

Classification of Anosov systems

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In an Anosov diffeomorphism, at every point the tangent space splits into two parts, one exponentially contracting and the other exponentially expanding. Despite the strong conditions of the definition and promising initial advances in the field, a full classification of Anosov systems has eluded researchers in dynamical systems.

In this mini-course, I will go over the classification results of Franks and Manning for Anosov diffeomorphisms on tori and nilmanifolds, and the results of Brin for systems which obey a specific “pinching” condition. To avoid too much algebra, I will focus as much as possible on systems on tori instead of the more general nilmanifolds and infranilmanifolds.

The course will consider the results of Verjovsky, Plante, and others as to when an Anosov flow is the suspension of a diffeomorphism. If there is enough time, I will also discuss recent related advances in partially hyperbolic systems.

References

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