Chapter 7
Mathematics Education and Student Values: The Cultivation of Mathematical Wellbeing

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Introduction

For a long time the association between student affect and performance in school mathematics has been well known. However the intricacies of this association are still being teased out. In the current political environment when educational institutions are being explicitly asked to foster an environment that promotes student wellbeing, this association has taken on a new and significant meaning. School mathematics is not just about the performance of students, although some politicians and others in the community have tried to define it in this manner. Equally, "working mathematically" means far more than being good at a specified set of skills and more than being able to show mastery of various conceptual structures.

Experienced teachers do understand that the wellbeing of many students diminishes when they are asked to engage with mathematics learning. Underlying such engagement and hence mathematical performance, although not always recognized by all teachers in the hectic activity of a classroom, is a command of a specific language that holds the conceptualizing process together. Moreover, and of particular importance for this chapter are the values, and their language, embedded within mathematics and its pedagogy.

We will structure this chapter in four sections. First there will be a review of some of the relevant research literature related to affect, attitudes and emotions in mathematics learning and teaching. Second the discussion will then focus in on pertinent literature concerning values in mathematics education. Third these strands of the literature will be brought together with a new construct: mathematical wellbeing. Finally this construct will be used to elaborate a research and development agenda.

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The Impact of Values Education

Values in education in Australia became a focus for research and action in schools when the Federal Government in the early 2000s announced its intention of funding projects that highlighted the teaching of values. This initiative side-stepped the notion that any time a teacher teaches, values are being enacted, since there is interaction between people. Nevertheless this government action initiated a great flurry of activity, some of which is reported in this volume (see Toomey, Chapter 2, this handbook). Much of the implementation and research looked at obvious questions of how values are best taught and what influence context plays in such teaching; for example, formal teaching contexts versus non-formal situations. It appears that most situations that were developed in schools were cross-curricular by nature. That was understandable since the nine values[1] highlighted in later government publications (DEST, 2005), and which became the focus for funding, are clearly drawn from societal values, none of which would be denied as being important in young peoples' formation. Nevertheless many educators were sceptical of such a listing to be the focus for values education (Jones, 2009). As one participant of a focus group called to discuss the implication of the government policy for university teacher education courses put it "These are motherhood statements. No one is going to disagree about teaching kids these values. BUT the key is what you mean by them and that will rely on the teaching contexts developed".[2]

One issue then is, are these or other values clear to the students as they go about the business of their normal routines of school life? If values are foundational to students' wellbeing,[3] then certainly they need to become aware of them and how they influence their normal living. Given that for students their schooling revolves around "going to classes", do they experience values that they recognize in those class times? It would seem to be counterproductive to the development of values if the only time that they became important was at special times, such as when completing cross-curricula projects, unless such projects are the norm for a particular school.[4] So, can the values that are targeted be embedded in the normal teaching of school subject areas? There seems to be no doubt that this is so, although the aim of many government-funded projects seemed to suggest that the explicit teaching of values in other than normal teaching was what was needed.

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[1] Integrity; Freedom; Responsibility; Respect; Doing your best; Honesty and trustworthiness; Fair go; Care and compassion; Understanding, tolerance and inclusion.

[2] This was one of the four focus groups at four different universities that Clarkson was asked to chair as part of the Australian Council of Deans Committee Project to promote the teaching of values. Comments such as this one were voiced in all groups.

[3] "Wellbeing refers to students' physical, social and emotional well-being and development. . .these elements are integral rather than incidental to learning. Learners will find it difficult to engage with learning programs, if they are distracted by significant physical, social and emotional issues" (Catholic Education Office, Archdiocese of Melbourne, 2008, p. 1).

[4] Most Australian schools run their teaching days through specific subject teaching.

However there is a class of values that was not embedded within the current general debate on the teaching of values to young people in school and which we will argue also is imperative for students' wellbeing. These are values that are central to the formation of particular subject areas, and guide their scholars in how they should act and think if they are to become practitioners at a deep level of the subject. Students might be able to answer test items correctly, but unless they know what the values are, and can enact them, they may have difficulty comprehending the deep structure, nuances and understandings that come with being a geographer, linguist or scientist. Furthermore, unless they are aware of such values, as well as the procedures of manipulating knowledge, the experience of connectedness, purposefulness of learning, some sense of control over your learning, producing productive work, engagement with others and the task, all aspects of what the research literature suggest link to students' wellbeing, will not be present (Chapman, Toomey, Cahill, Davis & Gaff, 2007). We argue that this is true for mathematics, although an unexpected implication for some. Indeed the learning of the values embedded within mathematics will start at a very early age of schooling, whether teachers, students or parents are aware of this implicit values learning.

In this chapter we explore some of the research literature on this issue that began in the early 1990s, although we acknowledge that there were precursors to this issue that pre-figured some of what we debate today. We discuss what values are embedded in this mathematical aspect of our culture and how they differ from attitudes, beliefs and other affective dimensions that have been explored in education. We also reflect on some of the research that has looked at teachers attempting to explicitly teach these values, and hence in Lovat's terms impinge directly on teacher effectiveness (Lovat, 2005). We will argue that given mathematics takes up a substantial period of a student's time when at school, their knowledge of and knowing acquisition of the embedded mathematical values will impact on the whole of their wellbeing, but in particular on a new construct that we describe, their "mathematical wellbeing". But we begin by taking a brief broad view of affect in mathematics education research, and in particular the most common variable, that of attitudes. This will help position the explicit research undertaken on values, which then follows. We bring the chapter to an end by moving beyond the notion of values per se and suggest a new emotional domain, paralleling Bloom's cognitive and affective domains (Bloom, Engelhart, Furst, Hill & Krathwohl, 1956; Krathwohl, Bloom & Masia, 1964), in association with the new construct of mathematical wellbeing (MWB).

Affect and Attitudes in Mathematics Education Research

Prior to the 1980s, there was a general understanding that attitudes of students to their mathematical study were important, and much research was conducted. However normally the final aim of this research was to understand how attitudes impinged on students' understandings and how attitudes impeded or enhance

performance, rather than for its own sake. During the 1980s and the 1990s, a change occurred. One key point that possibly marks the end of this change was the chapter by McLeod (1992) in the first handbook published by the influential National Council of Teachers of Mathematics in the United States. Affective issues, rather than the catch-all term of attitudes used until then, began to be seen as important in their own right. Hence by 2004 in summing up of the "Working Group" sessions on "Affect and Mathematics" at the International Congress of Mathematics Education in Copenhagen, a key comment was that we certainly need to be studying students' knowledge of mathematics, but there was also a need for the study of affective issues in their own right, not just into the impact they have on performance or understanding (Clarkson & Hannula, 2008). However the two-way relationship between knowledge and affect was also an important area of research. Hence there was a "triple bottom line", to borrow a business term, for this research area. In some ways this reflects recognition of the different cognitive and affective domains that Bloom and his co-workers had mapped out in the 1950s and the 1960s, although the affective domain was little used in mathematics education. The two-way influencing between the two however is perhaps new. Further detailed exploration of these issues will be undertaken in later sections of this chapter.

During the 1990s and the 2000s, it is true to say that research in this area has also become more nuanced. "Attitudes" are seen as important, but as noted above this term is no longer used as the summary or "catch-all" term. The broader term "affect" is now in common use. Therefore students' beliefs and values on the one hand and their feelings and emotions on the other have also become the foci for research. Attitudes are often now seen as an intermediate area between these extremes. Although these affective notions are often simply positioned in diagrams to indicate relationships between them, rarely is there discussion as to whether there is any movement between these identified affect notions, and if so, what might be the cause. This is an area of research that needs to be undertaken.

Another change is that this area of research now concentrates both on the students and on their teachers. In particular, pre-service education students are often used as participants in research projects. Thus there is far more differentiation in present-day research. There is now recognition that different key participants in the learning classroom may hold different values or beliefs, and the interplay between these is important. In the past there seemed to be far more research devoted to the attitudes of students only. Assumptions were made that of course students would model their values or attitudes or other affective disposition on those of the teacher in mathematics classrooms. In the last few years, these associations have been problematized and are the focus of study.

Before briefly reviewing some of the relevant research literature, it is instructive to note the place and role of affect in curricular documents that teachers work from, as well as other professional avenues that mathematics teachers consult. Statements discussing the importance of students having a positive attitude, disposition, motivation, confidence or just plain "liking" of mathematics were easy to find in a variety of state education system curriculum documents. Indeed it would be a surprise if such statements were not to be found in such documents. An electronic search was also

made of the presentations given at the 2005 Australian Association of Mathematics Teachers' conference. Again a variety of papers clearly suggested that having a positive disposition to mathematics for both students and teachers was important. Finally many articles in professional journals, which give teachers interesting topics or approaches that will hopefully capture the attention of students and promote positive attitudes to mathematics, were easy to find (for example, Hekimoglee, 2005; Shallcross, 2005). Hence, it is reasonable to suppose that the mathematics teaching profession regards positive attitudes towards mathematics as important.

Rather than complete a comprehensive search of worldwide literature, we inspected the research undertaken in Australasia in the last four years, an important and influential subset of the worldwide mathematics education research. The first notable point to make is the lack of research in this area. Although a variety of journals and conference proceedings were inspected, there was relatively little indication that ongoing deep research into attitudes and other affective variables was taking place. Apart from some focus on values and beliefs (dealt with in the next section), by far the main variable considered in the research was that of attitudes. In the main the notion of attitudes was mentioned as important within research reports and at times was included in the rationale for the particular study. Rarely was attitude itself isolated as an issue for which data were collected, however, and nor was it mentioned in the conclusion of such studies. The impression is gained that although students' and teachers' attitudes are of some importance, other constructs should and do take centre stage. In the reporting of research results, attitudes are at best an afterthought to ensure that there is some roundedness to the results. One wonders whether key terms that drive research on student learning and mathematics teaching, such as notions of "community of practice" and "the zone of proximal development", are really thought of as more than theoretical notions. If researchers really considered what it takes to have a "community of practice" that actually functions well, or a situation where students' "zones" become key teaching parameters, then for the participants to possess attitudes to each other and their mathematical activities that are positive, becomes self-evidentially crucial. This review of the literature on attitudes suggested however that researchers are not seeing this crucial assumption that they are making and leaving under-examined. However, even with this rather sombre assessment of the research on attitudes, there have been snippets in recent research that are well worth noting.

Leder and Groternboer (2005) comment at some length on the role that attitudes play in relation to beliefs, values, feelings and the like. In reviewing the literature, they suggest that for them attitudes are an intermediary category, more stable than emotions and feelings, but not as stable as beliefs and/or values. In reading this short but useful introduction to the area of research several notions come to mind. The first centres on whether there is always a potential developmental path from feelings towards beliefs. Clearly some feelings do transform into deeper belief, but whether there is a set path, which may or may not include attitudes, would be interesting to research in the mathematics context. Further it might be that researchers need to take more care in this area when engaging in self-reporting studies. How the issue of whether respondents are reporting their more surface emotions and feelings, or

whether the deeper attitudes and indeed beliefs and values are influencing responses, in self-reports is rarely noted as an issue by researchers. Even for observational and interviewing data, it seems to take careful preparation, insight and persistence by the researchers to discern whether respondents are going beyond feelings and emotions. These notions are complex in themselves, as is the interplay between them. In these comments we have to some degree played down feelings and emotions in preference to attitudes, beliefs and values. However this may not be appropriate, as we argue more fully later in this chapter. Many of us in everyday circumstances act on our surface perceptions, rather than deep well-thought-through beliefs. This may be reflecting an undervaluing of emotions and feelings, as seems to be happening at present in the research community. Hence, we may not be capturing all that is needed. Thus the interplay and perhaps transition between all of these notions need further research.

Of the few studies that looked at attitude and mathematics as a core aspect, there were few surprises about what was studied and the results. Hence, Handal and Bobis (2004) found that when using a novel teaching approach, students' attitudes initially increased, but then tended to fall away to give no lasting change. Beswick, Watson and Brown (2006) found that the students' attitudes to mathematics fell as they progressed through school. The authors also noted that many of the teachers they were working with were ambivalent and uncertain regarding the pedagogy they should use in teaching mathematics, but interestingly they had in general quite positive beliefs about the mathematics they actually had to teach. Clearly teachers can and indeed do have quite varying attitudes to subject matter and pedagogy. Nisbet and Grinbeek (2004) commented that it is often assumed, naively by many, that teachers change in a simple linear fashion, including changing their attitudes. But the authors, reflecting on their results, pointed out that change is always complex, multifaceted and indeed takes a long time. Finally Cooper, Baturo, Warren and Grant (2006) reminded us that indigenous approaches to attitude and motivation are important, and these need to be integrated into teaching to suit the cultural milieu. In particular indigenous conceptions of these personal attributes vary quite considerably, compared to Western notions, and hence the way they operate in classrooms may well be very different. It seems that this is a crucial area that needs much further and urgent attention.

Finally it is worth noting the results of an international study as it impinges on Australia. Australian students participating in the PISA study have shown that two aspects of student beliefs, namely, mathematics self-efficacy and self-concept, correlated positively with mathematical literacy performance. Multilevel analysis "suggests that the factors that may have the greatest influence on Australian students' mathematical literacy, as assessed in PISA, are the attitudes and beliefs of students, which stand out above any of the other factors incorporated into the model" (Thomson, Cresswell & de Bortoli, 2004, p. 203). However, below-average self-efficacy means were reported both by higher performing countries (compared to Australia) such as Finland, the Netherlands and Korea and by lower performing nations like Brazil and Thailand (Thomson et al., 2004, figure 7.12). This suggests that the direction of beliefs may not be a determinant of performance as might be assumed. Asian students (from Japan, Korea, Hong Kong and Macau)

reported the four lowest mathematics self-concept scores, the other self-belief variable, amongst the nations surveyed. From a socio-cultural standpoint, this result may have implications for Western nations which are experiencing high numbers of Asian immigrants.

Interestingly there is another idea, related to affective issues, that does not seem to have been canvassed at all in the mathematics education research literature, which would be worthwhile to explore. It was noted above that one of the relationships explored at length has been how attitudes impact on understanding and performance in mathematics. However there are other relationships that might also be important. At present there is much discussion in the education literature and in public as to why there are so few students who continue with the serious (undertaken for its own sake and not to gain bonus points for tertiary entry, as happens in some Australian education systems), non-compulsory study of mathematics in the last years of secondary school and go on into university. In this discussion much has been made of the mathematics curriculum, students' performance in mathematics and the lack of adequately qualified mathematics teachers throughout secondary schools. All these seemingly do have an impact. However a little-considered variable in this important debate is students' attitudes to mathematics and its teaching. Khoo and Ainley (2005) have shown that students' general attitude to schooling does impact on whether students begin tertiary education, but also that students' performances on literacy and numeracy tests have little impact. Interestingly, these trends seem to have been set by the time students were in the middle years of secondary schooling, well before the final years of school, which are often regarded as the time students make up their minds about what they will do with their lives. It may well be that students have already made up their mind about at least what might be foundational issues by then. An analogous study with mathematics as the focus, rather than general schooling, may well be useful.

Indeed this type of research needs to be broadened beyond whether students continue to tertiary studies, to the impact of their mathematical studies through life. This would ask whether affective issues, rather than their cognitive performances, or the interplay between these sets of issues, have any impact on people's lifelong learning disposition. We return to this theme in the last section of this chapter. Before dealing with the future however we now turn to a specific analysis of the central affect issue for this chapter, values and the closely related notion of beliefs, as they are understood in mathematics education research.

Values in Mathematics Education

Unlike attitudes, the role of values in mathematics education has been explicitly acknowledged and researched only over the last two decades or so, although the nature of values has meant that they would have underpinned the different studies in this field all along. An explicit focus on values as they relate to the learning and teaching of mathematics can be traced back to Alan Bishop's (1988) book *Mathematical Enculturation: A Cultural Perspective on Mathematics Education.*

In this publication, Bishop proposed that the development of mathematics in the Western civilization has demonstrated its valuing of three pairs of complementary values, each corresponding to one of the three components of culture as described by White (1959). Thus, the valuing of *rationalism* and *objectivism* in Western mathematics would reflect the ideological component of cultures, while the valuing of *control* and *progress* reflects the sentimental (attitudinal) component, and that of *openness* and *mystery*, the sociological component. This culturally laden nature of mathematical values is evident in similar research in other cultures. For example, Xu and Wang (2008) contrasted the valuing of *rationalism* in Western mathematics with the valuing of *artistry* in Chinese mathematics. Similar discussions are represented at length in Zhang and Wan (2006).

In the mid-1990s, Bishop's (1996) conception of values that are relevant to mathematics pedagogy saw a need to categorize values, namely, mathematical values (which are related to the discipline), mathematics educational values (related to the pedagogy of the discipline) and general educational values (related to general educational objectives). Later, Seah's (2004) research with immigrant teachers of mathematics led to a proposal of another category that plays out in the professional lives of teachers, which he called the organizational values as these reflect what the schools or local educational authorities regard as important.

How does educational research regard values in mathematics education? According to Bishop (1999), "... values in mathematics education are the deep affective qualities which education fosters through the school subject of mathematics" (p. 2). In this definition, Bishop's regard for the pervasiveness of values was evident in his assertion that values are more internalized than conceptual and procedural knowledge. One way of interpreting this development from affective qualities to values can be found in Seah (2004), when he proposed "values represent an individual's internalisation, "cognitisation" and decontextualisation of affective constructs (such as beliefs and attitudes) in her socio-cultural context" (p. 43).

Another approach to understanding values is in terms of its operation. Seah (in press) conjectured that regardless of their sources (that is, affective or otherwise), values constitute an individual's soft knowledge that underlies the individual's command of hard knowledge, which in turn has both cognitive (e.g., mathematical thinking) and affective (e.g., mathematical wellbeing) components. That is to say, cognitive and affective functions that are activated in one's engagement with mathematics and other things are mediated by the relevant values one espouses.

The experience gained from the various values research studies that were conducted over the last decade or so – and the accompanying informal communication and interaction amongst some of the researchers involved in these studies – had been helpful also in supporting an increasingly clear picture of the nature of difference between values and beliefs. These two terms are sometimes used interchangeably in daily speech, yet Bishop and Seah (2008) assert the following about them:

> (They) have based ... [their] current research on the theoretical notion that values operate, and are revealed, when choices are made .... The essential difference between values and

> beliefs is that one may hold various beliefs but it is when one must make choices that one's values come into play (p. 131).

That is, if a belief is concerned with what one considers to be true, then what one values would represent the emphasis and importance one accords to the related belief(s), and indeed the behaviour that flows from the value rather than the background beliefs (Clarkson & Bishop, 2000). As an example, we may believe that it is true that student use of graphic calculators or CAS frees up time for learners to engage in higher order thinking activities in mathematics. The value, on the other hand, is a manifestation of what we value personally, be it *technology*, *higher order thinking* or *efficiency*, or indeed all three. That is to say, the emphasis we place on one or more of these values influences, and in turn are influenced by, our beliefs about student use of graphic calculators or CAS.

Thus, alternative research findings have been emerging, which distinguish amongst the several concepts, such as attitudes, beliefs and values as noted in the previous section, that had traditionally been regarded collectively as affective in nature (see, for examples, Krathwohl et al., 1964; McLeod, 1992), providing insights to the stance adopted in the literature that values are more internalized modes of other affective modes. The distinction above between beliefs and values beyond their relative positions on a cognition–affect continuum is an example. The underlying influence of values in the articulation of attitudes and beliefs, as well as of cognitive processes, is a research strand that further highlights the implicit power of values. In addition to Seah's (in press) idea of soft knowledge mentioned above, there is also Mandler's (1989) theory that perceives emotions as an expression of values.

Research into values in mathematics education can generally be regarded as having been developing in two different directions. One is concerned with the fostering of "desirable" civic, ethical and moral values in the younger generations through mathematics learning (see, for example, Seah & Kalogeropoulos, 2004; Wong, 2005). The other direction relates to ways in which mathematics learning (including performance) might be enhanced through the teaching of values (see, for example, Seah, Atweh, Clarkson & Ellerton, 2008, for a review of research conducted in recent years in Australasia). It appears that between these two research directions, interest amongst mathematics education researchers has been understandably more evident in the latter. In the current discussion of the mathematical wellbeing of students, and of how this relates to a sense of mathematical wellbeing in society, it is thus the research into how values optimize mathematics learning and teaching that would be reviewed below. This review will now look specifically at the knowledge we have currently regarding what teachers, wider institutions, society and indeed students' value in the contexts of mathematics, mathematics pedagogy and school education. It is envisaged that this approach will enable us to better understand how interactions between participants in the mathematics learning and teaching process, and between them and their socio-cultural context, might contribute to a sense of wellbeing in mathematics learning or teaching. It is our thesis that this aspect of affective health is a key factor of the overall experience of wellbeing in an individual.

Teachers' Values

To many students, the mathematics teacher is the "public face" of discipline. Thus, it is reasonable to suggest that it is the teacher's own values about mathematics and mathematics pedagogy that play such a crucial role in shaping students' wellbeing in discipline and in its learning. The *Values and Mathematics Project* [VAMP] acknowledged this and sought to investigate the extent of control teachers have over their own portrayal of values in mathematics lessons in Australia. A challenge faced in this values research project, which took place in the late 1990s and the early 2000s, was the observation of a lack of shared vocabulary amongst teachers with which to discuss values and their role in (mathematics) education (Clarkson et al., 2000b).

What teachers value in their practice appears to be affected by pre- and in-service teacher education programmes, as well as by the intended curricula. A 2005 survey of 2924 teachers of mathematics and 612 heads of mathematics in Australia found that "irrespective of their tertiary background or years of teaching experience, teachers [of mathematics] valued ongoing professional development" (Harris & Jensz, 2006, p. 20), and that "in general, teachers [of mathematics] valued their disciplinary grounding and the practical aspects of their education studies" (p. 28). Another study researching with a smaller group of teachers echoed this observation: "teachers' values in the classroom are shaped to some extent by the values embedded in each subject, as perceived by them" (Bishop, Clarke, Corrigan & Gunstone, 2005, p. 158). Yet, teachers' values were also found to be shaped in some ways by official curriculum statements. In the *Values in Science and Mathematics Education* project, it was found that features of mathematics pedagogy that were valued by the primary and secondary teachers reflected the ideals of constructivist, reform-oriented mathematics lesson planning, as opposed to those of more "traditional" pedagogical practices.

Teachers' values as they are espoused in their mathematics teaching have "a powerful (negative or positive) influence on students' affect" (Frade, Carneiro & Faria, 2008, p. 11). The research team had adopted VAMP's conception of values as being beliefs in action (e.g., Clarkson et al., 2000a) in their work in Brazilian urban secondary schools. Also, teacher identification in questionnaires of what they value was also found to be different from what was interpreted through the lesson visits, thus echoing VAMP's concern over the validity of self-identification of values.

Immigrant mathematics teachers' values with regard to mathematics, mathematics pedagogy and education in general were investigated in Australia by Seah (2004). Through the professional experiences of teachers coming from different cultures, a sense of what the mathematics education system in the state of Victoria valued could be identified. This study was also interested in the ways in which these immigrant teachers negotiated about perceived differences in values between cultures, and it was found that each participating teacher adopted a variety of strategies in such situations depending on the context they were in. Five main responsive strategies were recorded (see Seah, 2005), with the first three (that is, status quo,

assimilation and accommodation) signalling an increasing acceptance of the "other" culture. The other two approaches, namely the amalgamation or appropriation, represent the embracing of the essence of the different cultures in one's worldview, decisions and actions. These two approaches thus have the tendency to generate new cultures as a result of the interaction amongst the different values from different cultures (see Seah, 2005, for details).

A parallel project to VAMP was undertaken in Taiwan that worked not only with teachers as VAMP did but also with students. Chin and Lin (2000) found that the values subscribed to by teachers were identified and matched with the values of their respective students. The shared experiences of teachers and their students in the mathematics classroom developed and shaped what the Taiwanese research team called the pedagogical identities of one another. This seemed to give firm evidence that the influence of teachers on students' value formation is a real effect.

In summary, teachers at times find it hard to articulate the values they are teaching or espousing in mathematics classrooms, but there is no doubt that they are teaching values whether they do so consciously or not. The Taiwanese work also assures us that students are aware of the values espoused by their teachers, and the teachers' values do impact on those that students are learning. However, as indicated earlier, teacher values are not the only factor in play.

Institutional Values

Classroom discourse and interactions are continually shaped and reshaped by institutional and societal phenomena. There is potential for "desirable" institutional values to be introduced to students through school mathematics, at the same time optimizing the quality of the mathematics learning experience for students. Wong's (2005) conception of a *NE* × *ME matrix* illustrates one such example, in which *NE* refers to the teaching in Singapore of its nationhood and values in National Education and *ME* refers to the mathematics education program. It is noteworthy that Wong established a case for the culture dimension to complement the existing knowledge and process dimensions of mathematics education, wherein this third dimension relates to "things *about* mathematics, rather than mathematics itself .... [and which] includes a strong value element" (p. 6).

An Australian study by Ingvarson, Beavis, Bishop, Peck & Elsworth (2004) studied the significance of the role played by mathematics departments in secondary schools in Australia, with a view to seeing how the departments structured and influenced the teaching by the individual mathematics teachers in the department. This is a relatively unexplored area for research, but as they say: "Student learning is typically affected most directly by the quality of opportunities for learning that individual teachers can provide. However, the quality of teaching is in turn affected by a wide variety of conditions at the school (and community[5]) level"

[5]Our addition in brackets.

(p. 15). Their research uncovered a wide variety of departmental structures, both formal and informal, together with an equally wide variety of significant values fostered in those departments. For example, at one school the interviewed teachers "see their goal as creating a mathematics learning community, which they call 'a Mathematics Network'. One example of this is that some staff run extra after-school mathematics problem solving sessions two days a week, sometimes based around the Mathematics Olympiads" (p. 60). In another school, there are no weekly meetings, but plenty of informal ones, mainly about mathematics. "We have to interact because of the common tests in the school". Also one afternoon after school once a term the staff has a meeting followed by wine, etc. It is then that they discuss new texts, some calculator programs, and it's also a time for sharing interests. "We're a consistent, coherent group with basically a program in mind, that we can execute, and there are no slackers .... those who can't hack it don't want to be in it". "You have to build the ethos" (p. 63).

They also compared schools at which students scored high in the PISA international assessment with those where the students scored low, and then looked at the level of consensus within a mathematical department within these schools. The results suggest that principals and department heads in "high PISA" schools were more likely to report a high degree of consensus amongst staff about standards for quality teaching and learning mathematics (see Table 7.1).

The extent to which textbooks play an institutional role varies from one education system to the next. It is fair to say, however, that whether the survival of textbooks is determined by market forces in one culture or the publication of textbooks is subject to governmental approval in another, textbooks are often written with the intended curriculum in mind. That is, to varying degree, textbooks portray institutional values of different education systems. Seah and Bishop (2000), for example, analyzed two sets of Years 7 and 8 textbooks used in Singapore and two sets from the same year levels in Melbourne, Australia, to compare and contrast the mathematical and mathematics educational values that were represented. Dede (2006) in Turkey used the same framework to analyze eight 6th and 7th primary mathematics textbooks, arriving at similar findings as Seah and Bishop (2000). The only two differences were in the ways in which Turkish textbooks emphasized *openness* more than the complementary value of *mystery*, and their greater emphasis on the mathematics educational value of *accessibility* over *specialism*.

Table 7.1 Comparison of PISA ranking of schools and schools' self-ranking (see Ingvarson et al., 2004, p. 44)

	School		
PISA Rank	Low	High	Total
Low	5	5	10 (37%)
High	0	17	17 (63%)
Total	5 (18%)	22 (82%)	27 (100%)

$\chi^2 = 10.4$: Sig. $p = 0.003$.

Societal Values

The various values that are enacted by different participants of the mathematics learning and teaching process are also mediated by what are valued in the respective communities and societies within which the lessons take place. One way to see this influence is to note the disruption that sometimes occurs when members of different societies interact. Galligan's (2005) ethnographic research with Australian lecturers teaching university preparatory mathematics overseas, for instance, has revealed how the conflicting values they experienced were grounded in the respective Hong Kong and Australian cultures. This prominence of societal values has meant that mathematics teachers' professional wellbeing can be affected when they teach in foreign lands, such as when they migrate. Given the common misconception amongst teachers that mathematics is culture-neutral and mathematics teaching does not involve (cultural) values (Seah, 2004), it can be expected that migrating mathematics teachers will be hit hardest since they are likely to be least prepared for value differences in the ways in which mathematics is effectively taught in different cultures.

Such disruption does not necessarily lead to an impasse. Seah's (2004) work with immigrant teachers of mathematics in Australia revealed that value differences, as perceived by these teachers, could be negotiated in a variety of ways depending on the context in which the value differences took place. The variety of ways of negotiating value differences represents varying degrees of espousing one value and rejecting the other, as well as different ways in which the values are synthesized to draw out desired features inherent amongst the values concerned. This knowledge is especially significant in this discussion because the teachers' negotiation strategies represent alternative ways in which differences of values might be resolved and, in doing so, teachers' professional wellbeing can be reclaimed. There are certainly direct implications for students' mathematical wellbeing.

In particular, the negotiation strategies of status quo, assimilation and accommodation represent the different extents to which values attributed to one societal culture are embraced over values attributed to another societal culture. Existing cultures are redefined and reconstructed, however, through the strategies of amalgamation and appropriation; in this sense, these two strategies are more culturally productive as the teachers' perception of value dissonance is addressed, and professional and mathematical wellbeing is reclaimed.

Research into values by the mathematics education research community might have been more active if the community had not experienced what Clarkson (2005) called a lack of a shared language with which to discuss values. This phenomenon may well reflect the relatively young history of mathematics educational research into values. Nevertheless, related research conducted around the world to date has highlighted the real possibility of mathematical, mathematics educational and educational values impacting on the quality of learning and teaching through the ways in which they shape individual student's mathematical wellbeing. This brings us back to consider students' values in the next section.

Students' Values

An understanding of what mathematics students value in their learning process constitutes our fourth dimension of capturing and incorporating relevant wisdom of practice in mathematics education research. As Keitel (2003) has said; "Their [students] explication can serve as a base for [the] interpretation of classroom practices and outcomes" (p. 4).

An analysis of children's drawing representing situations when they were learning mathematics well in Victoria, Australia, showed that young children valued the learning of mathematics and numeracy concepts in out-of-school contexts as much as within schools. The participating students were also valuing concepts beyond *number* and *computation* (Bishop & Clarke, 2005). Another study, also conducted in Victoria, collated primary school students' drawings of their individual impressions of effective mathematics lessons in schools. The findings revealed that in the primary school years, these lessons featured a co-valuing of *fun*, (teacher) *experience*, *board work* and (teachers' explicit) *explanation/instruction* by both students and their teachers (Seah, 2007). In particular, 67% of the 118 students related the valuing of *fun* in mathematics lessons in which they learnt particularly important. Similar feedback from pre-service primary school teachers' recall of their experiences as mathematics learners in Australia and Singapore were sought (see Seah & Ho, in press), and *fun* is once again the most highly valued feature of effective mathematics learning. Yet, "enjoyment or fun is rarely connected to doing mathematics, in contrast to other school subjects" (Keitel, 2003, p. 5). The high school student participants in Chin and Lin's (2000) study also identified *fun* as one of the few values they acted on in evaluating assessment items. What do these findings imply for the facilitation of more effective mathematics pedagogy then? If mathematics and/or its teaching can be made to be more fun and enjoyable, what will that look like in ways which are sustainable? What implications does this have for learning mathematics in depth? How can we guard against students feeling increasingly immune to what are potentially fun and enjoyable mathematics learning activities, such that this value is continually sought after, but rarely experienced?

Keitel's (2003) analysis of what German high-ability secondary students articulated in post-lesson interviews revealed the values held by them with regard to mathematics learning. These included *application*, *enjoyment/fun*, *mystery*, *assessment* and *collaboration*. What is significant about this study is that some of the values which the students identified, namely *application*, *enjoyment/fun* and *collaboration*, were not perceived by them to be similarly valued by the education system. Equally significantly, "they demonstrate that the "same" teacher's script is very differently understood and experienced by students, and that there are also commonalities and differences in inventing ways for living with their struggles" (Keitel, 2003, p. 4).

With the above consideration of some of the pertinent research on values in mathematics education, we can now turn to a new construct that builds on this work.

The Development of Mathematical Wellbeing

Having explored the relevant ideas associated with affect in general, and attitude and values in particular as they relate to mathematics education, we are ready to bring these lines of research together and propose the notion of mathematical wellbeing (MWB) as an idea which we believe impacts crucially on aspects of mathematics learning. It would certainly be useful for teachers and educators generally to have an understanding of this idea, since in our view it has the potential to link cognitive, affective and emotional educational objectives, something that is lacking in our field.

As noted earlier in this chapter, there was a time when the aim of learning mathematics was only perceived as a cognitive objective – when explanations of mathematical achievement or non-achievement were couched purely in cognitive terms. Later came the acceptance of the necessity to consider affective objectives also, not just as an adjunct to the cognitive but also as a "driver" of mathematical learning or non-learning. Central to a deeper understanding of the role played by the affective aspects was the teasing out of what some of these aspects actually were. We have argued here and elsewhere that a pivotal aspect was in the notion of values. We now propose that a consideration of wellbeing offers a new way of recognizing the importance of the emotional aspects of mathematics learning, and where the emotional can also be the driver of affect and cognition. Our conception of MWB is a developmental one, based on the ideas from a taxonomic stage approach developed by Benjamin Bloom and his co-workers (Bloom et al., 1956). They originally proposed a 6-level cognitive domain structure. This was then added to with a 5-level affective domain (Krathwohl et al., 1964) (see Table 7.2). A third domain was originally proposed that dealt with the kinaesthetic development of students, but this was never published.

Although much research has tended to concentrate on one or the other of these two domains, and hence has driven something of a wall between them, in the original grand plan these domains were seen as working in concert with each other, and together being useful in describing and explaining learners' overall abilities as they related to a specific area of learning. For example, it is clear that students use the thinking skills embedded within the cognitive domain to make decisions related to the affective domain. This does not mean however that students move in a related lock-step fashion through the levels of both domains. Nevertheless, clearly there is a relationship between the domains. It is hard to imagine many students operating at level 5 in the cognitive domain, but operating at only level 2 in the affective domain when considering a particular area of study. The reverse scenario is also difficult to imagine for many students.

But before proposing an MWB construct, we believe a gap in the literature needs to be filled. The MWB construct cannot just be derived from the cognitive and affective domains developed by Bloom and his colleagues. An emotional component is also needed. Hence we first propose, on the way to developing an MWB construct, a third emotional domain (see Table 7.2). This is followed by our proposal of the MWB construct (Table 7.3) before, in the following section exploring tentatively the relationship between the two.

Table 7.2 The three domains: cognitive, affective and emotional

Cognitive	Affective	Emotional
The cognitive continuum begins with the student's recall and recognition of *Knowledge* (1.0),	The affective continuum begins with the students merely *Receiving* (1.0) stimuli and passively attending to them. It extends by more actively attending to them, and by	The emotional continuum begins with having some *Feelings* (1.0) towards a school subject, recognizing that in its presence some feelings are created, and these
It extends through his/her *Comprehension* (2.0) of the knowledge,	*Responding* (2.0) to stimuli on request, willingly responding to these stimuli and taking satisfaction in this responding,	grow to become the student's behavioural and emotional *Responses* (2.0) to the situation, either positively or negatively, which then
his/her skill in *Application (3.0)* of the knowledge that is comprehended,	his/her *Valuing (3.0)* the phenomenon or activity so that he/she voluntarily responds and seeks out ways to respond, by acceptance of a value (3.1), by preference for a value (3.2) and by commitment (3.3) to a value	leads to a regular and *Expressed reaction (3.0)* in the context of the school subject, together with an active searching for a range of pleasurable experiences, or an avoidance of any non-pleasurable experiences, with an increasingly
and the skill in *Analysis* (4.0) of situations involving this knowledge, his/her skill in *Synthesis* (5.0) of this knowledge into new organizations, and finally	and through *Conceptualization* (4.1) of each value responded to, so that there is an	*Conscious* (4.0) awareness and rationalization of the feelings experienced and the choices made, leading to
a skill in *Evaluation* (6.0) in that area of knowledge to judge the value of materials and methods for given purposes.	*Organization*of these values into systems (4.2), and finally organizing the value complex into a single whole, leading to a *Characterization* (5.0) of the individual.	an *Organized* (5.0) set of life-style choices, maximizing the satisfactory experiences and minimizing the unsatisfactory.

A continuing major concern for teachers is when students seem to develop an over-powering negative set of feelings for a particular set of situations in their school. This clearly is often exemplified with students' feelings in mathematics sessions. In essence, this is an overpowering emotional response to the situation. Indeed, by the time some students are in the middle of primary school, it seems that they have become fixated at the second alternative of level 3 of the emotional domain (Table 7.2) with the exhibition of avoidance behaviour towards mathematics. We suggest that the third domain may give teachers some guidance in gauging students'

Table 7.3 Stages of the construct "Mathematical Wellbeing"

Stage 1: Awareness and acceptance of mathematical activity At this first stage the learner is aware of mathematics, not as a coherent body of knowledge but as a collection of mathematical activities. There is an awareness of the different nature of mathematics from other subjects/topics at school. The learner recognizes a mathematical activity as different from a language or a sport activity and it is accepted as a worthwhile pursuit. The learner feels comfortable in the mathematical learning context, although he/she has a passive acceptance of such experience and is disinclined to seek them out.
Stage 2: Positively responding to mathematical activity At this stage, mathematical activity invokes a positive response. More than just acceptance of the activity, here there is a welcoming of it and some pleasure in its pursuit and in its achievement. This pleasure develops feelings of self-confidence and positive self-esteem, which reinforce the acceptance and worthwhileness of mathematical activity in general.
Stage 3: Valuing mathematical activity At this stage the learner appreciates and enjoys mathematical activity to the extent that there is an active seeking out of those activities and of people with whom those activities can be shared. Awareness grows of the human development of mathematical knowledge and of one's place in the mathematical scheme of things. The learner reaches acceptably high (to them) levels of mathematical competence.
Stage 4: Having an integrated and conscious value structure for mathematics At this stage the person has developed an awareness of their appreciation of mathematics, of how and why they value it, and where that valuing might lead them in the future. The learner is confident in their level of skill and competence and in their ability to judge their own strengths and weaknesses.
Stage 5: Independently competent and confident in mathematical activity At this stage the learner is a fully independent actor on the mathematical stage. Sufficiently independent to be able to hold one's own in mathematical arguments at various levels, the learner is able to criticize other's arguments from well-rehearsed criteria.

emotional responses to such situations. We also wonder whether they may also be able to overtly plan learning situations, being more conscious of their students' potential emotional responses by using the emotional domain that we propose.

Using this three-domain stage structure, we can now summarize the main ideas of MWB, again as a developmental model. This is summarized in Table 7.3.

These then are the stages that we propose a learner progresses through on his/her way to consolidating a state of mathematical wellbeing. Recognizing the importance of these different stages is crucial for all teachers because they will occur at different ages for different learners. Teachers may see some aspects of cognitive development, but crucially fail to see the relationship of those with the learner's affective or emotional wellbeing. Again, an emotional growth may only be superficial or related to some other aspect of the mathematical learning situation, such as the behaviour of other learners, since we know that the roles of significant others in the classroom can be influential for cognitive, affective or indeed emotional reasons.

Relating Mathematical Wellbeing to Cognitive, Affective and Emotional Educational Objectives

Just as important as the stages in this portrayal of MWB development is the idea of how the MWB provides a framework for seeing links between the cognitive, the affective and the emotional aspects of mathematical educational objectives. An approach might be to compare these three domains of educational objectives against the development of mathematical wellbeing as presented in Table 7.3. There, as a student develops his/her mathematical wellbeing, we can see features of cognitive, affective and emotional objectives being satisfied at each stage. Thus, for example, a student who has been feeling a greater sense of MWB might be beginning to respond more positively to mathematical activities (that is, stage 2) either knows or comprehends the mathematical demands inherent in these activities or tasks (cognitive domain), at the same time that the act of responding is reflective of both affective and emotional educational objectives. The attainment of mathematical wellbeing, then, accompanies student developments in the cognitive, affective and emotional domains. At the same time, the interaction between mathematical wellbeing and growth in cognitive, affective and emotional objectives leads to these factors feeding on each other, given that a positive sense of wellbeing can stimulate self-confidence and foster enabling attitudes, and so on. In fact, in recognition of student (dis-)engagement being a prominent issue in mathematics classrooms, and to the extent that student engagement with mathematics relates to the state of an individual's mathematical wellbeing, there is a strong case for academic consideration of the ways in which the cognitive, affective and emotional growth of students in their working with mathematics can contribute to the many benefits brought about by the corresponding development of their individual MWB. That is to say, if the interaction between MWB and cognitive/affective/emotional growth is indeed so mutually dependent and influential, then our ongoing quest to confront student dis-engagement (one of many manifestations of MWB) takes on new meanings, highlighting the importance of adopting a multi-prong approach involving cognitive, affective *and* emotional considerations.

The role played by values in this process of facilitating MWB is likely to be considerable, given its significance in the affective domain, and also in the cognitive and emotional domains. The central place of values within the affective educational objectives is relatively explicit in the ways in which the Taxonomy has been articulated; Levels 3 through 5 are concerned with the development within the learner of a value complex that characterizes the individual. In the emotional domain, on the other hand, the involvement of values is more clearly expressed through the organization of personal life-style choices in Level 5, but it is also reasonable to perceive of the personal valuing process taking a part in the Level 4 operation of rationalizing feelings and subsequent decision-making. Similarly, as a student grows cognitively, he/she acquires the competence to value materials and methods and can make decisions as to their quality related to given tasks, represented by Level 6 of the cognitive domain.

Students attain cognitive, affective and emotional educational objectives to different degrees at every stage of their mathematics learning. At any one time, the different domains shape, and are shaped by, the student's state of MWB. The higher order functioning of the different domains – a capacity expressed as the ultimate educational objectives in the respective taxonomies – involves the evaluation and fine-tuning of an individual's value system, which in turn results in an increasingly integrated and conscious value system (see Table 7.3) that regulates MWB. This higher order functioning constitutes a student's life skill, which he/she then takes from school in order to contribute to the economic and social life of society. In other words, a more successful school (and mathematics) education equips the student with this level of independence and (mathematical) wellbeing to deal with the many types of issues one comes across in life. At this level of personal growth, values and (mathematical) wellbeing are both stable enough to be regulating, and fluid enough to allow for adjustments as new insights and experiences "make one wiser"!

This relationship between values and wellbeing had been investigated in contexts which are not directly related to mathematics education. Eckersley (2004), for example, discussed how cultural values such as *individualism* and *materialism* affect a person's wellbeing. Amin, Yusof and Haneef's (2006) study with nearly 3000 students in Malaysia had revealed the impact of social values on human behaviour, which then affects personal wellbeing. Douglas (2005), then chairperson of the research and development group "Australia 21", called for a need to enact four major value shifts in people's lives so that health and wellbeing within communities could be sustained.

Our central point is that the fostering of positive MWB needs to take into account how individual values have been – and continue to be – shaped and harnessed. As an example, the successful cultivation of a student's valuing of *resilience* and *hard work* presents the student with opportunities to respond to intellectually challenging instances in his/her mathematics learning experience with a positive and enabling MWB. Alternatively, a student's valuing of *creativity* will likely support a more positive approach to novel mathematical problem situations, boosting his/her MWB in the process. On the other hand, one could picture a senior student who has developed over many years a relatively negative MWB, sadly not an unusual state of affairs in schools. Her higher order cognitive, affective and emotional functioning has shaped this outcome. However, this outcome need not be regarded as a given that is fixed for life. Our argument for this perspective of the interaction between MWB and cognitive/affective/emotional educational objectives has provided us with an insight into how we can capitalize on creative and purposeful designs of learning activities to stimulate such a student's independent critique and reviewing competence to potentially redefine his/her MWB. We acknowledge that this is highly complex, but at this level of functioning, the learner is well-equipped to evaluate cognitively, rationalizing her emotional experience and decisions, in ways which characterize her being and wellbeing.

Developing a Research Agenda

Earlier in this chapter, we proposed a new construct (MWB). To build that construct we needed to develop a new emotional domain. We have indicated why we believe the MWB construct is needed and briefly speculated about how the underlying processes operate. The perspectives we propose are both empowering (for teachers and learners alike) and exciting. Part of the excitement resides in the many opportunities that these perspectives open up for researching. There seems to be at least three areas on which it would be worthwhile for future research to concentrate:

1. investigating the construct;
2. the utility of the construct; and
3. the development of students' MWB.

We have identified a number of research questions within these three areas that we anticipate fellow researchers and educators may wish to explore:

1. *Investigating the construct*

   a. How valid is the construct of MWB in terms of its recognizability by teachers and researchers within and between the stages of development?
   b. What types of behaviour do teachers imagine students will engage in at the different stages?
   c. What is the theoretical relationship between the three domains of educational objectives and MWB?
   d. Is MWB necessarily a developmental construct?

2. *The utility of the construct*

   a. How might teachers be able to use the MWB stages with good effect in the mathematics classroom?
   b. Given an imaginary piece of dialogue at a parent–teacher evening, can teachers gauge the parents' assessment of the students' MWB stages?
   c. To what extent do teachers and researchers agree on a student's MWB stage?
   d. To what extent can students place themselves within the MWB stages?
   e. How do teachers know, and just as importantly how do students know, when they are fixated at a particular stage?

3. *The development of students' MWB*

   a. How does students' MWB develop in mathematics classrooms?
   b. What influence does peer pressure have in students' progression through the MWB?
   c. What impact do factors beyond the classroom have on MWB, and what is their relative importance, compared to factors located in the classroom?
   d. How might teachers facilitate the cultivation of students' MWB?

e. Is the scaffolding that a teacher may use the same for transition between all stages of the MWB?

The last two research questions (3d and 3e) are rather distinct from the previous ones. Unlike the preceding research questions, which are relatively theoretical in nature, these last questions particularly explore the specific actions that might be taken by teachers to put in practice the various theoretical and philosophical understandings of the cultivation and development of MWB in students. As the reader might have inferred from our discussion so far, we take the view that by the nature of their work, teachers are constantly cultivating and influencing students' developing MWB, through the ways in which they value various aspects of their professional tasks and their interactions with students, through the ways in which they plan and execute their lessons and through the ways in which they respond to critical incidents in the day-to-day functioning within their mathematics classrooms. Rather, in proposing these last two research questions, we are encouraging research activities to focus on exploring how the cultivation of students' MWB might be conducted more purposefully and consciously by teachers. Hence, we elaborate questions 3d and 3e with the following:

- How might we tap into current MWB academic and practical knowledge to develop MWB positively amongst students?
- What teacher knowledge might be harnessed to support this task?
- How might student attainment of cognitive, affective and emotional education objectives be guided by teachers in ways that facilitate positive and empowering MWB amongst students?

Conclusion

It is a common but for us an unpleasant experience that we are still not used to after teaching mathematics between 20 and 50 years: You are at a social gathering and it becomes known that you are or have taught mathematics. Most of the adults then start to recall their negative experiences of doing mathematics at school. Most sad are older people recalling experiences from long ago. With the varying types of nuances and emphases that they use to relate their story, it becomes clear that these and presumably other related experiences are still hurting. This is not a positive advertisement for their mathematical wellbeing.

The MWB construct as proposed in this chapter overlaps with the meaning given to the more general construct of educational wellbeing (footnote 3 of this chapter). The MWB highlights the fundamental place of students' emotional development, an issue that has only gradually acknowledged, although only implicity, in recent years within the mathematical education teaching and research community. By building this construct, we have tried to give depth to this issue and sought to provide a

possible way forward where students' emotions can now be explicitly addressed when doing mathematics.

When speculating on the notions of lifelong learning, we argued sometime ago that although many students master the procedures of mathematics, be these up to mid-secondary school or through to undergraduate university level, rarely is it that any sense of mathematical achievement ultimately stays with people (Clarkson et al., 2001). Rather, it is their emotional feelings about mathematics, and the sense they made of the inherent mathematical values, that linger on through their lives. We now reformulate this speculation and suggest that those few students who are able to positively respond to mathematical activity (Stage 2 and further of Table 7.3) are the ones who do not look back on their mathematical experiences in school as times they wish they had missed. These are people who move into a long-lasting sense of mathematical wellbeing, and hence presumably can be that much more productive within their society and more constructive as they talk to their acquaintances and children about the good times of doing mathematics. Clearly, there is much to be done in mathematics classrooms to reinvent these places of learning. A refocusing on values and emotions, we believe, will help to bring a greater sense of mathematical wellbeing to our students.

References

Amin, R. M., Yusof, S. A., & Haneef, M. A. M. (2006). Values, social problems and balanced development in Malaysia. *Journal of Socio-Economics, 35*, 151–163.

Beswick, K., Watson, J., & Brown, N. (2006). Teachers' confidences and beliefs and their students' attitudes to mathematics. In P. Grootenboer, R. Zevenbergen & M. Chinnappan (Eds.), *Identities, cultures and learning spaces* (pp. 68–75). Sydney, NSW: Mathematics Education Research Group of Australasia.

Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht: Kluwer Academic Publishers.

Bishop, A. J. (1996, June 3–7). *How should mathematics teaching in modern societies relate to cultural values — some preliminary questions*. Paper presented at the Seventh Southeast Asian Conference on Mathematics Education, Hanoi, Vietnam.

Bishop, A. J. (1999). Mathematics teaching and values education: An intersection in need of research. *Zentralblatt fuer Didaktik der Mathematik, 31*(1), 1–4.

Bishop, A., & Clarke, B. (2005). Values in maths and science: What can we learn from children's drawings? In J. Mousley, L. Bragg, C. Campbell, B. Doig, J. Gough, S. Groves, P. Hubber, H. Mays, & G. Williams (Eds.), *Mathematics: Celebrating achievement* (pp. 35–41). Victoria: Mathematical Association of Victoria.

Bishop, A., Clarke, B., Corrigan, D., & Gunstone, D. (2005). Teachers' preferences and practices regarding values in teaching mathematics and science. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Research, theory and practice* (Vol. 1, pp. 153–160). Sydney, Australia: Mathematics Education Research Group of Australasia.

Bishop, A. J., & Seah, W. T. (2008). Educating values through mathematics teaching: Possibilities and challenges. In M. H. Chau & T. Kerry (Eds.), International perspectives on education (pp. 118–138). London: Continuum.

Bloom, B., Engelhart, M., Furst, E., Hill, W., & Krathwohl, D. (1956). *Taxonomy of educational objectives: Cognitive domain*. London: Longmans.

Catholic Education Office, (Archdiocese of Melbourne). (2008). *Student wellbeing: Central to learning and school improvement* (Rev. ed.). Melbourne, VIC: Author. Available at http://www.edfac.unimelb.edu.au/swap/resources/publications/downloads/SW_Research_1_revised.pdf.

Chapman, J., Toomey, R., Cahill, S., Davis, M., & Gaff, J. (2007). Clusters and learning networks: A strategy for reform in values education. In D. Aspin & J. Chapman (Eds.), *Values education and lifelong learning: Principles, policies, programmes* (pp. 315–345). New York: Springer.

Chin, C., & Lin, F. L. (2000, August 9–14). *Values and values statement emerged in students' preferences on test items: A case study from mathematical induction*. Paper presented at the History and Pedagogy of Mathematics 2000 Conference, Taipei, Taiwan.

Clarkson, P. C. (2005). *Researching the language for rational explanations in mathematics teaching and learning*. Paper presented at the Australian Association for Research in Education Conference, Melbourne, Australia. Retrieved March 5, 2007, from http://www.aare.edu.au/04pap/cla04891.pdf.

Clarkson, P. C., & Bishop, A. (2000). Values and mathematics. In A. Ahmed, J. Kraemer, & K. Williams (Eds.), *Cultural diversity in mathematics education* (pp. 239–244). Chichester, UK: Horwood Publishing.

Clarkson, P., Bishop, A. J., FitzSimons, G. E., & Seah, W. T. (2000a). Challenges and constraints in researching values. In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000* (Vol. 1, pp. 188–195). Sydney, NSW: Mathematics Education Research Group of Australasia.

Clarkson, P. C., Bishop, A. J., FitzSimons, G. E., & Seah, W. T. (2000b). Methodology challenges and constraints in the VAMP project. In W. S. Horng & F. L. Lin (Eds.), *Proceedings of the HPM 2000 Conference: History in mathematics education: Challenges for a new millennium* (Vol. 2, pp. 155–162). Taipei, Taiwan: National Taiwan Normal University.

Clarkson, P., Bishop, A., FitzSimmons, G., & Seah, W. T. (2001). Lifelong learning and values: An undervalued legacy of mathematics education? In G. FitzSimons, J. O'Donoghue & D. Coben (Eds.), *Adult and lifelong learning in mathematics* (pp. 37–46). Melbourne, VIC: Language Australia.

Clarkson, P. C., & Hannula, M. (2008). Student's motivation and attitudes towards mathematics and its study. In M. Niss (Ed.), *ICME-10 Proceedings* (pp. 399–401). Copenhagen: IMFUFA, Department of Science, Systems and Models, RosKilde University.

Cooper, T., Baturo, A., Warren, L., & Grant, E. (2006). Aim high – beat yourself: Effective mathematics teaching in a remote indigenous community. In J. Novetná, H. Moraová, M. Krátká & N. Stehliková (Eds.), *Proceedings of the 30th conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 369–376). Prague: International Group for the Psychology of Mathematics Education.

Dede, Y. (2006). Values in Turkish middle school mathematics textbooks. *Quality and Quantity, 40*, 331–359.

DEST. (2005). *National framework for values education in Australian schools*. Canberra, ACT: Australian Government Department of Education, Science and Training (DEST). Available at http://www.curriculum.edu.au/values/default.asp?id=8757.

Douglas, B. (2005, October 17–18). *People, health and wellbeing: Will we make the necessary value shifts in time?* Paper presented at the Manning Clark House Forum Making Canberra Sustainable, Canberra, Australia.

Eckersley, R. (2004). *Well and good: How we feel and why it matters*. Victoria, Australia: Text Publishing.

Frade, C., Carneiro, M., & Faria, D. (2008). *Culture and affect: Two studies about influences of teachers' values on students' affect*. Paper presented at the 11th International Congress on Mathematical Education, Monterrey, Mexico.

Galligan, L. (2005). Conflicts in offshore learning environments of a university preparatory mathematics course. In H. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 25–32). Melbourne, VIC: International Group for the Psychology of Mathematics Education.

Handal, B., & Bobis, J. (2004). Teaching mathematics thematically: Teachers' perspectives. *Mathematics Education Research Journal, 16*, 3–18.

Harris, K. L., & Jensz, F. (2006). *The preparation of mathematics teachers in Australia: Meeting the demand for suitably qualified mathematics teachers in secondary schools*. Victoria, Australia: Australian Council of Deans of Science.

Hekimoglee, H. (2005). Romance of a mathematician: Celebrating St. Valentine's Day in a mathematics class. *Australian Senior Mathematics Journal, 19*, 39–43.

Ingvarson, L., Beavis, A., Bishop, A., Peck, R., & Elsworth, R. (2004). *Investigation of effective mathematics teaching and learning in Australian secondary schools*. Camberwell, VIC: Australian Council for Education Research.

Jones, T. M. (2009). Framing the framework: Discourses in Australia's national values education policy. *Educational Research Policy and Practice, 8*, 35–57.

Keitel, C. (2003). *Values in mathematics classroom practice: The students' perspective*. Paper presented at the Conference of the Learners' Perspective Study international research team.

Khoo, S., & Ainley, J. (2005). *Attitudes, intentions and participation*. Camberwell, VIC: Australian Council for Educational Research.

Krathwohl, D. R., Bloom, B. S., & Masia, B. B. (1964). *Taxonomy of educational objectives: The classification of educational goals: Affective domain*. New York: David McKay.

Leder, G., & Grooternboer, P. (2005). Affect and mathematics education. *Mathematics Education Research Journal, 17*, 1–8.

Lovat, T. (2005, May). *Values education and teachers' work: A quality teaching perspective*. Keynote address at the National Values Education Forum, Australian Government Department of Education Science and Training, National Museum, Canberra. Available at http://www.curriculum.edu.au/verve/_resources/Values_Conf_020505_forum_address_Lovat.pdf

Mandler, G. (1989). Affect and learning: Causes and consequences of emotional interactions. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 3–19). New York: Springer-Verlag.

McLeod, D. (1992). Research on affect in mathematics education: A reconceptualization. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). New York: MacMillan.

Nisbet, S., & Grimbeek, P. (2004). Primary teachers' beliefs and practices with respect to compulsory numeracy testing. In I. Putt, R. Faragher, & M. McLean (Eds.), *Mathematics education for the third millennium: Towards 2010* (pp. 406–413). Sydney, NSW: Mathematics Education Research Group of Australasia.

Seah, W. T. (2004). *The negotiation of perceived value differences by immigrant teachers of mathematics in Australia*. Unpublished PhD Dissertation, Monash University, Victoria, Australia.

Seah, W. T. (2005). Negotiating about perceived value differences in mathematics teaching: The case of immigrant teachers in Australia. In H. Chick & J. L. Vincent (Eds.), Proceedings of the 29th conference of the International Group for the Psychology of Mathematics Education (Vol. 4, pp. 145–152). Melbourne, VIC: PME.

Seah, W. T. (2007). Qualities co-valued in effective mathematics lessons in Australia: Preliminary findings. In J. H. Woo, H. C. Lew, K. S. Park, & D. Y. Seo (Eds.), *Proceedings of the 31st conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 161–168). Seoul, South Korea: International Group for the Psychology of Mathematics Education.

Seah, W. T. (in press). *Conceptualising values as a soft learning: Implications for devoloping values in mathematics education*. Paper presented at the National seminar on values in mathematics and science education, Kuala Lumpur, Malaysia.

Seah, W. T., Atweh, B., Clarkson, P. C., & Ellerton, N. (2008). Socio-cultural factors in mathematics teaching and learning. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W-T. Seah & P. Sullivan (Eds.), *Research in mathematics education in Australasia 2004 –2007* (pp. 223–254). Rotterdam: Sense Publishers.

Seah, W. T., & Bishop, A. J. (2000, April 24–28). *Values in mathematics textbooks: A view through two Australasian regions*. Paper presented at the 81st Annual Meeting of the American Educational Research Association, New Orleans, LA. (ERIC Document Reproduction Service No. ED 440 870).

Seah, W. T., & Ho, S. Y. (in press). Values operating in effective mathematics lessons in Australia and Singapore: Reflections of pre-service teachers. *Proceedings of the 33rd conference of the International Group for the Psychology of Mathematics Education*. Thessaloniki, Greece: International Group for the Psychology of Mathematics Education.

Seah, W. T., & Kalogeropoulos, P. (2004). Victorian curriculum reform 2004 and values: The role for mathematics education. In B. Tadich, S. Tobias, C. Brew, B. Beatty, & P. Sullivan (Eds.), *Towards excellence in mathematics* (pp. 441–448). Brunswick, Australia: Mathematical Association of Victoria.

Shallcross, D. (2005). Building the bell rock lighthouse. *Australian Mathematics Teacher, 61*, 36–40.

Thomson, S., Cresswell, J., & de Bortoli, L. (2004). *Facing the future: A focus on mathematical literacy among Australian 15-year-old students in PISA 2003*. Camberwell, Australia: Australian Council for Educational Research.

White, L. A. (1959). *The evolution of culture*. New York: McGraw-Hill.

Wong, K. Y. (2005, July 20–23). Add cultural values to mathematics instruction: A Singapore initiative. Paper presented at the 4th Asian Mathematical Conference Singapore.

Xu, N. N., & Wang, X. C. (2008). Cultural values researching on the near modern mathematics education of China . *Journal of Mathematics Education, 17*, 8–10.

Zhang, W. Z., & Wan, X. (2006). *Cultural traditions and the modernisation of mathematics education*. Beijing, China: Beijing University Press.

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