

Appendix. A Pulping, Handsheet Formation and Physical Testing

Appendix A.1 Fibre raw material

Laboratory made pulps were used in this project. Firstly, it is easier to control the quality of the laboratory made pulp compare to commercial pulp. It can also avoid any effects on the pulp that may result from commercial pulping process. Radiata pine, which is the major long fibre resource used in Australia, was chosen as the fibre raw material for this project. In the preliminary experiment, recycled fibres from commercial plaster board were also used. The recycled pulp was washed in a container with a 200 mesh screen on its bottom to remove fines and fillers before it was used for making handsheets and doing fibre analysis. This pulp was a mixture of different fibre resources and only for the purpose of demonstration of the new technique for measuring fibre cross-sectional dimensions directly in paper. It was not used any more in later experiment.

Wood chips of radiata pine, collected from a pulp mill in Australia, were mixed, air dried and stored in a laboratory of Australian Pulp and Paper Institute. These woodchips were cooked using kraft batch pulping process to obtain pulps with different kappa number.

The wood chips selected for this project were cooked in the Australian Pulp and Paper Institute laboratory pulping rig using the kraft cooking process. Pulps with two different kappa number were cooked.

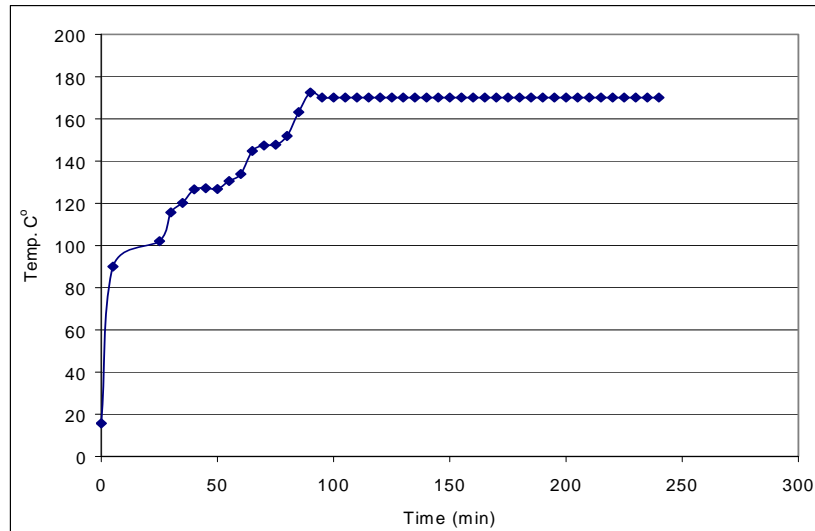
Appendix A.2 Pulping for low kappa pulp

The cooking conditions used for cooking low kappa pulp are presented in TableA.1. The kappa number was tested in accordance with AS201m-86: 1990. Figure A.1 gives the cooking temperature profile used for making the low kappa pulp.

Table A.1. Cooking conditions for low kappa pulp

Wood chips (O.D)	4530g (each batch)
Liquid ratio	8.35
EA	18% (on wood, as Na ₂ O)
Sulfidity	30%
Yield	45.6%
Kappa No.	30

Figure A.1 Cooking temperature profile



Appendix A.3 Pulping for high kappa pulp

A two-stage cooking process, which simulates the cooking conditions of the pulp mill, was used for making high kappa pulp. The chemicals were split into two equal portions as shown in Table 3.2. Figure A.2 gives the cooking temperature profile used for making the high kappa pulp. This pulp was not used in further experimental work because there were a large quantity of shives in it.

Table A.2 Cooking conditions for high kappa pulp

Wood chips (O.D)	4530g (each batch)
Liquid ratio	9.01
EA	7.5% (on wood, as Na ₂ O, for initial and second stages)
Sulfidity	30%
Yield	65.8%
Kappa No.	95

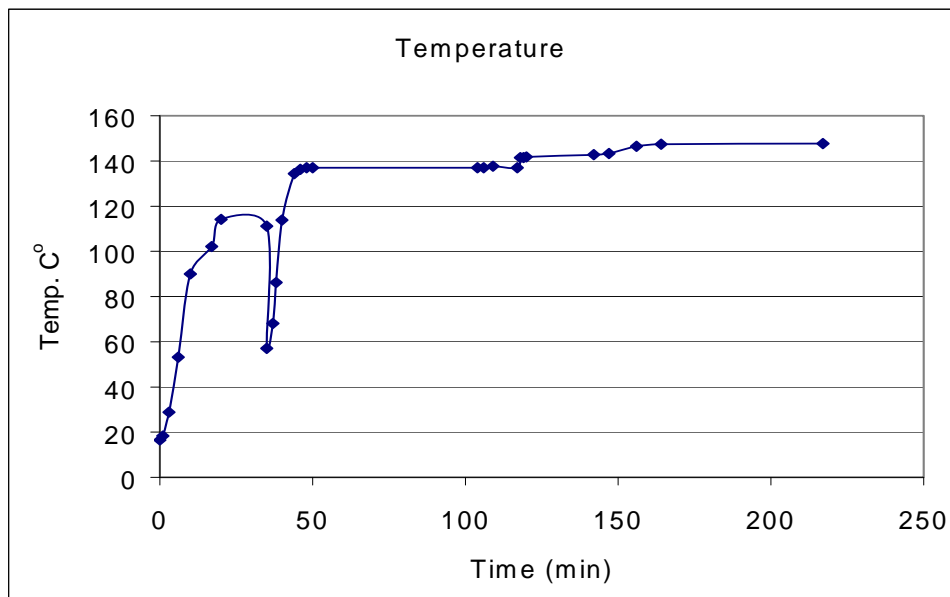


Figure A. 2 Temperature profile for cooking the high kappa pulp

Appendix A.4 Handsheet formation

Handsheets of 60g/m^2 from different pulp fractions were made on a Moving Belt Sheet Former {Xu, 2000 #840}. The handsheets were formed at a very low pulp consistency (0.06%) in order to get good formation of the paper. The size of the square handsheets used is $220\text{mm}\times 220\text{mm}$. All of the handsheets were dried under restrained in a conditioned room (23 C° and 50% humidity).

Appendix A.5 Physical property test

- The sheet grammage was tested following AS/NZS 1310.405s-92: 1991.
- The tensile strength of paper with normal strength was tested using a Lorentzen & Wettre, Alwetron TH1 following AS/NZS 1301.404s-81: 1991.
- The tensile strength of paper made from the unrefined high kappa pulp and the fractionated fractions from this pulp was too low to be able to test using the Lorentzen & Wettre, Alwetron tensile tester. The tensile strength of handsheets from these pulp fractions was tested using an Instron Tensile Tester at a constant elongation of 0.166mm/sec .
- The sheet thickness and apparent density were tested following AS/NZS 1301.426s-94: 1992.
- The zero span tensile strength of paper was tested using a Pulmac Zero Span tester in accordance with the method comes with the machine.
- The scattering coefficient of paper was measured at two different light wavelengths, viz. 557nm and 700nm , using a Colortouch Brightness tester.

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Table A.3. Summary of physical properties of handsheets from the low kappa pulp

Sample	Apparent density (kg/m ³)	Tensile index (kN.m/kg)	Zero span tensile index (kN.m/kg)	Scattering coefficient (m ² /kg)	RBA _{N2}	RBA _{sc}
L ₀ P ₁	286	34.5	143.1	23.17	0.21	0.30
L ₀ P ₂	375	37.0	145.7	22.19	0.24	0.33
L ₀ P ₃	509	46.0	148.6	18.87	0.30	0.43
L ₀ P ₄	590	48.0	143.8	16.00	0.36	0.52
L ₀ P ₅	596	50.0	147.3	14.30	0.40	0.57
L ₁ P ₁	338	29.8	145.5	22.56	0.26	0.32
L ₁ P ₂	406	33.8	149.2	20.71	/	0.37
L ₁ P ₃	541	42.8	151.9	18.03	0.31	0.45
L ₁ P ₄	590	47.0	151.4	15.62	0.37	0.53
L ₁ P ₅	613	48.7	147.7	14.32	/	0.57
L ₂ P ₁	373	29.1	141.2	22.31	0.31	0.33
L ₂ P ₂	411	28.7	140.1	20.89	/	0.37
L ₂ P ₃	434	29.5	148.4	19.95	0.35	0.40
L ₂ P ₄	609	43.1	148.2	15.69	/	0.53
L ₂ P ₅	573	37.6	141.5	16.43	0.38	0.50
L ₃ P ₁	370	22.3	154.3	21.25	/	0.36
L ₃ P ₂	425	27.4	142.2	20.12	/	0.39
L ₃ P ₃	473	27.3	158.8	19.05	/	0.42
L ₃ P ₄	621	39.3	165.6	15.35	/	0.54
L ₃ P ₅	632	41.2	158.0	15.28	0.43	0.54
AcP ₁	218	21.2	127.5	24.43	0.18	/
AcP ₂	322	26.8	131.8	23.90	0.24	/
AcP ₃	392	32.7	132.2	22.85	0.32	/
AcP ₄	487	42.0	127.9	19.66	0.34	/
AcP ₅	651	56.5	134.3	16.34	0.44	/
RejP ₁	193	9.8	121.4	22.21	0.08	/
RejP ₂	218	12.8	129.4	22.21	0.18	/

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RejP ₃	306	16.8	136.1	21.51	0.21	/
RejP ₄	380	22.1	136.2	20.96	0.28	/
RejP ₅	574	31.4	142.9	16.25	0.36	/

Appendix B Measurement of fibre cross-section on glass slides

Appendix B 1 Slide preparation

A drop of diluted suspension of pre-stained pulp fibres was taken and placed on a clean glass slide. The slide was then put into an oven for 2 minutes to dry the fibres. A few drops of immersion oil were placed on the dried fibres and a cover slip was placed on top. The slide assembly was put into a container where a vacuum was then applied for about 1 hour to expel the air trapped in the fibre and allow the immersion oil thoroughly penetrate into the fibre structure. The slide assembly was then taken out of the container and put another drop of immersion oil on top of the cover slip, and it is ready for analysis in the confocal microscope.

Appendix B 2 Imaging and image analysis

A 60×oil immersion lens was used to capture the image. The cross-sectional images of fibres were scanned perpendicular to the longitudinal axes of the fibres to avoid elongation of the image. The image frame acquired was 50×50 μm with a resolution of 512×512 pixels.

The images of fibre cross-sections were then analyzed using a custom macro written in the software OPTIMAS 6.1. {Xu, 1997 #213} (see Appendix A2). The macro performs the following steps for each fibre to determine the cross-section dimensions:

- (1) Locates a fibre in the multi-image file.
- (2) Extracts the region of interest (ROI) around the fibre.

- (3) Applies filtering operations to the ROI to improve image quality. A variation of the algorithm by {Xu, 1997 #213} was used to reduce the amount of change to the original image associated with using averaging filters and threshold {Conn, 1999 #1136}.
- (4) Reduction of the image from 256 bit grey scale image to a binary (black and white) image where the fibre is classed as white and the background black.
- (5) Direct measurements of total fibre cross-section area, fibre wall cross-section area, outside fibre perimeter, fibre wall thickness, fibre width and fibre thickness.

Appendix. C Other Measurement

Appendix C.1 Measurement of fibre length

The fibre length of the pulp samples used in this project was measured using a Kajaani FS200 optical fibre length analyser. The Kajaani fibre length analyzer measures single fibres that are drawn from a diluted 0.01% consistency through a narrow capillary, under suction. The fibres pass through a beam of polarized light. These fibres, being birefringent to polarized light, create an interval of birefringence as they pass through. This interval is related to the fibre length of the fibres. This method is fast as it can measure thousands of fibres in a short space of time. Up to 35 length categories can be measured and the software produces statistical information on the fibre distributions.

Table C.1. Summary of fibre length of different pulp fibres measured by Kajaani FS

200

Pulp Fibre		Mathematic length	Weighted length	Fines (<0.1mm)		
				P (%)	W (%)	
Low kappa pulp	L ₀	1.78	3.14	10.06	0.29	
	L ₁	1.44	2.53	7.01	0.25	
	L ₂	1.22	2.10	6.18	0.27	
	L ₃	1.08	1.80	4.48	0.22	
	Accepts (AA)	2.41	3.34	2.59	0.05	
	Rejects (RR)	2.22	2.98	1.09	0.03	
High kappa	Pine 1	2.16	3.10	2.78	0.06	
	Feed	2.28	3.10	2.60	0.06	
	Pine 2	Accepts	2.29	3.17	2.23	0.05
	Rejects	2.38	3.42	0.81	0.02	

Appendix C.2 Measurement of free surface area of paper

The nitrogen adsorption is an important technique has been used to measure fibre surface area. Haselton [Haselton, 1954 #1079][Haselton, 1955 #1080] is the first to propose the use of gas adsorption as a method of measurement of the surface area and bonded area of papers. After comparing bonded areas calculated by four methods he concluded that a value closest to the true bonded area could be obtained by applying a correction factor to the regular optical values. When nitrogen adsorption is employed for area studies, the surface area accessible to nitrogen molecules of approximately 4.3Å thickness (N₂ diameter is 16.3 sq Å) is determined. The bonded area in the external and internal portions of fibres is believed to be separated by distances of 4 to 5 Å or less. Therefore, it is reasonable to believe that nitrogen is not absorbed on areas involved in bonding and that adsorption methods may provide an excellent tool for the measurement of the unbonded internal and external area of cellulosic materials. The Brunauer, Emmett and Teller (B.E.T.) theory [Brunauer, 1938 #1082] was used to calculate the volume of gas required to form a monolayer on a given adsorbent. The BET equation is given as:

$$\frac{p/p_0}{v(1-p/p_0)} = \frac{1}{v_m c} + \frac{c-1}{v_m c} (p/p_0) \quad \text{C. 1}$$

where p/p_0 is relative pressure, v is the volume of adsorbed gas (cm³/g STP) per gram of adsorbent at a certain relative pressure, v_m (cm³ STP) is the volume of gas per gram sample required to form a monolayer, c is a constant.

The plot of $\frac{p/p_0}{v(1-p/p_0)}$ against p/p_0 should therefore be a straight line with slope

$s = (c-1)/v_m c$ and intercept $i = 1/v_m c$. Solution of the two simultaneous equations gives v_m and c . Multiplying the number of gas molecules corresponding to v_m by the cross-sectional area of each molecule gives the area of the solid accessible to the gas.

$$A = \frac{Nv_m L}{22,400} \times 10^{-20}$$

C. 2

Where A (m^2/g) is the specific area of the sample, N is Avogadro's number and L (m^2) is the molecular cross-sectional area of the gas absorbed.

The isotherm for this calculation was obtained by the nitrogen adsorption method using a Micromeritics ASAP (Accelerated Surface Area and Porosimetry) 2010. The sample weight for each measurement was about 2g. Each data point of the BET area was the average of two duplicate measurements.

Table C.2. Summary of BET area for handsheets and spray dried fibres

Sample	BET area of sheets (m^2/kg)	BET area of spray dried fibres (m^2/kg)
L ₀ P ₁	716	905
L ₀ P ₂	687	
L ₀ P ₃	632	
L ₀ P ₄	583	
L ₀ P ₅	544	
L ₁ P ₁	683	920
L ₁ P ₃	636	
L ₁ P ₅	582	
L ₂ P ₁	636	921
L ₂ P ₃	602	
L ₂ P ₅	570	
L ₃ P ₅	525	/
AcP ₁	873	950
AcP ₂	805	
AcP ₃	722	
AcP ₄	697	
AcP ₅	594	

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RejP ₁	872	1064
RejP ₂	783	
RejP ₃	754	
RejP ₄	683	
RejP ₅	606	

Appendix.D-Macro used in this thesis

Appendix D.1 Optimas macro for measuring mass centre of fibre in paper cross-section

```
/******
```

Optimas macro for calculating the position of a fibre
in a paper X-section by describing with the mouse.

author: A. Conn

APPI (c) 2000

25-Oct-2000

```
*****/
```

```
/* set pixel scale here */
```

```
REAL COM_rMicron_per_pixel = 100.0/512.0;
```

```
/* Open configuration file */
```

```
OpenConfiguration("c:/optimas6/Config/JiHong.cfg");
```

```
Calibrate (One_Pixel_per_Micro);
```

```
/* clear screen of lines, areas etc. */
```

```
/* set ROI to full screen */
```

```
SelectFullScreen();
```

```
ClearScreen();
```

```
/* user creates area to get the centre of mass */
```

```
SetExport(ArCenterOfMass,1,TRUE); /* set "To DDE" */
```

```
CreateArea(); /* user creates an area */
```

```
Extract(); /* get the data object */
```

```
CreatePoints(ArCenterOfMass); /* create a point at the center */
```

```
FSP_Centre = ArCenterOfMass*COM_rMicron_per_pixel;
```

```
/* write results to screen */
```

```
MacroMessage("\nFibre Centre (um) = ",FSP_Centre);
```

```
/* write results to center.dat file */
```

```
    fh = OpenFile ("c:/optimas6/center.dat");
```

```
    PositionFile (fh, 0L, 2); /* position to end of file */
```

```
    a = ToText (FSP_Centre): "\n";
```

```
    WriteFile (fh, a);
```

```
    CloseFile (fh);
```

```
/* end with clean up */
```

```
ObjectWildcardList("COM_.*", 2);
```

Appendix D.2 Optimas macro for measuring fibre dimensions in paper cross-section

```
/******
```

Optimas macro for calculating the Area, position and bounding
box of a fibre in a paper X-section via thresholding.

author: A. Conn

APPI (c) 2000

25-Oct-2000

```
*****/
```

```
/* set pixel scale here */
```

```
REAL FSP_rMicron_per_pixel = 100.0/512.0;
```

```
/* Open configuration file */
```

```
OpenConfiguration("c:/optimas6/Config/JiHong.cfg");
```

```
Calibrate (One_Pixel_per_Micro);
```

```
/* set ROI to full screen */
SelectFullScreen();
ClearScreen();

/* save image to List */
ROItoList( "Xsection");

/* user selects Region of interest to be processed */
StatusBar = "Select a Region of Interest around a single fibre Xsection ...";
SelectROI();

/* image filtered to improve contrast and smoothness */
Filters( SharpenMed );
MedianFilter(,5);

/* Automatic threshold - Search for threshold in minimum near region of mean */
BOOLEAN FSP_bLightObjects = TRUE; /* find light objects */
INTEGER FSP_iForegroundLimits = 1 : 95; /* ignore bottom 1% and top 5% */
REAL FSP_rEndPercents =
(REAL)( FSP_iForegroundLimits[0] : (100 - FSP_iForegroundLimits[1]) );
if (FSP_bLightObjects)
FSP_rEndPercents = FSP_rEndPercents[1:0];
Histogram(); /* force update of ArROIHistogram */
REAL FSP_rAutoThresh =
GetAutoThreshold(ArROIHistogram,5,,FSP_rEndPercents,ActiveLuminanceRange);
Threshold( FSP_rAutoThresh[FSP_bLightObjects+1,]); /* set new threshold */

/* fine tune threshold manually */
StatusBar = "Manually fine tune the threshold ...";
Threshold();
```

```
/* Binarise to bitmap */
INTEGER FSP_hInitialBitmap = BitmapCreate();

/* auto create areas */
CreateArea(,TRUE); /* clear existing, autcreate areas */

/* user selects area (outside of fibre) of interest */
StatusBar = "Select the outside boundary of the fibre ...";
AutoExtract = FALSE; /* no auto extract on select */
MultipleMode = FALSE; /* single mode is better for simple selecting */
/* while user does not select one */
while ( !(FSP_hID = Select( , FALSE)) ) /* prevent edit/deletes */
    if ( !Prompt("None selected, try again?") ) /* give more tries */
        pause(); /* if user cancels, stop the macro */

/* reset extract modes */
//AutoExtract = TRUE; /* no auto extract on select */
//MultipleMode = TRUE; /* single mode is better for simple selecting */

/* clear other Areas */
ClearProtect( FSP_hID, TRUE); /* protect 'em */
ClearScreen();
ClearProtect( FSP_hID, FALSE); /* unprotect 'em */

/* Screen Area to Bitmap - process to give fibre of interest*/
INTEGER FSP_hAreaBitmap = ActiveImage.BitmapCreate(,,,0);
BitmapProcess (FSP_hAreaBitmap, FSP_hInitialBitmap, 8, ); /* AND Bitmaps */
BitmapProcess (FSP_hAreaBitmap, , 22,); /* copy result to image */
ClearScreen();

/* calculate Area from white pixels */
Histogram();
```

```
FSP_Area = (REAL)ArROIHistogram[255] *
FSP_rMicron_per_pixel*FSP_rMicron_per_pixel;

/* Create Area and Points */
SetExport( mArBoundingROI,1,TRUE); /* set "To DDE" */
SetExport(mArCenterOfMass,1,TRUE); /* set "To DDE" */
CreateArea( , TRUE);
MultipleExtractAll (TRUE);
FSP_Centre = mArCenterOfMass[0..2]* FSP_rMicron_per_pixel;
CreatePoints(mArCenterOfMass[0..2]);

/* set ROI to full screen */
SelectFullScreen();

/* replace original image */
ArithmeticOp("copy", "Xsection");

/* draw BoundingROI around Area and calculate sides*/
ROI = mArBoundingROI[0..4];
FSP_Width = (mArBoundingROI[2] - mArBoundingROI[0])*FSP_rMicron_per_pixel ;
FSP_Height = (mArBoundingROI[1] - mArBoundingROI[3])*FSP_rMicron_per_pixel ;

/* calculate % of fibre area in boundingbox */
FSP_AreaPercent = FSP_Area/FSP_Width/FSP_Height*100.0;

/* write results to screen */
MacroMessage("Fibre Area (um2) = ",FSP_Area,"\nFibre Centre (um) =
",FSP_Centre,"\nWidth (um) = ",FSP_Width,"\nHeight (um) = ",FSP_Height,"\nArea
(%) = ",FSP_AreaPercent);

/* write results to sp.dat file */
fh = OpenFile ("c:/optimas6/sp.dat");
```

```

        PositionFile (fh, 0L, 2); /* position to end of file */
        a = ToText (FSP_Area) : "\t":ToText (FSP_Centre): "\t":ToText
(FSP_Width): "\t":ToText (FSP_Height): "\t":ToText (FSP_AreaPercent): "\n";
        WriteFile (fh, a);
        CloseFile (fh);

/* end with clean up */
ObjectWildCardList("FSP_.*", 2);

```

Appendix D.3 Optimas macro for calculating major axis bounding box

```

/*****
Optimas macro for calculating the Area, position and bounding
box of a fibre in a paper X-section via thresholding.

author: A. Conn and Jihong He
APPI (c) 2001
25-July-2001
*****/

```

```

/* set pixel scale here */
REAL FSP_rMicron_per_pixel = 100.0/512.0;

/* Open configuration file */
OpenConfiguration("c:/optimas6/Config/JiHong.cfg");
Calibrate (One_Pixel_per_Micro);

/* set ROI to full screen */
SelectFullScreen();
ClearScreen();

```

```
/* save image to List */
ROItoList( "Xsection");

/* user selects Region of interest to be processed */
StatusBar = "Select a Region of Interest around a single fibre Xsection ...";
SelectROI();

/* image filtered to improve contrast and smoothness */
Filters( SharpenMed );
MedianFilter(,5);

/* Automatic threshold - Search for threshold in minimum near region of mean */
BOOLEAN FSP_bLightObjects = TRUE; /* find light objects */
INTEGER FSP_iForegroundLimits = 1 : 95; /* ignore bottom 1% and top 5% */
REAL FSP_rEndPercents =
(REAL)( FSP_iForegroundLimits[0] : (100 - FSP_iForegroundLimits[1]) );
if (FSP_bLightObjects)
FSP_rEndPercents = FSP_rEndPercents[1:0];
Histogram(); /* force update of ArROIHistogram */
REAL FSP_rAutoThresh =
GetAutoThreshold(ArROIHistogram,5,,FSP_rEndPercents,ActiveLuminanceRange);
Threshold( FSP_rAutoThresh[FSP_bLightObjects+1,]); /* set new threshold */

/* fine tune threshold manually */
StatusBar = "Manually fine tune the threshold ...";
Threshold();

/* Binarise to bitmap */
INTEGER FSP_hInitialBitmap = BitmapCreate();

/* auto create areas */
```

```
CreateArea(,TRUE); /* clear existing, autocreate areas */

/* user selects area (outside of fibre) of interest */
StatusBar = "Select the outside boundary of the fibre ...";
AutoExtract = FALSE; /* no auto extract on select */
MultipleMode = FALSE; /* single mode is better for simple selecting */
/* while user does not select one */
while ( !(FSP_hID = Select( , FALSE)) ) /* prevent edit/deletes */
    if ( !Prompt("None selected, try again?") ) /* give more tries */
        pause(); /* if user cancels, stop the macro */

/* reset extract modes */
//AutoExtract = TRUE; /* no auto extract on select */
//MultipleMode = TRUE; /* single mode is better for simple selecting */

/* clear other Areas */
ClearProtect( FSP_hID, TRUE); /* protect 'em */
ClearScreen();
ClearProtect( FSP_hID, FALSE); /* unprotect 'em */

SetExport(mArBoundingBox,1,TRUE); /* set "To DDE" */
SetExport(mArMajorAxisAngle,1,TRUE); /* set "To DDE" */
MultipleExtractAll(TRUE); /* get the data object */
CreateArea(mArBoundingBox);

ClearProtect( FSP_hID, TRUE); /* protect 'em */

Box = mArBoundingBox[0..7];
Box_Width = Sqrt(Pow((ArBoundingBox[2] -
ArBoundingBox[0]),2)+Pow((ArBoundingBox[3] -
ArBoundingBox[1]),2))*FSP_rMicron_per_pixel ;
```

```
Box_Height = Sqrt(Pow((ArBoundingBox[0] -
ArBoundingBox[6]),2)+Pow((ArBoundingBox[7] -
ArBoundingBox[1]),2))*FSP_rMicron_per_pixel ;

/* clear other Areas */
ClearProtect( FSP_hID, TRUE); /* protect 'em */
ClearScreen();
ClearProtect( FSP_hID, FALSE); /* unprotect 'em */

/* Screen Area to Bitmap - process to give fibre of interest*/
INTEGER FSP_hAreaBitmap = ActiveImage.BitmapCreate(,,,0);
BitmapProcess (FSP_hAreaBitmap, FSP_hInitialBitmap, 8, ); /* AND Bitmaps */
BitmapProcess (FSP_hAreaBitmap, , 22,); /* copy result to image */
ClearScreen();

/* calculate Area from white pixels */
Histogram();
FSP_Area = (REAL)ArROIHistogram[255] *
FSP_rMicron_per_pixel*FSP_rMicron_per_pixel;

/* Create Area and Points */
SetExport( mArBoundingROI,1,TRUE); /* set "To DDE" */
SetExport(mArCenterOfMass,1,TRUE); /* set "To DDE" */
CreateArea( , TRUE);
MultipleExtractAll (TRUE);
FSP_Centre = mArCenterOfMass[0..2]* FSP_rMicron_per_pixel;
CreatePoints(mArCenterOfMass[0..2]);

/* set ROI to full screen */
SelectFullScreen();
```

```
/* replace original image */
ArithmeticOp("copy", "Xsection");

/* draw BoundingROI around Area and calculate sides*/
ROI = mArBoundingROI[0..4];
FSP_Width = (mArBoundingROI[2] - mArBoundingROI[0])*FSP_rMicron_per_pixel ;
FSP_Height = (mArBoundingROI[1] - mArBoundingROI[3])*FSP_rMicron_per_pixel ;

/* calculate % of fibre area in boundingbox */
FSP_AreaPercent = FSP_Area/FSP_Width/FSP_Height*100.0;
Box_AreaPercent = FSP_Area/Box_Width/Box_Height*100.0;

/* write results to screen */
MacroMessage("Fibre Area (um2) = ",FSP_Area,"\nFibre Centre (um) =
",FSP_Centre,"\nWidth (um) = ",FSP_Width,"\nHeight (um) = ",FSP_Height,"\nArea
(%) = ",FSP_AreaPercent,"\nBoxWidth (um) = ",Box_Width,"\nBoxHeight (um) =
",Box_Height,"\nBoxArea (%) = ",Box_AreaPercent,"\ MajorAxisAngle (degree) =
",ArMajorAxisAngle);

/* write results to sp.dat file */
fh = OpenFile ("c:/optimas6/sp.dat");
PositionFile (fh, 0L, 2); /* position to end of file */
a = ToText (FSP_Area) :"\t":ToText (FSP_Centre):"\t":ToText
(FSP_Width):"\t":ToText (FSP_Height):"\t":ToText (FSP_AreaPercent):"\t": ToText
(Box_Width):"\t": ToText (Box_Height):"\t": ToText (Box_AreaPercent):"\t":ToText
(ArMajorAxisAngle):"\n";
WriteFile (fh, a);
CloseFile (fh);
```

```
/* end with clean up */  
ObjectWildcardList("FSP_.*", 2);
```

Appendix D.4 MatLab macro for simulation of load distribution along the fibre with bond breakage

```
%Evaluate n contacts direct stress transfer  
clear  
  
syms d1 d2 d3 d4 d5 d6 d7 d8 d9 d10 d11 d12 d13 d14 d15 d16 d17 d18 d19 d20 d21 D  
sum1;  
syms d22 d23 d24 d25 d26 d27 d28 d29 d30 d31 d32 d33 d34 d35 d36 d37 d38 d39 d40  
d41 d42 d43 d44 d45 d46 d47 d48 d49 d50 d51 d52 d53 d54 d55 d56 d57 d58 d59 d60;  
syms d61 d62 d64 d65 d66 d67 d68 d69 d70  
add=0 %Counter for separating different simulation sets in excel spreadsheet. Later to  
set to no. of fibres+2  
bdd=0 %conter for r position  
Favmax=0 %max average load  
%Sample identifiers and comments  
SampleID='SL0'  
Comment='Increase strain, remove broken bonds random angle and position. Force=-  
2(d/lc)EAcos2th'  
nsimulations=2 % no. of separate simulations  
  
%constants for calculation  
fibrelength=1572;  
b=48.4;  
c=1.62; %Weibull distribution parameters  
  
lc=5*b; %length before crossing fibres bonded into matrix
```

```
E=40*10^9; %Fibre elastic modulus
A=200*10^-12; %Fibre cross-sectional area
halfc=0.52; %Fraction of partial contacts
Width=31.0e-6; %fibre width
sbs=3e6; %shear bond strength per unit area
R=1.35; %Fractional spreading on contact
spread=ddeinit('excel','data.xls'); %Write the calculation constants to the data file
ddepoke(spread,'r1c1',Comment);
ddepoke(spread,'r2c1','Half fibre length (microns)');
ddepoke(spread,'r3c1',fibrelength);
ddepoke(spread,'r4c1','Crossing fibre length until locked in matrix');
ddepoke(spread,'r5c1',lc);
ddepoke(spread,'r6c1',strcat('Sample: ',SampleID));
ddepoke(spread,'r7c1','Weibull contact distribution parameters');
ddepoke(spread,'r8c1',b);
ddepoke(spread,'r8c2',c);
ddepoke(spread,'r9c1','Shear bond strength per unit area');
ddepoke(spread,'r10c1',sbs);

ddepoke(spread,'r2c6','Fibre elastic modulus');
ddepoke(spread,'r3c6',E);
ddepoke(spread,'r4c6','Fibre cross-section area');
ddepoke(spread,'r5c6',A);
ddepoke(spread,'r6c6','Fibre width');
ddepoke(spread,'r7c6',Width);
ddepoke(spread,'r8c6','Percentage of fractional contacts');
ddepoke(spread,'r9c6',halfc);
ddepoke(spread,'r2c11','Spreading of bonded area');
ddepoke(spread,'r3c11',R);

%Generate fibres with random positions, angles and full or partial contacts
```

```

%Generate positions from Weibull distributions
x(1)=b*(-log(1-rand))^(1/c)/2; %Generates the first fibre
%Division by 2 as calculating from middle of fibre
sum2=x(1);
for j=2:60;
    sum2=sum2+b*(-log(1-rand))^(1/c);
    if sum2<fibrelength; % Generate fibre until latest fibre exceeds fibre length, then stop. i
is total fibre no.
        x(j)=sum2;
        i=j;
    end;
end;
%Generate random angles
for j=1:i;
    th(j)=pi/2-asin(rand);
end;

%Generate full or partial contacts
for j=1:i;
    if rand<halfc;
        bondstrength(j)=0.5*R*Width^2/sin(th(j))*sbs;
    else;
        bondstrength(j)=R*Width^2/sin(th(j))*sbs;
    end;
end;

for j=1:i
    bondbreak(j)=0;
end

count=1
interval=0.001;

```

```
eps=0;

while eps<0.013; %network/matrix strain
eps

'Simulation No.:',count
clear Dr F Fseg D sum1 sum2
D=[d1 d2 d3 d4 d5 d6 d7 d8 d9 d10 d11 d12 d13 d14 d15 d16 d17 d18 d19 d20 d21
d22 d23 d24 d25 d26 d27 d28 d29 d30 d31 d32 d33 d34 d35 d36 d37 d38 d39 d40 d41
d42 d43 d44 d45 d46 d47 d48 d49 d50 d51 d52 d53 d54 d55 d56 d57 d58 d59 d60 d61
d62 d64 d64 d65 d66 d67 d68 d69 d70];

for j=1:i;
fv(j)=-2/lc*cos(th(j))^2; %multiply these values by displacement to get force
end;

for j=i-1:-1:1;
sum1=0;
for k=i:-1:j+1;
sum1=vpa(sum1+fv(k)*D(k));
end;
D(j)=vpa(D(j+1)+(x(j+1)-x(j))*(eps-sum1));
end

sum1=0;

for j=1:i;
sum1=vpa(sum1+x(j)*fv(j)*D(j));
end;

z=vpa(sum1-eps*x(i)-D(i));
```

```
y=solve(z,'D(i)');
double(y)
Dr(i)=double(y);
for j=i-1:-1:1;
    sum2=0;
    for k=i:-1:j+1;
        sum2=sum2+fv(k)*Dr(k);
    end;
    Dr(j)=Dr(j+1)+(x(j+1)-x(j))*(eps-sum2);
end;
%resubstitution to obtain expressions for the D(j) Dr is now the real expression

for j=1:i;
    F(j)=Dr(j)*fv(j)*E*A;
end;
%Calculate the forces applied at each contact point.

for j=1:i;
    sum2=0;
    for k=j:i;
        sum2=F(k)+sum2;
    end;
    Fseg(j)=sum2;
end;
%Calculate the force applied to each segment

%'check of calculation'
Fseg(1)*x(1)/E/A-eps*x(1);
Dr(1);
```

```
fibrestrain=double((eps*x(i)+Dr(i))/fibrelength);
Fav=fibrestrain*E*A;

if Fav>Favmax
    Favmax=Fav;
end
Favmax;
r=Fseg(1)/fibrestrain/E/A;
%r calculated from maximum load in first segment divided by average load along whole
fibre.

xr=rot90(x,3);
thr=rot90(th,3);
Drr=rot90(Dr,3);
bsr=rot90(bondstrength,3);
Fr=rot90(F,3);
Fsegr=rot90(Fseg,3);
Datamat=[xr thr bsr Drr Fr Fsegr];

% Write the results of the simulation to the Excel spreadsheet
ddepoke(spread,strcat('r',num2str(11+add),'c',num2str(11)),'External matrix strain');
ddepoke(spread,strcat('r',num2str(12+add),'c',num2str(11)),eps);

ddepoke(spread,strcat('r',num2str(13+add),'c',num2str(11)),'Check of calculation');
ddepoke(spread,strcat('r',num2str(14+add),'c',num2str(11)),'Displacement of first contact,
d1');
ddepoke(spread,strcat('r',num2str(15+add),'c',num2str(11)),double(Dr(1)));
ddepoke(spread,strcat('r',num2str(16+add),'c',num2str(11)),'Total force in 1st segment');
```

```

ddepoke(spread, strcat('r', num2str(17+add), 'c', num2str(11)), Fseg(1));
ddepoke(spread, strcat('r', num2str(18+add), 'c', num2str(11)), 'Displacement due to force');
ddepoke(spread, strcat('r', num2str(19+add), 'c', num2str(11)), Fseg(1)*x(1)/E/A);
ddepoke(spread, strcat('r', num2str(20+add), 'c', num2str(11)), 'd1=Force-segment1*x1-
globalstrain*x1');
ddepoke(spread, strcat('r', num2str(21+add), 'c', num2str(11)), Fseg(1)*x(1)/E/A-eps*x(1));
ddepoke(spread, strcat('r', num2str(22+add), 'c', num2str(11)), 'Difference between two
calculations of d1');
ddepoke(spread, strcat('r', num2str(23+add), 'c', num2str(11)), double(Dr(1))-
(Fseg(1)*x(1)/E/A-eps*x(1)));

ddepoke(spread, strcat('r', num2str(25+add), 'c', num2str(11)), 'r: cald from max load in first
segment divided by avg. load along whole fibre');
ddepoke(spread, strcat('r', num2str(26+add), 'c', num2str(11)), r);
ddepoke(spread, strcat('r', num2str(27+add), 'c', num2str(11)), 'Fibre strain');
ddepoke(spread, strcat('r', num2str(28+add), 'c', num2str(11)), fibrestrain);

ddepoke(spread, strcat('r', num2str(11+add), 'c', num2str(3)), strcat('Simulation no.:
', num2str(count)));
ddepoke(spread, strcat('r', num2str(12+add), 'c', num2str(3)), 'Contact position');
ddepoke(spread, strcat('r', num2str(12+add), 'c', num2str(4)), 'angle');
ddepoke(spread, strcat('r', num2str(12+add), 'c', num2str(5)), 'Bond strength');
ddepoke(spread, strcat('r', num2str(12+add), 'c', num2str(6)), 'Displacements');
ddepoke(spread, strcat('r', num2str(12+add), 'c', num2str(7)), 'force at contact');
ddepoke(spread, strcat('r', num2str(12+add), 'c', num2str(8)), 'Force in segment');
ddepoke(spread, strcat('r', num2str(13+add), 'c', num2str(3)), 'r', num2str(i+13+add), 'c', num2s
tr(8)), Datamat);
ddepoke(spread, 'r10c20', 'r');
ddepoke(spread, 'r10c21', 'Fav');
ddepoke(spread, 'r10c22', 'Favmax');
ddepoke(spread, 'r10c23', 'strain');
ddepoke(spread, 'r10c24', 'Simulation No.');
```

```
broken=0
%Check for broken fibres
for j=1:i;
    if bondstrength(j)>0;
        if bondstrength(j)<F(j);
            bondbreak(j)=1;
            th(j)=acos(0);
            bondstrength(j)=0;
            broken=1;
        end
    end
end
bondbreakr=rot90(bondbreak,3);
ddepoke(spread,strcat('r',num2str(12+add),'c',num2str(9)),'Bonds broken');
ddepoke(spread,strcat('r',num2str(13+add),'c',num2str(9)),'r',num2str(i+13+add),'c',num2s
tr(9)),bondbreakr);
ddepoke(spread,strcat('r',num2str(11+bdd),'c',num2str(20)),r);
ddepoke(spread,strcat('r',num2str(11+bdd),'c',num2str(21)),Fav);
ddepoke(spread,strcat('r',num2str(11+bdd),'c',num2str(22)),Favmax);
ddepoke(spread,strcat('r',num2str(11+bdd),'c',num2str(23)),eps);
ddepoke(spread,strcat('r',num2str(11+bdd),'c',num2str(24)),count);

if broken==0
    eps=eps+interval
end
if Fav<0.5*Favmax
    eps=0.013
end
broken=0
```

if i<19 %calculates how much to displace write statements so each simulation is
separated in excel

add=22+add;

else

add=i+3+add;

end

count=count+1;

bdd=bdd+1;

end

end

Appendix E. Direct Load Transfer Theory

An analytical solution for the load distribution in a fibre in a network

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Abstract

A new method is presented which allows the load-distribution in one fibre in a loaded network to be analytically calculated. Only the case of a fibre oriented in the direction of the applied load is considered. The method assumes that the force applied to the fibre at a fibre-fibre contact (a crossing point) arises from the displacement of the fibre crossings compared to the external strain field. This paper shows that the displacements at all the contacts can be expressed in terms of the displacement at the final contact and consequently how this can be used to calculate the displacement at the final crossing point. An expression for force transfer at a contact is developed, which assumes that force is transferred into the fibre of interest by direct axial loading, rather than shear. Each crossing fibre is assumed to be rigidly connected into the surrounding matrix after a given distance. The expression for force and the new analytical solution was used to derive force distributions along a fibre for a number of examples. The calculated force distribution along the fibre is sensitive to the exact geometry, for example, the number, location, orientation of the set of crossing fibres.

Introduction

An important factor affecting the mechanical properties of a fibre network under load is the stress-distribution along the length of a fibre in a network. This distribution has been solved analytically only for the relatively simple case of the shear lag model [1-3], where all of the fibres are assumed to be the same and to cross each other at 90° . The force on the fibre from a contact is assumed to be proportional to the difference between the local displacement of the contact point and the overall displacement due to the applied strain.

However, the stress-distribution along a fibre has not been solved analytically for a network with distributions of fibre properties and orientation, although it is possible to calculate using finite element models. The problem is difficult to analyse because force can be applied at any or all fibre crossings and any force both produces and is caused by displacement relative to the applied strain. The force distribution along the fibre will also depend on the exact geometry of the crossing fibres

This purpose of this paper is to present an analytical solution to this problem. A simple model of stress transfer at a fibre crossing is also developed and the model and the analytical solution will be combined to calculate some stress-distributions for some specific examples.

Theory Part 1 Stress Distribution along a fibre in a network

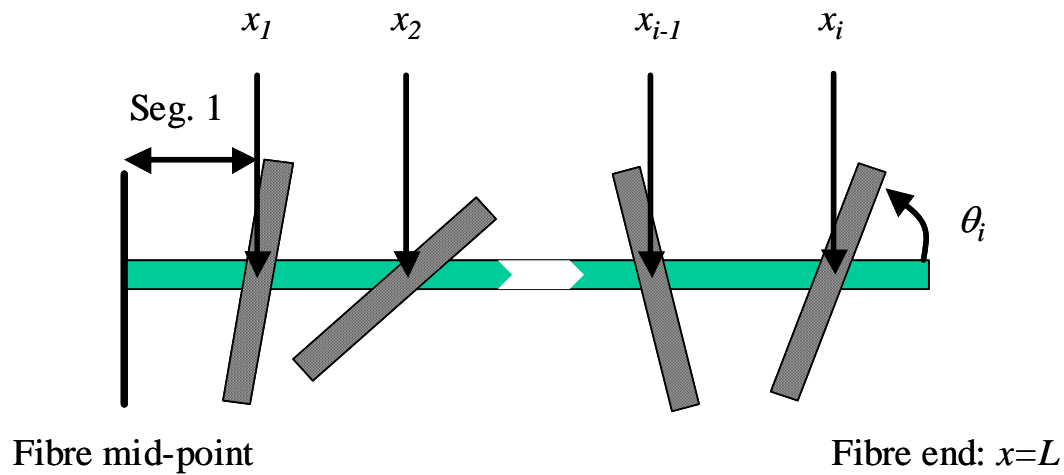


Figure 1 Unstrained half fibre of length, L , with i crossing fibres.

Figure 1 shows half of a fibre, which is connected to a fibre network by i crossing fibres. The centre of the fibre at $x=0$ set as the reference point. A strain of ε is applied to external network in the direction of the fibre axis. It is assumed that force is transferred into the fibre by displacements of the fibre-crossings from their equilibrium position for the applied network strain. The displacement of the j^{th} crossing fibre is designated δ_j . The forces that develop at the j^{th} crossing are some function of δ_j and the angle of the crossing fibre to the fibre axis, θ_j .

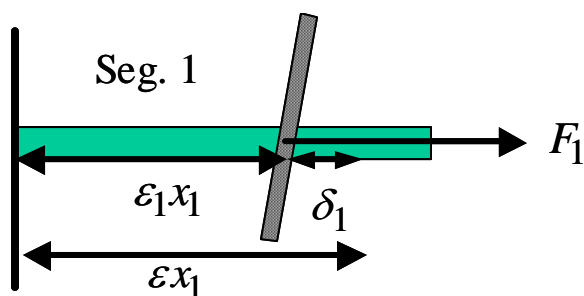


Figure 2. Displacement of crossing point from equilibrium position in the applied global strain field.

The idea is illustrated in Fig. 2, which shows segment 1 of the loaded fibre from Fig.

1. This has a strain in the first segment of ε_1 which differs from the network strain, ε , displacing the crossing point from the equilibrium position by $\delta_1 = (\varepsilon_1 - \varepsilon)x_1$.

The force which has produced the strain ε_1 is the sum of the forces developed at all i crossing fibres and therefore

$$\delta_1 = \frac{1}{EA} x_1 \sum_{j=1}^{j=i} F_j - \varepsilon x_1 \quad 1$$

where E and A are the elastic modulus and cross-sectional area, respectively, of the fibre. The forces that develop each contact point are some function of δ and the angle of the crossing fibre to the fibre axis, θ .

The displacement at the second contact is

$$\delta_2 = \frac{1}{EA} \left(x_1 \sum_{j=1}^{j=i} F_j + (x_2 - x_1) \sum_{j=2}^{j=i} F_j \right) - \varepsilon x_1 - \varepsilon (x_2 - x_1) \quad 2$$

which simplifies to

$$\delta_2 = \frac{1}{EA} \left(x_1 F_1 + x_2 \sum_{j=2}^{j=i} F_j \right) - \varepsilon x_2 \quad 3$$

Extending this process it can be shown that for n^{th} and $n-1^{\text{th}}$ contacts that

$$\delta_n = \frac{1}{EA} \left(\sum_{j=1}^{j=n-1} x_j F_j + x_n \sum_{j=n}^{j=i} F_j \right) - \varepsilon x_n \quad 4$$

$$\delta_{n-1} = \frac{1}{EA} \left(\sum_{j=1}^{j=n-2} x_j F_j + x_n \sum_{j=n-1}^{j=i} F_j \right) - \varepsilon x_{n-1} \quad 5$$

and eqn. 5 can be rewritten as

$$\delta_{n-1} = \frac{1}{EA} \left(\sum_{j=1}^{j=n-1} x_j F_j + x_n \sum_{j=n}^{j=i} F_j \right) - \varepsilon x_n + (x_n - x_{n-1}) \left(\varepsilon - \frac{1}{EA} \sum_{j=n}^{j=i} F_j \right) \quad 6$$

or

$$\delta_{n-1} = \delta_n + (x_n - x_{n-1}) \left(\varepsilon - \frac{1}{EA} \sum_{j=n}^{j=i} F_j \right) \quad 7$$

Thus the displacement at each contact can be expressed in terms of the forces (which in turn are a function of the displacements) developed at all the contacts further along the fibre towards the end as well as the displacement of the next contact along the fibre. Thus δ_{i-1} , can be written in terms of δ_i ; δ_{i-2} can be written in terms of δ_{i-1} and thus in terms of δ_i , and a similar chain can be developed such that each of the displacement can be expressed in terms of δ_i , the displacement at the final crossing nearest the fibre end. The displacements at all the fibre crossings can be expressed in terms of δ_i and as δ_i is given by

$$\delta_i = \frac{1}{EA} \left(\sum_{j=1}^{j=i} x_j F_j \right) - \varepsilon x_i \quad 8$$

then it is possible to solve this equation to determine δ_i and thus to uniquely determine the displacements at all crossings, provided that F can be expressed as a function of displacement.

In the next section, a simple expression for force is developed assuming direct axial stress transfer from the crossing fibres. This expression and eqns 7 and 8 are then used to calculate stress distributions for several examples of sets of crossing fibres.

However, it should be emphasised that equations 6 and 7 apply to any mode of stress transfer and the work that follows is included to illustrate how the solution is applied.

Theory Part 2- A model for force transfer at fibre contacts

The model assumes that stress transfer takes place directly from direct transfer of axial loads in the crossing fibres and that the crossing fibre has a distance, l_c , before it is rigidly bonded into the surrounding network or matrix. Figure 3 shows such a single fibre contact, before and after the matrix is strained. For this analysis, the reference point, $x = 0$, is set at the position of the crossing point in the unstrained system.

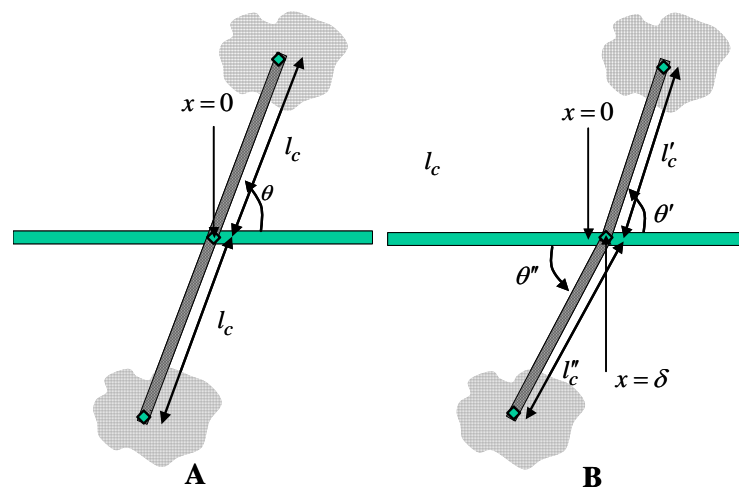


Figure 3 Part A: A crossing fibre, with a crossing angle of θ and a distance of l_c until it is firmly bonded into the surrounding matrix. The point at which the fibre crosses is $x = 0$. Part B: The same fibre crossing after the matrix has been strained by ε and the crossing point has also moved a distance of δ from $x = 0$.

A network strain of ε , in the direction of the fibre axis is applied to the crossing fibre's contacts in the surrounding matrix. To keep the calculation simple, it is assumed that the matrix is rigid, except under the application of the external strain, ε .

This means that loads developed in the crossing fibre will not cause the anchor points in the matrix to move.

Under the matrix strain of ε , the RHS and LHS contacts into the matrix are displaced by $+\varepsilon l_c \cos \theta$ and $-\varepsilon l_c \cos \theta$, respectively. As a result of other strains in the fibre of interest, the contact point is also displaced by δ . Following this displacement, the length of the RHS and LHS of the crossing fibre are not the same. If the new lengths are denoted l'_c (RHS) and l''_c (LHS), then

$$l'^2_c = l^2_c \sin^2(\theta) + (l_c \cos(\theta)(1 + \varepsilon) + \delta)^2 \quad 9$$

$$l''^2_c = l^2_c \sin^2(\theta) + (l_c \cos(\theta)(1 + \varepsilon) - \delta)^2 \quad 10$$

and the forces in the two crossing fibre segments are $F'_a = (l'_c / l_c - 1) E_c A_c$ and

$F''_a = (l''_c / l_c - 1) E_c$, where E_c is the modulus of the crossing fibre and A_c is the cross-sectional area. The net force along the fibre of interest is

$F = F'_a \cos(\theta') - F''_a \cos(\theta'')$ which is equal to

$$F = -2E_c A_c \cos^2(\theta) \delta / l_c \quad 11$$

provided that $\varepsilon \ll 1$ and $\delta \ll l_c$. This equation does not imply that the crossing fibre is not under load if $\delta = 0$. Rather, both segments are under load, but if $\delta = 0$, then the two forces exactly cancel each other out.

Calculation of sample stress distributions

To illustrate the application of eqns. 7 and 8, the equations for three fibre crossings will now be given in combination with the force expression given by eqn. 11. To

simplify these equations it is assumed that the fibre and the crossing fibres are identical and thus $E_c = E$ and $A_c = A$. The three fibres crossing the fibre of interest do so at positions x_1 , x_2 and x_3 and at angles θ_1 , θ_2 and θ_3 , respectively. The relevant equations then become:

$$\delta_2 = \delta_3 + (x_3 - x_2) \left(\varepsilon + 2(\delta_3 / l_c) \cos^2 \theta_3 \right) \quad 12$$

$$\delta_1 = \delta_2 + (x_2 - x_1) \left(\varepsilon + 2(\delta_3 / l_c) \cos^2 \theta_3 + 2(\delta_2 / l_c) \cos^2 \theta_2 \right) \quad 13$$

$$\delta_3 = 2x_3 (\delta_3 / l_c) \cos^2 \theta_3 + 2x_2 (\delta_2 / l_c) \cos^2 \theta_2 + 2x_1 (\delta_1 / l_c) \cos^2 \theta_1 - \varepsilon x_3 \quad 14$$

and both δ_1 and δ_2 can ultimately expressed in terms of δ_3 and eqn. 14 is solvable to obtain δ_3 in terms of x_1 , x_2 , x_3 , θ_1 , θ_2 and θ_3 . Therefore the displacement at each contact (and the corresponding force) can be calculated.

Figure 5 shows calculated force distributions for several cases for a half fibre with 10 fibre crossings. In each case the following constants were used for the calculation:

$$E = E_c = 30 \text{ GPa}, \quad A = A_c = 200 \times 10^{-12} \text{ m}^2, \quad \text{a half fibre length, } L, \text{ of } 1000 \text{ } \mu\text{m} \text{ and a}$$

network strain of 3%. Also common to each case is that the 10 contacts are spaced 100 μm apart at 100, 200, 300...1000 μm from the fibre mid-point. In each example in Figure 5, the load is plotted at the mid-point between contacts. Straight lines link the points from the same series as a guide to the eye, although these are not strictly correct, as the load actually changes in steps at each fibre crossing.

In Case 1, $l_c = 300 \text{ } \mu\text{m}$ and the angle of fibre crossing is 57.3° , which is the average crossing angle for randomly distributed fibre network. The calculated average strain along the fibre is 2.45%. The fibre would have a uniform strain equal to the external

matrix strain of 3% if the entire length of the fibre had a tensile load of 0.18N. It can be seen from Figure 5, that the force on the fibre only approaches this load near the middle of the fibre ($x = 0$) and that the load imparted to the fibre by each contact smoothly increases, the further the crossing fibre is from the centre. Case 2, uses the same conditions as above, except that $l_c = 150 \mu\text{m}$. The overall calculated strain in the fibre of interest has then risen to 2.65% and the cumulative force at each crossing point is higher than in Case 1, although the overall shape of the distribution is the same. The shape of the distribution is also similar to what would be produced by the shear-lag model.

Case 3, uses the same conditions as case 1, except that the fibre angles are varied and for the fibres 1-10 are $[57.3^\circ, 57.3^\circ, 57.3^\circ, 90^\circ, 57.3^\circ, 57.3^\circ, 10^\circ, 80^\circ, 90^\circ]$. The calculated overall strain in the fibre is then 2.12%. Figure 5 shows that over half of the final maximum load in the fibre is applied at the 8th fibre crossing. Case 4 is then calculated by assuming that this stress concentration has caused this bond to break. The overall calculated strain has then fallen to 1.62%. It can be seen that with the 8th fibre crossing removed, the other load-bearing crossing fibres have partially compensated for the missing fibre as the load applied at each of these other crossings has increased. The data calculated for case 3 and 4 shows that if the fibre properties and orientations are randomly distributed, then the development of stress will be quite non-uniform.

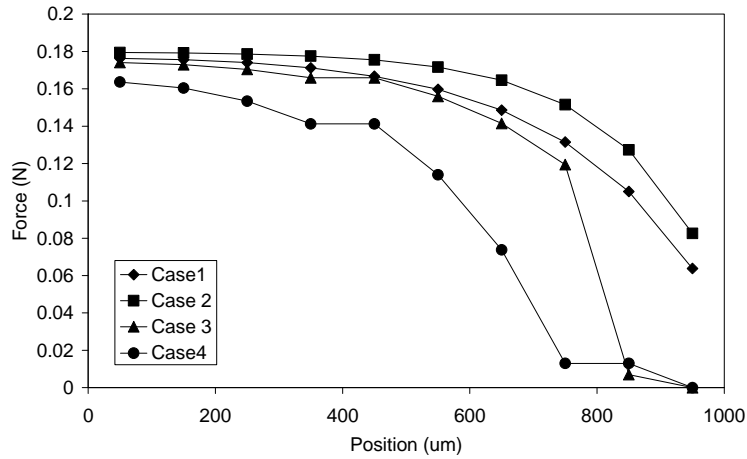


Figure 4. Calculated force distributions along the fibre length

Conclusions

A new solution has been developed which allows the displacements, compared to the applied strain field, at different bonds along a fibre of interest to be all expressed in terms of the displacement at the last bond before the end of the fibre. This allows, provided force is known as a function of the displacement, the displacement of the final crossing to be calculated, from which the force distribution along the whole length of the fibre can be determined. A model for direct force transfer from axial loading of the crossing fibre was also developed. The crossing fibres in this model were connected to a matrix, which was assumed to be rigid except for the strain due to the applied stress. The new method was used to derive the stress distribution along the length of the fibre of interest for several cases. It was shown that the exact geometry of the set of crossing fibres has a strong influence on the calculated stress field.

References

- [1] COX, H.L., "The Elasticity and Strength of Paper and Other Fibrous Materials", *British Journal of Applied Physics*, 3(3):72-79 (1952).
- [2] PAGE, D.H. and SETH, R.S., "The Elastic Modulus of Paper Ii. The Importance of Fiber Modulus, Bonding, and Fiber Length", *Tappi*, 63(6):113-116 (1980).
- [3] ASTROM, J., SAARINEN, S., NISKANEN, K., and KURKIJARVI, J., "Microscopic Mechanics of Fiber Networks", *Journal of Applied Physics*, 75(5):2383-2392 (1994).