Determination of Load-Bearing Element Length in Paper using Zero/Short Span Tensile Testing

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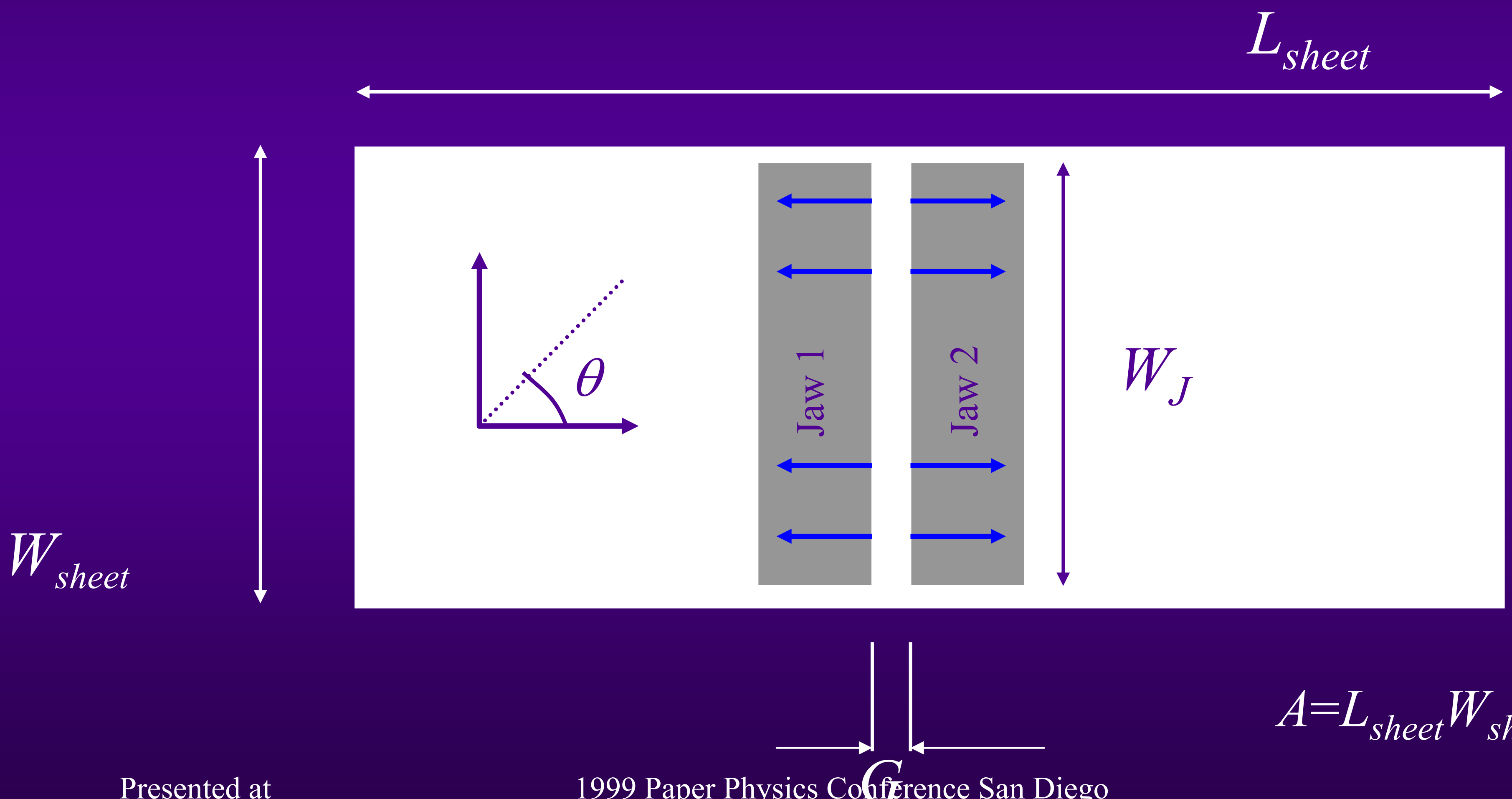
Introduction

- Paper strength
- Effect of drying treatment
- Defects in fibres
- Load-bearing element

Load-bearing element

- NOT a fibre
  - Fibres can be made up of many elements
  - Joined by kinks etc
- Properties:
  - Length, l
  - Cross sectional area, C
  - Young's modulus, E

Test and Sample Dimensions



$$A = L_{sheet} W_{sheet}$$

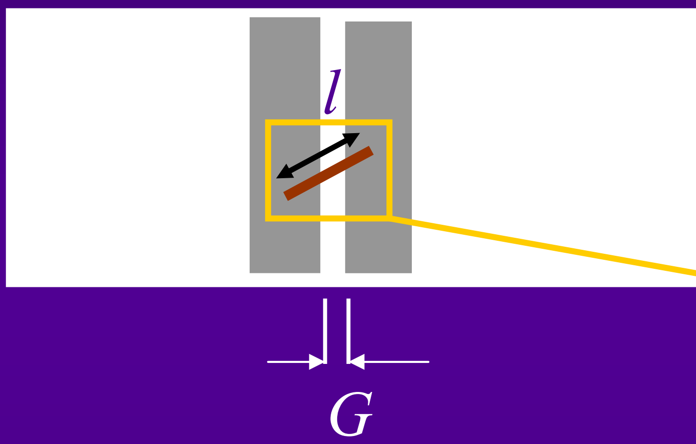
Probabilities

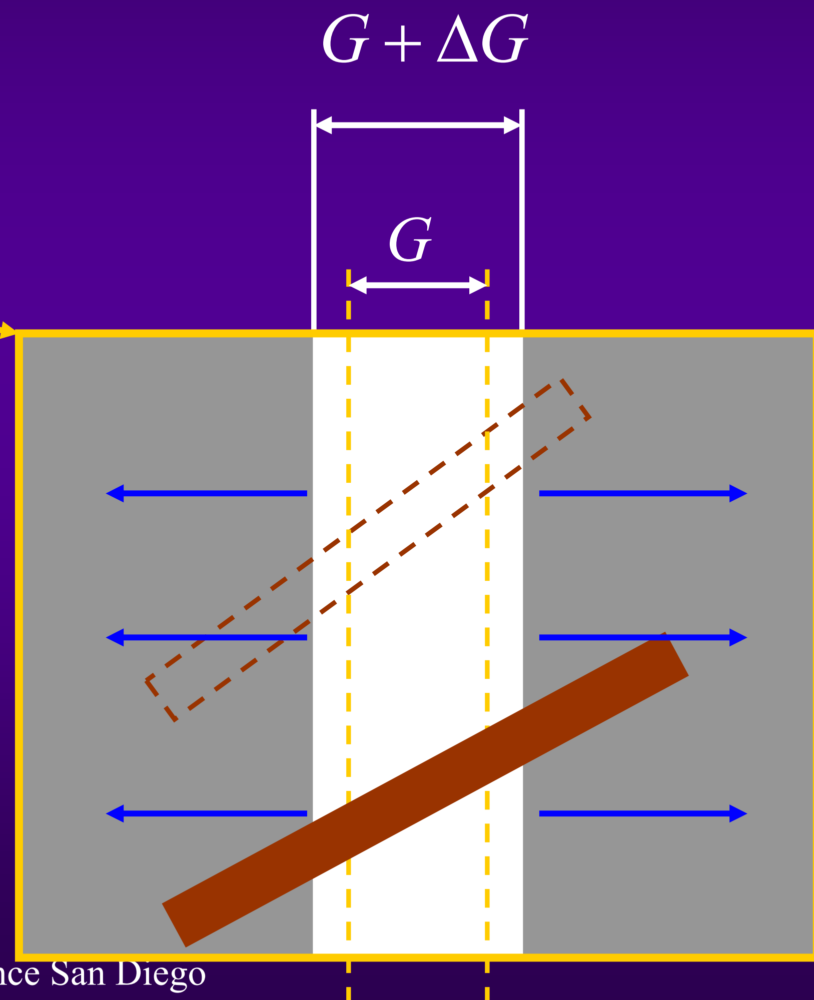
- Single load-bearing element in sheet, length, l, angle θ
- P_1: probability that element is gripped by Jaw 1
- P_2: probability of element gripped by Jaw 2 if also gripped by Jaw 1
- Assumption: W_{sheet}, L_{sheet}, W_j much greater than l.

$$P_1 = \frac{l \cos \theta}{L_{sheet}} \frac{W_j}{W_{sheet}}$$

$$P_2 = 1 - \cos \theta \quad G / l$$

Short span test





Overall strain: $\varepsilon = \dfrac{\Delta G}{G}$

Load on single fibre

At strain, ε, force on fibre spanning both jaws is

$$F_f = EC\varepsilon \cos^2 \theta$$

and component in loading direction is

$$F_f = EC\varepsilon \cos^3 \theta$$

Average contribution by randomly located, oriented fibre is $F_f P_1 P_2$ or

$$F_{av} = (1 - \overline{f_c}) EC\varepsilon \frac{lW_j}{A} \frac{2}{\pi} \int_0^{\cos^{-1}(G/l)} \left(\cos^4 \theta - \frac{G}{l} \cos^3 \theta \right) d\theta$$

where $\overline{f_c}$ is fraction of fibres not bearing load due to out of plane curl

Integratin g get

$$F_{av} = (1-\overline{f_c})EC\varepsilon\frac{lW_j}{A}\frac{2}{\pi}\left[-\frac{1}{12}\left(\frac{G}{l}\right)^3\sqrt{1-\left(\frac{G}{l}\right)^2} + \frac{3}{8}\arccos\frac{G}{l}-\frac{7}{24}\frac{G}{l}\sqrt{1-\left(\frac{G}{l}\right)^2}\right] \quad (1)$$

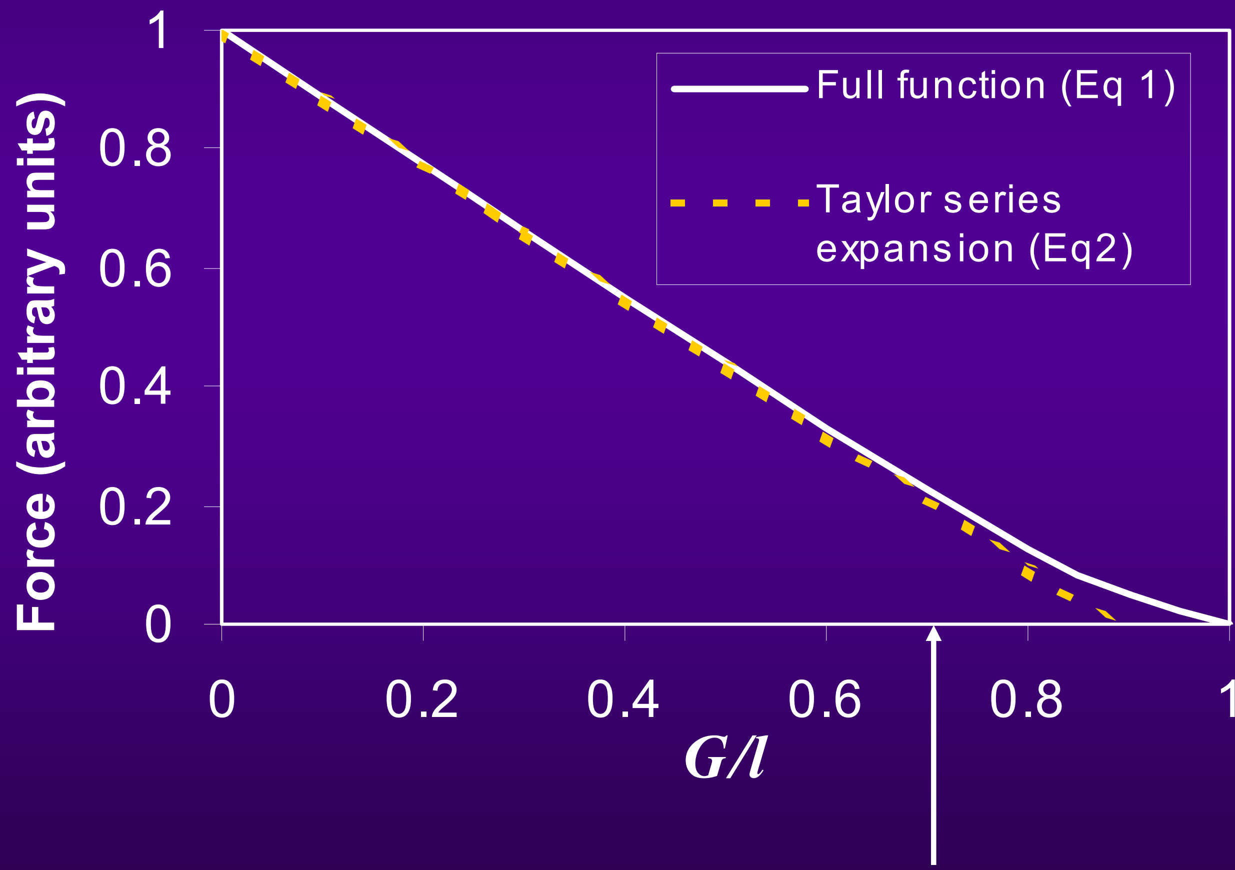
Use Taylor series expansion obtain :

$$F_{av} = (1-\overline{f_c})EC\varepsilon\frac{W_j}{A}\frac{3}{8}\left[l-\frac{32}{9\pi}G\right] \quad (2)$$

Assumption s

$G < 0.7l$	Random orientatio n	No fibre - fibre bonding

Comparison between equations 1 and 2



Approximate limit for accuracy of Taylor's series approximation

Total force

- For gap, *G*, if have *I(G)* load bearing elements with $l > 32G/9\pi$
- then the total force, *F*, is

$$F(\varepsilon, G) = I(G)(1-\overline{f_c})\,\varepsilon\frac{W_j}{A}\left[\frac{3}{8}\overline{E(G)C(G)l(G)} - \frac{32}{9\pi}\overline{E(G)C(G)}G\right]$$

- where $\overline{E(G)C(G)l(G)}$ is the average of the *I(G)* elements

Average length of elements

$$\overline{l(0)} = -\frac{32}{9\pi} \frac{F(\varepsilon_{frac},0)}{\left.\dfrac{dF(\varepsilon_{frac},G)}{dG}\right|_{G=0}}$$

- Assumptions:
  - $\overline{E(0)C(0)l(0)} \approx \overline{E(0)C(0)}\,\overline{l(0)}$
  - $\overline{E(G)C(G)},\ l(G)$ independent of G if $G << l$

Validity of Assumptions

- Major assumption: $\overline{E(0)C(0)l(0)} \approx \overline{E(0)C(0)}\;\overline{l(0)}$
- Test: artificial distributions of fibre properties
- Result: K<1 (always) if longer fibres are stiffer

$$\overline{E(0)C(0)}\;\overline{l(0)} = K\,\overline{E(0)C(0)l(0)}$$

Furnish	Relative number of fibres	EC	L	$\overline{E(0)C(0)l(0)}$	$\overline{E(0)C(0)}\;\overline{l(0)}$	K
Hardwood	0.5	1.0	0.6	0.9	0.875	0.972
	0.5	1.5	0.8			
Hardwood & softwood	0.8	1.0	0.5	1.2	0.96	0.8
	0.2	2.0	2.0			

Experimental Method

- Five samples
  - Unbleached Eucalypt NSSC pulp*
  - *E. globulus* kraft pulp- Laboratory pulped, oxygen bleached to kappa no. 17.9*
  - Unbleached brown mixed waste pulp for packaging grades*
  - Unbleached *P. Radiata* kraft pulp #1*
  - Unbleached *P. Radiata* kraft pulp #2, 45 kappa**

* Refigerated for up to 1 year before making handsheets.

**Collected from pulp mill brown-stock washer. Handsheets made immediately.

Experiments

- Handsheet preparation
  - British Standard Handsheet machine
  - Not refined in PFI mill
- Zero/short span tests
  - Pulmac zero span tester
  - Span: 0.0, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6 mm
- Fibre length measurements
  - Kajaani FS 200
- Sheet tensile strength

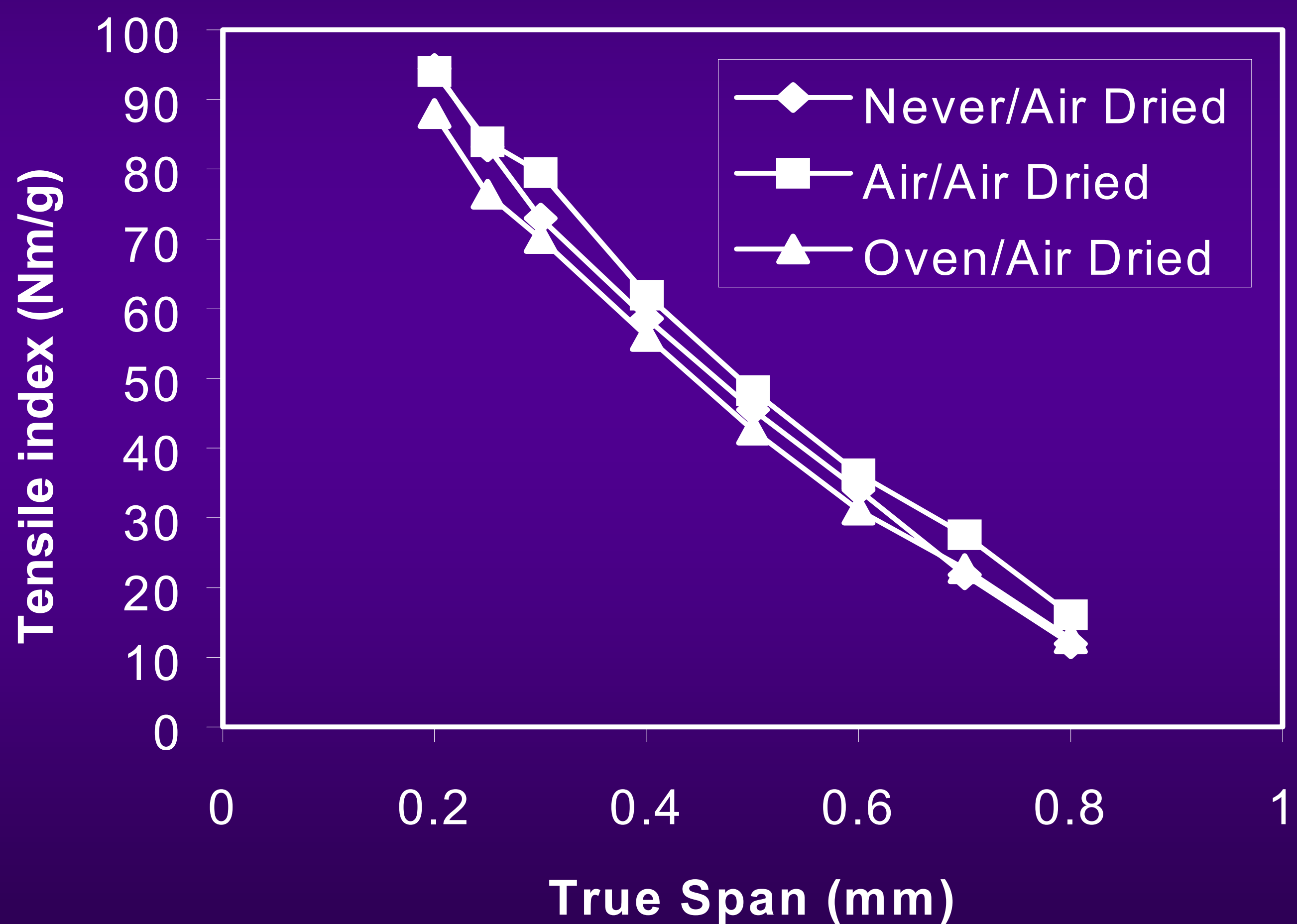
3 Drying Conditions

- 1. Make handsheets; air-dry under restraint
  - Labelled never/air dried
- 2. Make handsheets; air dry under restraint; reslush; make handsheets; air dry under restraint
  - Labelled air/air dried
- 3. Make handsheets; oven dry without restraint; reslush; make handsheets; air dry under restraint
  - Labelled oven/air dried

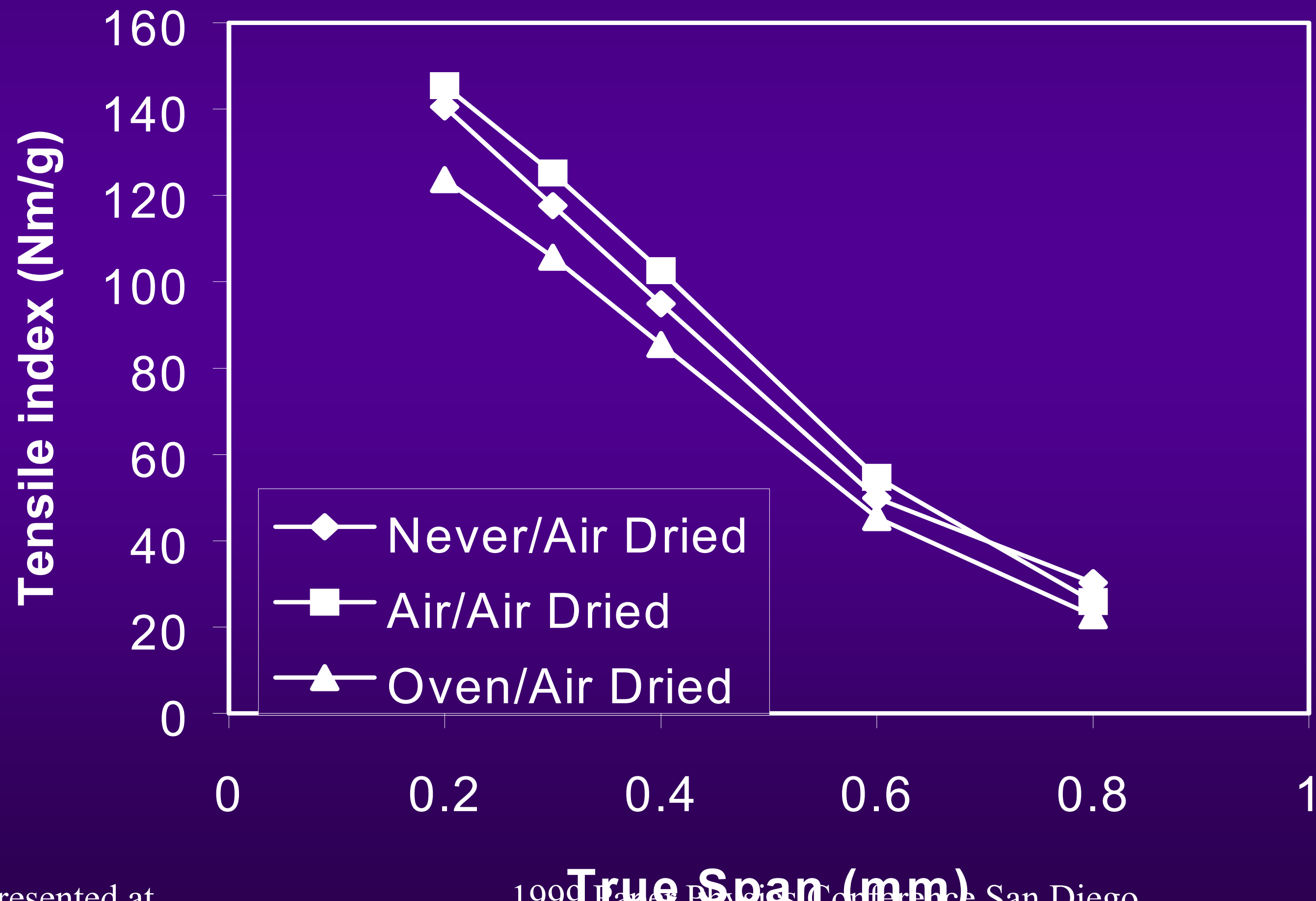
Results

- Zero/short span measurements
  - Plotted with residual span of 0.2 mm
- Tensile strength
- Fits of data to obtain load-bearing element length
- Comparison with measured fibre length

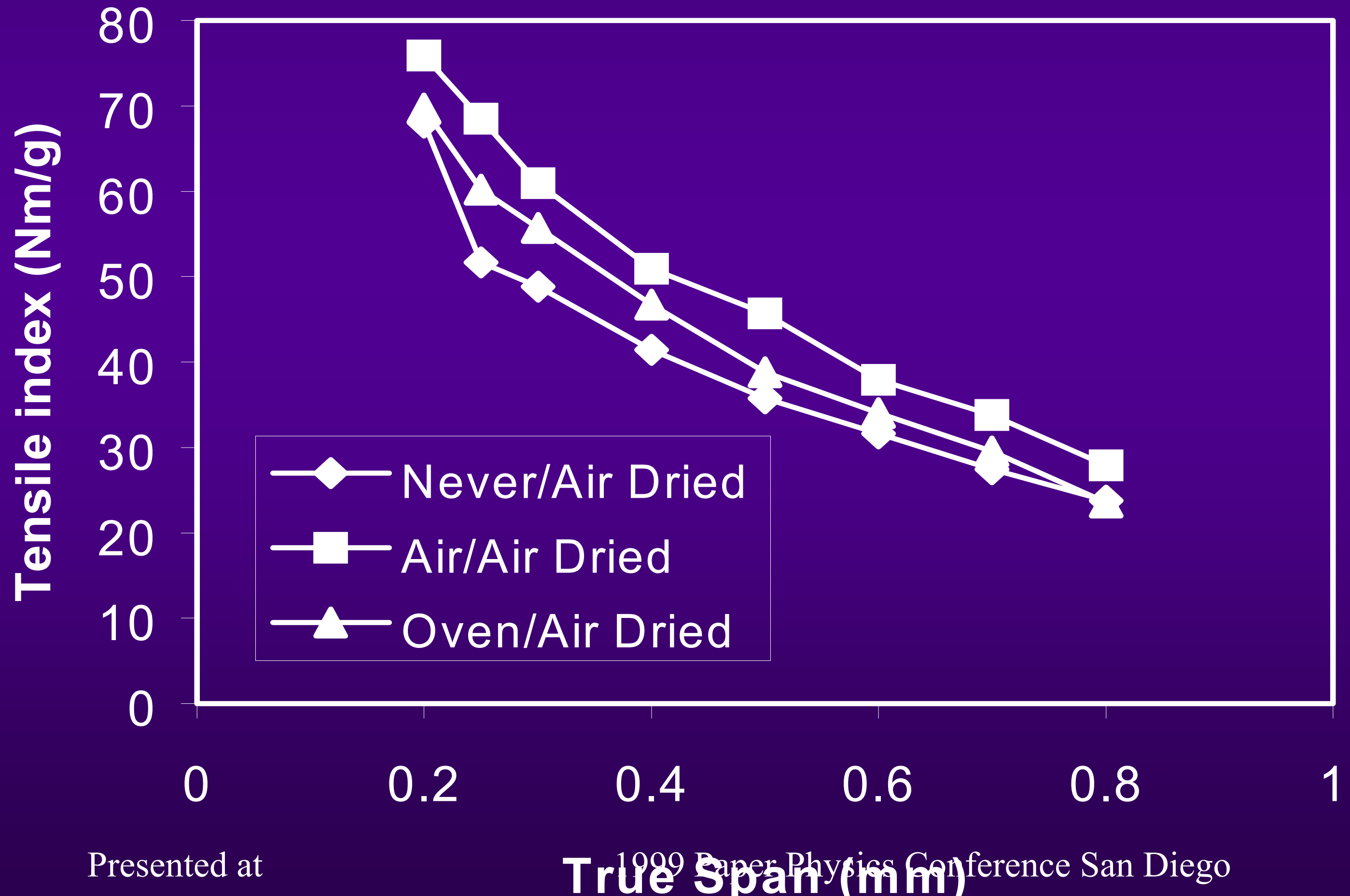
Zero/short span results: NSSC Eucalypt



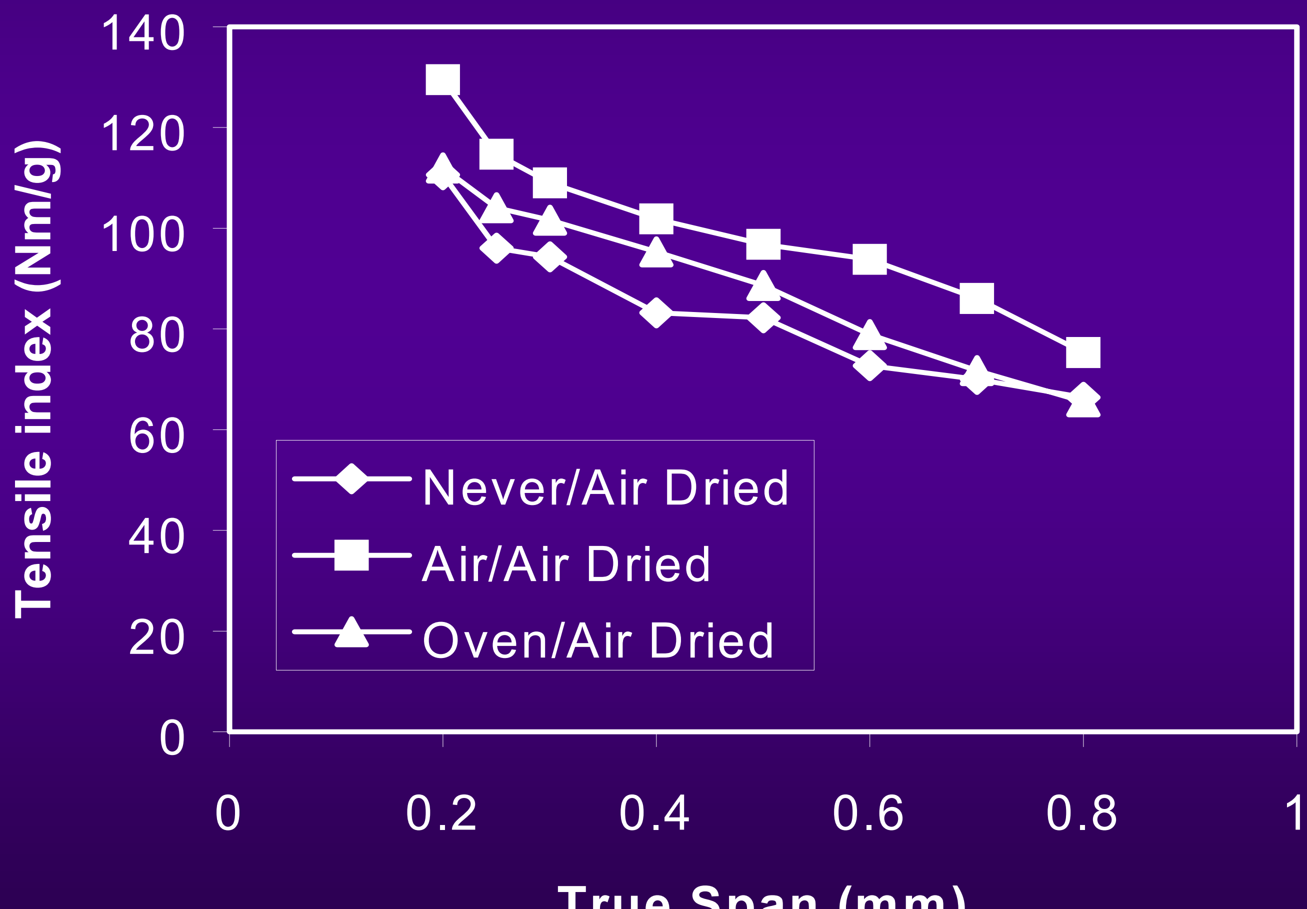
Zero/short span results: E. globulus kraft pulp



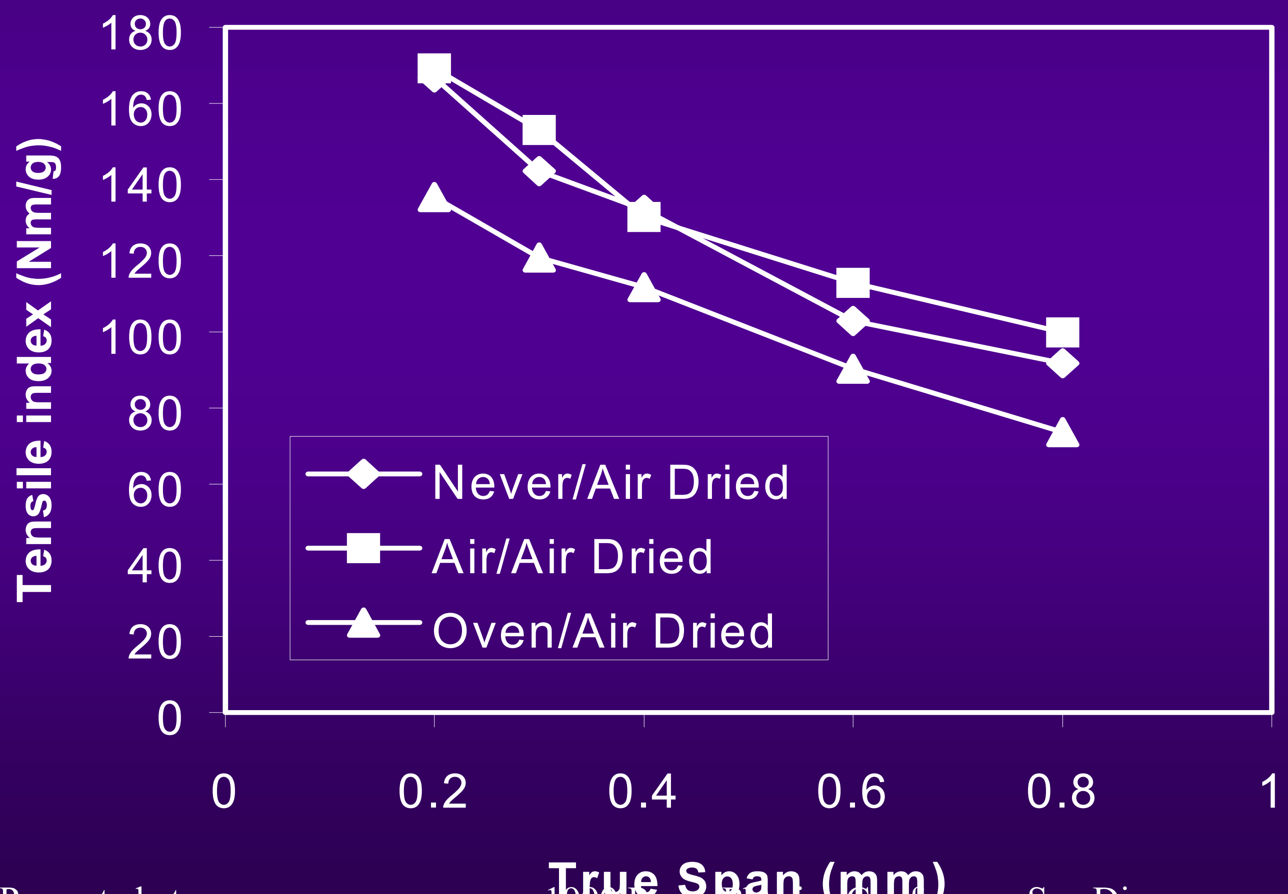
Zero/short span results: waste paper



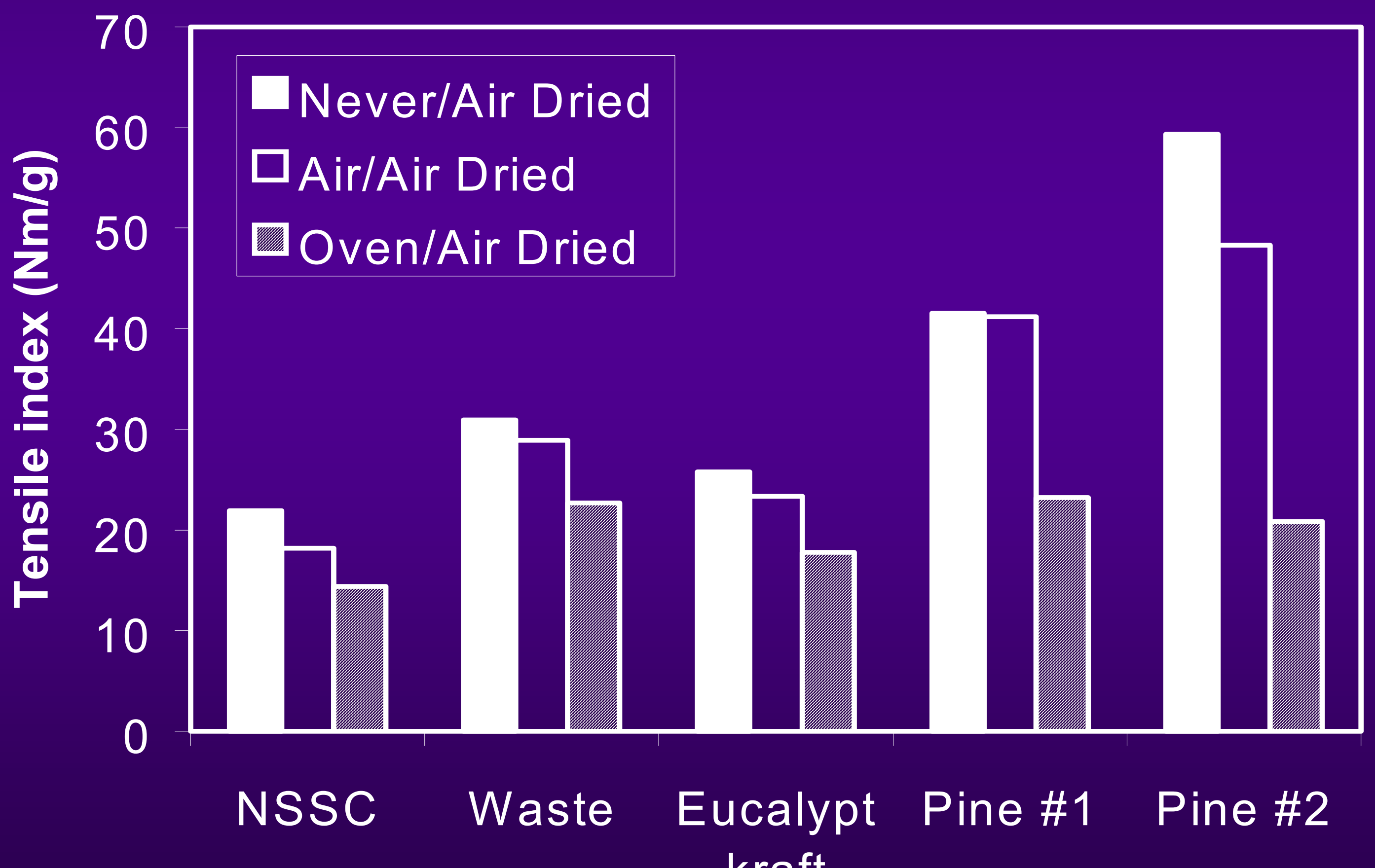
Zero/short span results: P. radiata kraft pulp #1



Zero/short span results: P. radiata kraft pulp #2



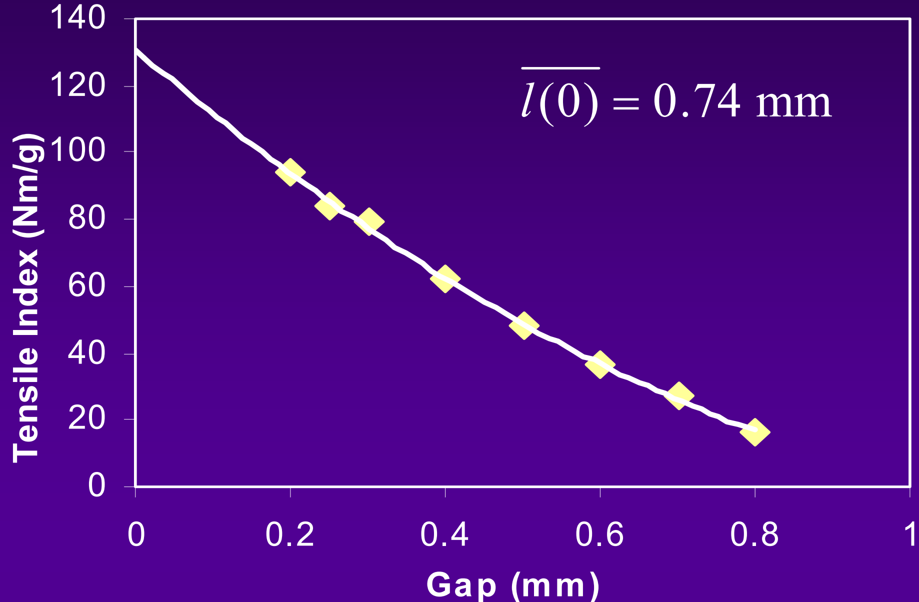
Sheet tensile strength under different drying conditions

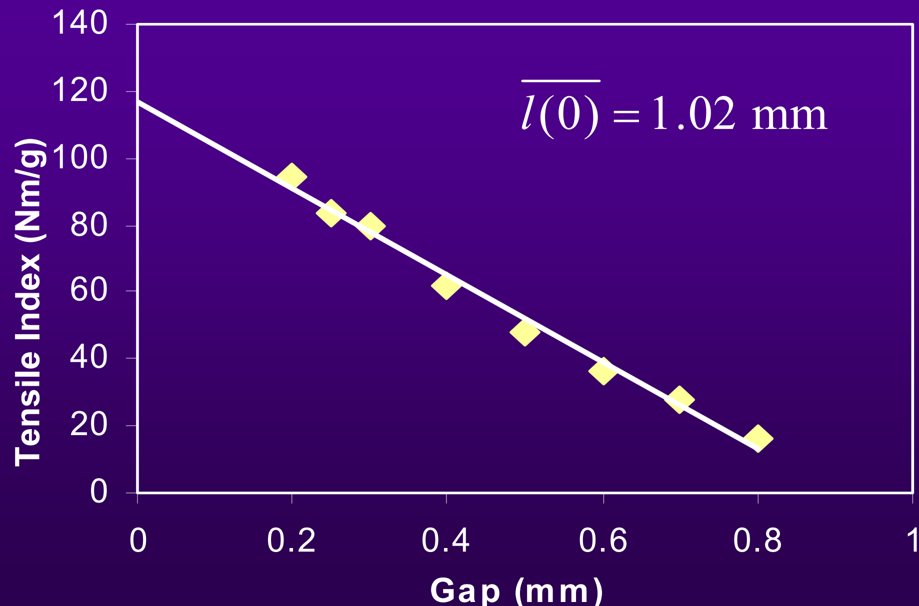


Fits to zero/short span data

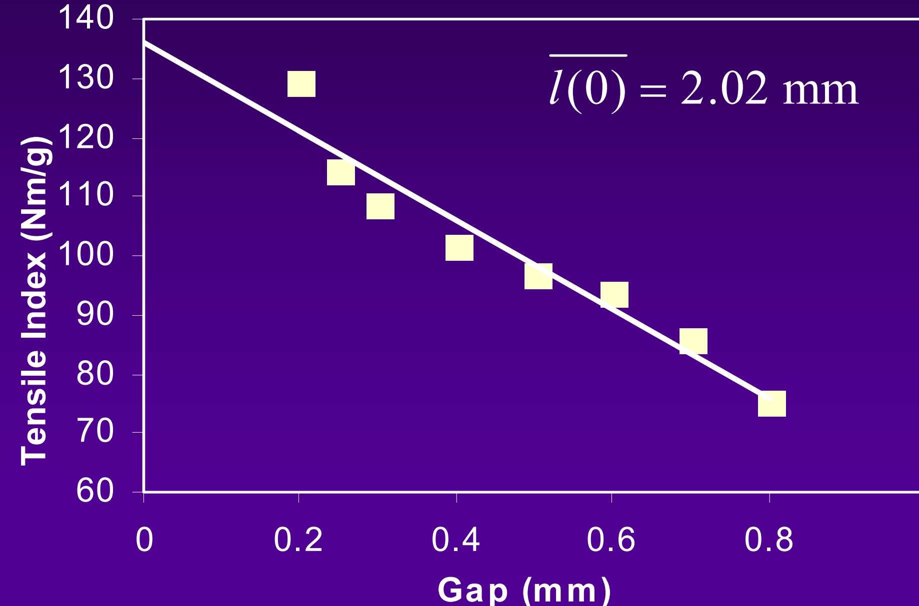
- Quadratic
- Linear (full data set)
- Linear (restricted data set)
- No residual span

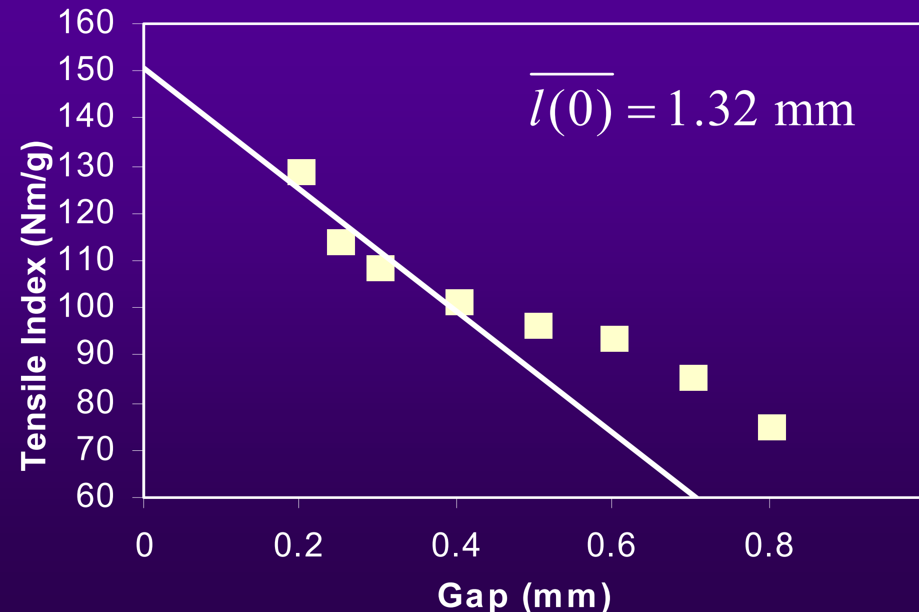
NSSC Eucalypt- Air/Air dried





Pine kraft #1- Air/Air dried





	Never/Air Dried			Air/Air Dried		
	Quad-ratic	Linear (0-0.4mm)	Linear	Quad-ratic	Linear (0-0.4mm)	Linear
Euc NSSC	0.61	0.82	0.97	0.74	0.90	1.02
Euc kraft	0.67	0.92	1.04	0.79	1.00	1.02
Waste	0.63	0.82	1.26	0.73	0.90	1.26
Pine #1	1.00	1.19	1.97	1.33	1.32	2.02
Pine #2	0.89	1.29	1.69	1.26	1.21	1.83

	Oven/Air Dried		
	Quad-ratic	Linear (0-0.4mm)	Linear
Euc NSSC	0.69	0.86	0.99
Euc kraft	0.74	0.96	1.02
Waste	0.75	0.92	1.23
Pine #1	1.75	1.82	1.88
Pine #2	1.24	1.50	1.71

Load-bearing element lengths (mm) determined by different fitting methods

Effect of residual span

Average load-bearing element lengths (mm) from fitting zero/short span data using different residual spans

	Never/Air Dried		**Air/Air Dried**		**Air/Oven Dried**	
	Residual 0.2 mm	No Residual	Residual 0.2 mm	No Residual	Residual 0.2 mm	No Residual
Euc NSSC	0.61	0.57	0.74	0.62	0.69	0.57
Euc kraft	0.67	0.57	0.79	0.65	0.74	0.62
Waste	0.63	0.57	0.73	0.67	0.75	0.65
Pine #1	1.19	0.97	1.32	1.10	1.82	1.59
Pine #2	1.29	1.07	1.21	0.99	1.50	1.28

	Never/Air Dried			Air/Air Dried		
	Arithme-tic (FS 200)	Load-bearing element length (fit)	Length weighted (FS 200)	Arithme-tic (FS 200)	Load-bearing element length (fit)	Length weighted (FS 200)
Euc NSSC	0.63	0.61	0.81	*0.63*	0.74	*0.81*
Euc kraft	0.57	0.67	0.71	*0.57*	0.79	*0.71*
Waste	0.61	0.63	1.14	0.63	0.73	1.17
Pine #1	1.23	1.19	2.19	1.27	1.32	2.21
Pine #2	1.56	1.29	2.43	*1.56*	1.21	*2.43*

	Oven/Air Dried		
	Arithme-tic (FS 200)	Load-bearing element length (fit)	Length weighted (FS 200)
Euc NSSC	*0.63*	0.69	*0.81*
Euc kraft	*0.57*	0.74	*0.71*
Waste	0.59	0.75	1.10
Pine #1	1.18	1.82	2.15
Pine #2	*1.56*	1.50	2.43

Comparison of load-bearing element length (mm) with arithmetic and length weighted fibre lengths (mm)

Conclusions

- Load-bearing element length approximately the same as arithmetic fibre length for these pulps
- Drying treatment- no effect on average load-bearing element length

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