* [10] at $Re_\tau = 1.8 \times 10^3, 2.0 \times 10^4, 2.2 \times 10^5, 5.3 \times 10^5$; dashed line: log-law with $\kappa = 0.43$ and $B = 5.5$, obtained from LES.

inputs to the wall slip velocity at element edge nodes are taken as the average of the interior nodes on the either side of the edge. All LES reported presently were run for many mean-flow transit times. The bulk velocity u_b was obtained as a volume average.

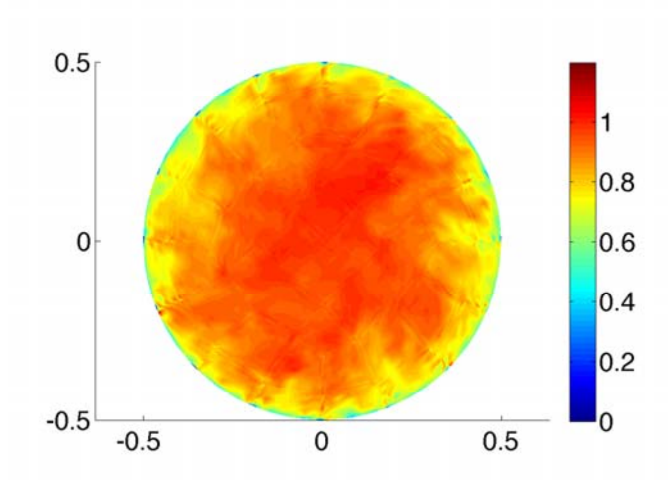

Figure 4: Instantaneous color-coded image of the instantaneous stream-wise velocity component. $Re_\tau = 2 \times 10^5$.

Results

For turbulent pipe flow the friction factor is defined as

$$f = -2 \left(\overline{\frac{dp}{dz}} \right) \frac{D}{\rho u_b^2} = 8 \left(\frac{\overline{u_\tau^2}}{u_b^2} \right), \qquad (1)$$

where $\overline{dp/dz}$ is the mean pressure gradient in the stream-wise direction. Figure 2 shows f versus Re_b compared with the experimental measurements of McKeon *et al.*[9]. The semi-empirical formula

$$f = 2\left[1/\kappa(\log(Re_D\sqrt{f/2}) - 1) + B\right]^{-2} \qquad (2)$$

was fitted to (f, Re_D) results from the LES to obtain estimates of the Kármán constant κ. We find $\kappa = 0.43$ and $B = 5.5$, which are comparable to values obtained by McKeon *et al.* [10], who report $\kappa = 0.42$ and $B = 5.6$. While our estimate of κ can be viewed as a postdictive result of the overall LES, our estimate of B is effectively determined by a related input parameter to the wall model [4].

In order to obtain mean velocity-profiles, raw data from nodes was first averaged over the stream-wise (Fourier) direction. Then, the stream-wise-averaged data was sampled at 40 evenly-spaced points along radial lines θ = constant. Once mean flow profiles are established, other turbulence statistics such as stream-wise turbulence intensities can also be calculated. The mean velocity profiles at $Re_\tau = 2 \times 10^3$, 2×10^4, 2×10^5, and 2×10^6 are shown in Figure 3 compared to the experimental super-pipe measurements of McKeon *et al.* [10]. Although our LES data shows inconsistency at $Re_\tau = 2 \times 10^4$, it captures the overall trend of the super-pipe data.

Figure 4 shows a color-coded map of the stream-wise velocity field at an instant in time for the LES at $Re_\tau = 2 \times 10^5$. The spectral-element mesh used presently is independent of Reynolds number, and hence increasing Reynolds number corresponds to increased under-resolution of the flow. Consequently, as Re_τ increases, the solution experiences stronger effects of errors associated with $C1$ discontinuity inherent in the spectral element method. These effects can be observed faintly along the element edges immediately normal to the wall in Figure 4.

Conclusions

The present LES extends the stretched-vortex subgrid-scale model, and the wall model [4] to the LES of pipe flow at extreme Reynolds numbers with values of order those of the superpipe experiments. Despite some effects of $C1$ discontinuities near the pipe wall, the present LES appear able to reproduce friction-factor-Reynolds number measurements at large Reynolds number and also the main features of the pipe-flow mean velocity profiles. It may be argued that this might be in part a result of the use of a log-like relationship to provide a slip velocity for the outer-flow LES. Cheng and Samtaney [2] recently reported the LES of channel flow at large Reynolds number using the stretched-vortex SGS model in the bulk of the flow and a similar wall model as used presently except that the slip velocity calculation is performed with a power-law rather than a log-like model. For the turbulent mean velocity profiles they find similar outer-flow results for both models including log-like behavior for the mean velocity profile.

A limitation of the present modeling approach is that the flow on viscous length scales very near the wall cannot be resolved by the LES. Nonetheless LES techniques with affective wall modeling appear to be capable of capturing the outer flow [5] including very large-scale motions (VLSMs[8]) comprising flow structures with streamwise extent of order $5-10$ pipe diameters. A hybrid simulation approach consisting of LES providing a description of both large and VLSM motions coupled to semi-empirical modeling of the near-wall motions [11, 7] offers the potential for future advances in our understanding of pipe-flow at extreme Reynolds numbers.

Acknowledgements

This work has been supported in part by the National Science Foundation under Grant CBET-1235605.

References

[1] Blackburn, H. M. and Sherwin S. J. Formulation of a galerkin spectral element-Fourier method for three-dimensional incompressible flows in cylindrical geometries. *Journal of Computational Physics*, **197**, 759–778, 2004.

[2] Cheng, W and Samtaney, R Power-law versus log-law in wall-bounded turbulence: A large-eddy simulation perspective. *Phys Fluids*, **26**, Art. 0117038, 2014.

[3] Chin, C., Ooi, A. S.H., Marusic, I. and Blackburn, H. M The influence of pipe length on turbulence statistics computed from direct numerical simulation data *Phys Fluids*, **22**, Art.115107, 2010.

[4] Chung, D. and Pullin, D. I. Large-eddy simulation and wall-modeling of turbulent chanel flow. *Journal of Fluid Mechanics*, **631**, 281–309, 2009.

[5] Chung, D. and McKeon, B. J. Large-eddy simulation of large-scale structures in long channel flow *Journal of Fluid Mechanics*, **661**, 341–364, 2010.

[6] Inoue, M. and Pullin, D. I. Large-eddy simulation of the zero-pressure-gradient turbulent boundary layer up to $Re_\theta = O(10^{12})$. *Journal of Fluid Mechanics*, **686**, 1–27, September 2011.

[7] Inoue, M., Mathis, R., Marusic, I. and Pullin D. I. Inner-layer intensities for the flat-plate turbulent boundary layer combining a predictive wall-model with large-eddy simulations. *Physics of Fluids*, **24**, Art.075102, 2012.

[8] Kim, K. C. and Adrian, R. J. Very large-scale motion in the outer layer. *Physics of Fluids*, **11**, 417–422, 1999

[9] McKeon, B. J., Swanson, C. J., Zagarola, M. V. Donnelly, R. J. and Smits A. J., Friction factors for smooth pipe flow. *Journal of Fluid Mechanics*, **511**, 41–4, 2004.

[10] McKeon, B. J., W. Jiang, J. Li, Morrison, J. F. and Smits, A. J.. Further observations on the mean velocity distribution in fully developed pipe flow. *Journal of Fluid Mechanics*, **501**, 135–147, 2004.

[11] Mathis, R., Hutchins, N., and Marusic I., A predictive inner–outer model for streamwise turbulence statistics in wall-bounded flows, *Journal of Fluid Mechanics*, **681**, 537–566, 2011.

[12] Moody, L. F. Friction factors for pipe flow. *Transactions of ASME*, **66**, 671–684, 1944.

[13] J. Nikuradse. J. Strömungsgestze in rauhen rohren. *V.D.I. Forschungsheft*, **361**, 1–22, 1933.

[14] Zagarola, M. V. and Smits, A. J. Mean-flow scaling of turbulent pipe flow. *Journal of Fluid Mechanics*, **373**, 33–79, 1998.