The upward transport of inclusions in Newtonian and power-law salt diapirs

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ABSTRACT

This paper studies the ability of salt diapirs to lift large inclusions of dense rocks (rafts). Dense inclusions will be lifted if salt in the diapir rises faster than the inclusions sink. The power-law rheologies of six different salts, and viscosities estimated for Newtonian salt are used to calculate the settling velocity of these inclusions as a function of their radii and density as well as the temperature of the salt. This is done using the known equation describing the velocity of solid spheres settling in unbounded power-law fluid. Two-dimensional numerical models are used to study the effect of the ellipticity of inclusions on their sinking velocity, as a function of the power-law exponent $n$. The calculations of the settling velocities of inclusions neglect several other factors discussed in the paper: (a) the ambient fluid (salt) is finite (bounded); (b) more than one inclusion translates in the salt; and (c) further strain-rate softening of salt is caused by its diapiric ascent. The results suggest that, whereas Newtonian salt of $10^{17}-10^{18}$ Pa s and the power-law salt of the Vacherie Dome would have to rise at unreasonably high speeds in order to lift large inclusions, most power-law salt diapirs would be capable of lifting inclusions of the sizes observed in the Iranian domes (up to 3-6 km$^2$) if these rise at geologically reasonable velocities and temperatures.

Introduction

In a recent paper, Gansser (1992) suggests that inclusions (rafts) in the Iranian salt domes are an enigma. The most important enigmatic aspect of such inclusions is their huge size. Inclusions of undisturbed Hormuz sediments and large bodies of basic volcanic rocks have been lifted by the salt and now accumulate on the dissolving surface of outcropping salt domes (Fig. 1). Inclusions of sedimentary rocks are commonly interpreted as competent beds, interlayered with the initial salt sequence, that have been disrupted by diapiric flow. Inclusions of basic igneous rocks common in salt diapirs in northern Spain, Arctic Canada and Iran, have been interpreted as either intruded into the salt or as interlayered lava flows (e.g., Talbot and Jackson, 1987; Gansser, 1997). Talbot and Jackson (1987) considered that salt might incorporate inclusions by flowing into fractures in the overlying rocks and magmatic-like stoping of the blocks. However, most inclusions in diapirs of different salt sequences in the Zagros Mountains and the Great Kavir in Iran are the same age as the salt and thus come from the same stratigraphic level. This led Jackson et al. (1990) to conclude that stoping is unlikely to be significant in the emplacement of these diapirs.

The negative buoyancy force acting on inclusions is defined as $F_b = \Delta \rho g v$, where $\Delta \rho$ is the density difference between inclusion and salt, $g$ is gravity acceleration, and $v$ is the volume of the inclusion. $F_b$ causes the inclusion to sink through the salt at a velocity controlled by the effective viscosity of the salt, $\eta_{eff}$, which is a function of the rheological properties, strain rate and temperature of the salt. If the inclusion sinks more slowly than the salt rises within the diapir, the net movement of the inclusion will follow the rise of the salt upwards. Thus, the principal parameters controlling the maximum negative buoyancy of the inclusions that rise in a diapir are both the ascent rate and the effective viscosity of the salt.
In the Iranian salt diapirs, large inclusions were carried vertically several kilometres to outcrop on the surface. Dense inclusions of stromatolitic limestone and dolomite, sandstone, granite gneiss, rhyolites and basic igneous rocks are common in the Zagros Mountains (e.g., Gansser, 1992). The largest known inclusions are 3–6 km$^2$ in area (Fig. 1; e.g., Gansser, 1960, 1992; Kent, 1979) and must have been lifted at least 5 km by the salt (Talbot and Jackson, 1987). Primary structures preserved in inclusions in the Great Kavir suggest that any displacement was by rigid body displacement (Jackson et al., 1990, p. 69).

The rise of thin interbedded layers of denser material was studied by Ramberg (1981, e.g., pp. 270–273, 303, 311–313 and 321–324) by means of centrifuge models of light silicone putty lifting and folding modelling clay layers. The present paper develops the discussion initiated by Talbot and Weinberg (1992) and uses experimentally determined rheological parameters of salt to calculate the maximum buoyancy of inclusions that could be lifted by diapiric salt rising at constant velocity and temperature. Any yield strength that salt may have is disregarded here. Salt creep may be approximated by that of a power-law fluid when coarse-grained salt deforms at high strain rates, or by that of a Newtonian fluid when fine-grained salt deforms at low strain rates (see Van Keken et al., 1993). In order to estimate the sinking velocity of inclusions through power-law or Newtonian salt, the inclusion geometries are simplified here to spheres sinking in infinite (unbounded) salt. The known velocity equation for solid spheres sinking through unbounded power-law fluid (e.g., Crochet et al., 1984) is rewritten to allow direct use of measured creep parameters of salt (Weinberg and Podladchikov, submitted).

The paper first presents a short review of the inferred velocity of diapirs and salt within the diapirs, and of experimental measurements of the rheology of salt. The equations used to calculate
the settling velocity of spherical inclusions through power-law fluids are then described. The velocities of inclusions settling through six salts of different power-law rheologies are calculated. These are compared to rates at which inclusions settle through Newtonian salt. The settling velocity of inclusions defines a minimum velocity for the salt to lift the same inclusions. A qualitative analysis of the influence on velocity of nonspherical inclusions is based on two-dimensional finite-difference numerical calculations. Other factors which affect the velocity are also discussed: the presence of lateral boundaries to the salt, several inclusions sinking simultaneously, and the effect of strain-rate softening of the salt due to diapiric ascent. The largest inclusion present in a diapir helps to constrain either the rheology of the diapiric material or its ascent rate, if one of them is known. The results indicate that the transport of large dense inclusions by power-law salt diapirs rising at velocities compatible with geological evidence is not enigmatic but rather to be expected.

Ascent velocity of salt within diapirs

The ascent rate of salt diapirs may vary considerably from diapir to diapir, and from stage to stage in the development of a particular diapir (Jackson and Talbot, 1986). Typical diapirs on the US Gulf Coast reach their maximum vertical growth rate of approximately 0.2 mm/a after a few million years and maintain that rate for 10−15 Ma (Jackson and Talbot, 1986). Although they later slowed, the salt diapirs of northwest Germany rose at approximately 0.1 to 0.5 mm/a (Jaritz, 1987). Estimates of the ascent rates of Mount Sedom in Israel, indicate velocities of 3 to 4 mm/a possibly since late Pliocene (Zak and Freund, 1980). The shape and steady-state height of the fountain of salt in Kuh-e-Namak diapir, perhaps driven by present active folding of the rock sequence in the Zagros Mountains (Iran), suggest that the Hormuz salt may be rising at 170 mm/a (Talbot and Jarvis, 1984). A similar approach applied to diapirs of the Great Kavir indicated salt ascent rates of approximately 10 mm/a (Jackson et al., 1990). Diapirs that vent through the surface (e.g., in Iran) rise faster than those still buried (e.g., on the US Gulf Coast and northern Germany), mainly because of decreased resistance by air or water compared to that of an overburden.

The rate of rise of salt within the diapir (gross rate) may differ from the rate of rise of the diapir’s crest (net rate). Several reasons may account for this: dissolution of the salt at the diapir’s crest, lateral spreading of salt in sheets, internal circulation in a spherical diapir (may cause salt velocities up to twice that of the net diapir ascent velocity; Schmeling et al., 1988), and no slip along the walls of salt rising in a pipe.

Salt rheology

The rheology of rock salt depends on the presence of water, magnitude of differential stress, strain rate, confining pressure, crystal size and proportion of impurities (e.g., Urai et al., 1986). Several laboratory studies have indicated that dry salt deforms by dislocation creep and behaves as a power-law fluid at high strain rates (e.g., Pfeifle et al., 1983; Handin et al., 1986; Urai et al., 1986):

$$\dot{\varepsilon} = A \exp \left(-\frac{E}{RT}\right)\sigma^n$$

where $\dot{\varepsilon}$ is the strain rate, $A$ the pre-exponential parameter given in Pa$^{-n}$s$^{-1}$, $E$ the activation energy given in kJ/mol, $R$ the gas constant, $T$ the temperature in K, $n$ the dimensionless power-law exponent, and $\sigma$ the stress in Pa. Values of the creep parameters $A$, $E$ and $n$ vary considerably for rock salt from different localities (see Table 1; e.g., Pfeifle et al., 1983; Handin et al., 1986). Traces of brine in confined salt deforming

| Table 1 Steady-state creep parameters for natural rock salt (listed in Handin et al., 1986, after Pfeiffer et al., 1985) |
|-----------------|-----------------|-----------------|
| $A$ (MPa$^{-n}$ s$^{-1}$) | $n$ | $E$ (kJ/mol) |
| Richton Dome (M) | 2.6 $10^{-2}$ | 5.0 | 82.3 |
| Permian Basin (TX) | 4.66 $10^{-3}$ | 4.50 | 72.0 |
| Paradox Formation (UT) | 1.53 $10^{-7}$ | 1.39 | 28.8 |
| Avery Island (LA) | 5.76 $10^{-8}$ | 4.10 | 33.6 |
| Salado Formation (NM) | 3.91 $10^{-6}$ | 4.90 | 50.2 |
| Vacherie Dome (LA) | 8.71 $10^{-3}$ | 2.22 | 62.9 |
at slow strain rates cause a change in the deformation mechanism from dislocation creep to solution-transfer creep in relatively fine grained salt (Urai et al., 1986). The salt then becomes weaker and behaves like a Newtonian fluid with a viscosity that is directly proportional to the cube of grain size ($\eta \sim d^3$). This high mobility of grain boundaries accounts for the fast flow observed in salt namakiers after rain (up to 0.5 m/day, Talbot and Rogers, 1980). Urai et al. (1986) proposed that natural deformation of rock salt in most diapirs is likely to occur in the transition region between dislocation creep and solution-precipitation creep. This causes difficulties in ascertaining the actual behaviour of salt in a given natural setting.

In order to avoid this problem, and to obtain a first approximation of the size of inclusions that salt diapirs are able to lift, both power-law and Newtonian rheologies of salt are used here. The large variation in rheologic parameters of power-law rock salt (Table 1) implies a wide variation in the effective viscosity of each salt and therefore the size of inclusions they are able to carry. Values of Newtonian viscosity of salt commonly used in the literature will also be used here for comparison.

The sinking of inclusions through power-law salt

The velocity $V$ of solid spheres slowly sinking in unbounded Newtonian fluid may be easily calculated by Stokes' equation:

$$V = \frac{2 \Delta \rho gr^2}{9 \eta}$$

where $r$ is the sphere's radius, and $\eta$ the viscosity of the ambient fluid.

The sinking velocity of solid spheres through power-law fluids has been thoroughly studied in the literature of fluid mechanics and chemical engineering (e.g., Crochet et al., 1984; Dazhi and Tanner, 1985; Kawase and Moo-Young, 1986). The equations in Crochet et al. (1984) may be rearranged to:

$$V = \frac{2}{9^n} \frac{(\Delta \rho g)^{n+1}}{K^n X^n}$$

where $K$ is given in Pa s$^{1/n}$, and $X$ is a known correction factor that depends only on $m = 1/n$ and may be calculated by:

$$X = 1.3(1 - m^2) + m$$

derived from the best fit of the values in Crochet et al. (1984). Equation 2 reduces to Stokes' equation (1) when $n = 1$.

To allow direct application of the rheological parameters of salt to calculate the velocities, we follow here Weinberg and Podladchikov (submitted) and rewrite eqn. (2) to:

$$V = \frac{1}{3} \frac{\Delta \rho gr^2}{\eta_{eff}} \left( \frac{1}{1.5X} \right)^n$$

where:

$$\eta_{eff} = \frac{K^n 6^{n-1}}{(\Delta \rho gr)^{n-1}}$$

and:

$$K^n = \frac{1}{Ae^{-E/RT}3(n+1)/2}$$

The significance of the effective viscosity $\eta_{eff}$ in eqn. (5) is that the sinking velocity remains essentially similar if the power-law ambient fluid is substituted by a Newtonian fluid with the same viscosity. The advantage of eqn. (5) is that $\eta_{eff}$ of the power-law fluid is calculated from the buoyancy stress of the sinking sphere, without the necessity of assuming a strain rate in advance.

Results

The results of calculations using the rheologies of Table 1 in the equations above are shown in Figure 2. For a given salt rheology and ambient temperature (assumed to be constant), the rate at which inclusions sink depends on their negative buoyancy. Thus, the curves in Figure 2 represent the minimum diapiric velocity necessary to lift inclusions of the corresponding buoyancy, at the given temperature. Calculations were also carried out for Newtonian salt of $10^{17}$–$10^{18}$ Pa s (e.g., Van Keken et al., 1993). The lifting capacity of Newtonian salt is generally lower than that of power-law salts (see Fig. 2), with the exception for the soft Vacherie Dome salt.
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Fig. 2. Sinking velocities of inclusions (left-hand column) and effective viscosity $\eta_{eff}$ of salt (right-hand column) as a function of inclusion radius for seven different salt rheologies. The values are given for different salt temperatures and two density differences, $\Delta \rho$, between inclusion and salt. Each line corresponds to an equation in the inset which describes the dependency of inclusion velocity or effective viscosity of salt as a function of radius for the given $\Delta \rho$ and $T$. The left-hand column shows that in order to lift an inclusion of a certain size, the diapir has to rise at velocities above the sinking velocities of that inclusion. Conversely, salt rising at a known velocity may lift all inclusions that sink slower than the salt rises. The right-hand column shows how the $\eta_{eff}$ of salt decreases with increasing inclusion radius. (a) Avery Island, Louisiana; (b) Salado Formation, New Mexico; (c) Vacherie Dome, Louisiana; (d) Paradox Formation, Utah; (e) Permian Basin, Texas; (f) Richton Dome, Mississippi; (g) Newtonian salt at 160°C with a constant viscosity of $10^{17}$ Pa s for halite grain size $d = 5$ mm; or $10^{18}$ Pa s for $d = 10$ mm (from Van Keken et al., 1993). The creep parameters for the six power-law salts (a–f) are listed in Table 1.
Fig. 2 (continued).
Sinking velocity of elliptical inclusions

Most natural inclusions are closer to tabular shape than spherical (e.g., Gansser, 1960, 1992; Kent, 1979) and may be more closely approximated by an ellipsoid. The settling velocity of ellipsoids in Newtonian fluids is known (Happel and Brenner, 1986, pp. 145–157). Here, the velocity of 2-D ellipses (infinitely long elliptical cylinders) settling in power-law fluids is studied by means of finite-difference numerical calculations using a computer code developed by Prof. Harro Schmeling and described by Weinberg and Schmeling (1992). Contrary to the earlier assumptions of the present paper, the inclusions in the numerical models translate through ambient fluid bounded by free-slip lateral walls five times wider than the horizontal radius of the elliptical inclusion (see insert in Fig. 3). Although the results presented here correspond to infinitely long elliptical cylinders translating perpendicular to the longest axis, the results allow a qualitative insight into the influence of ellipticity and $n$-value on the rate at which inclusions sink.

In Newtonian fluids, spherical cylinders sink most rapidly, whereas ellipses sinking parallel to the shorter axis (flat lying in Fig. 3) move faster than ellipses moving parallel to the longer axis (upright in Fig. 3). These results are in accordance with the results described by Happel and Brenner (1986) for ellipsoids sinking in Newtonian fluids. For power-law fluids, the situation is the reverse of the Newtonian case. An upright ellipse concentrates the buoyancy stress to narrow regions above and below it; this causes a decrease in the controlling effective viscosity of the ambient fluid and consequently increases the sinking velocities (Fig. 3). Conversely, a flat-lying ellipse will spread its buoyancy stress to larger volumes and sink more slowly. Although little is known about the 3-D shape of the inclusions (M. Jackson, pers. commun., 1993) they are thought to be horizontal and tabular because of their horizontal sedimentary and/or igneous layering (see Fig. 4; e.g., Gansser, 1960; Jackson et al., 1990). However, if this is so, the question arises as to how they remained horizontal when flowing with the salt streaming from horizontal beds into a vertical diapir. Why did they not rotate into steep dips? Moreover, any inclined slabs sinking through salt would tend to rotate towards vertical due to the decrease in drag. Perhaps the inclusions were vertical as they rose with the diapir and extruded at the surface, but fell towards horizontal because of salt dissolution. A more detailed study of the 3-D shape and orientation of the inclusions would further discussion of the flow inside salt diapirs. If however inclusions are actually horizontal and tabular, they behave qualitatively in the same way as a flat-lying ellipse, and may be expected to sink through power-law fluids more slowly than would spheres of the same buoyancy, and thus can be more easily lifted by diapiric salt.

Other influences on the sinking velocity of inclusions

Several factors other than the shape may also affect the sinking velocity of inclusions: (a) proximity of the inclusion to the lateral boundaries of the salt; (b) several inclusions sinking simultane-
Fig. 4. Mafic volcanic rocks capping a hillock in dome 22 in the Great Kavir, Iran (redrawn from Jackson et al., 1990). 1 = coarse-grained augite-dolerite; 2 = thin gypsum layer; 3 = irregular banded salt.

ously; (c) further softening of the salt due to strain rates unrelated to the sinking of inclusions, such as the diapiric rise of salt; and (d) yield strength of salt, if any.

The interaction of lateral boundaries of the diapir with the buoyancy stress of the inclusion is non-trivial and causes complex variation in the sinking velocity of inclusions (Weinberg and Podladchikov, submitted). Even though the large inclusions present in the Iranian domes form a significant proportion of the diapir map section, it was assumed above that the diapir enveloping the inclusion was infinite (unbounded). In a Newtonian salt diapir \( n = 1 \), no-slip contact with the wall rocks slows the settling velocity of inclusions. As the \( n \)-value of salt increases, the influence of these boundaries becomes less pronounced and the assumption of infinite ambient fluid becomes more realistic. The sinking of inclusions very close to the diapir’s walls will be slowed, in all cases, but so too will be the rise velocity of salt close to these walls. The velocity of inclusions is similarly affected if several inclusions sink simultaneously through salt. The velocity of a swarm of inclusions decreases if \( n \) is close to unity, and tends to remain unaffected for \( n \)-values between 1 and 3. For \( n > 3 \), the increased stresses imposed by the swarm of inclusions may cause both faster or slower velocity depending on the distance between inclusions. When the inclusions are dispersed, softening of salt will lead to faster velocities; when the swarm is very concentrated, softening of salt will be less important than the increase in drag on the inclusions, and the net effect is a decrease in the sinking rate (for further details see Chhabra, 1988; Weinberg and Podladchikov, submitted).

In the calculations above, it was also assumed that softening of power-law salt is only related to the strain rates around the sinking inclusion. However, stresses related to the ascent of the diapir will also soften the salt and interact in a non-linear way with the stress imposed by the inclusion. The resulting interaction is difficult to assess at present, but probably causes the inclusion to sink faster than sinking rates calculated without any other stresses.

In summary, most of the simplifications made to calculate the sinking velocity of inclusions (salt with no yield strength, unbounded salt diapir and sinking of single inclusions) minimise the capability of salt to lift inclusions. If, however, the \( n \)-value of salt is high \(( n > 3 )\) the walls of the diapir and the presence of several inclusions may in some cases accelerate the sinking of inclusions. A simplification that is likely to increase the lifting capacity of salt as compared to nature, is the assumption that there are no additional stresses apart from the inclusion’s buoyancy acting on the salt. This latter assumption may cause very large differences in the lifting capacity of salt.

**Discussion**

The maximum negative buoyancy of inclusions that a diapir may lift varies considerably due to large variations in the rheology, and thus the effective viscosity of salt (Fig. 2; Table 1). The largest inclusion reported from any salt diapir has a volume corresponding to a sphere of approximately 500 m radius (an inclusion of approximately 5 km² and 125 m thick). Figure 2 indicates that, if spherical, this inclusion would be lifted by diapirs of most measured rheologies rising at reasonable rates and temperatures. Only diapirs of Newtonian salt or of the Vacherie Dome salt would have to rise extremely fast to lift this spherical inclusion. However, a tabular inclusion would be lifted more easily by a power-law fluid than a spherical inclusion. At 160°C (used in the calculations for Newtonian salt), it would be necessary for salt of \( 10^{17} \) Pa s to rise faster than 100 mm/a, and salt of \( 10^{18} \) Pa s to rise faster than 10 mm/a (Fig. 2g). These results suggest that the long-standing enigma posed by the Iranian inclu-
sions arose only because the surrounding salt was taken to have unrealistically low Newtonian viscosity (e.g., $10^{16}$ in Jackson et al., 1990).

Given the maximum negative buoyancy of inclusions in a diapir, and an estimate of the salt velocity, the effective viscosity of the salt may be estimated from Figure 2. Conversely, if the rheology and temperature of the diapir can be assessed, the velocity of salt may be estimated with the help of the largest inclusion. However, if the salt in a diapir slows sufficiently at later stages of diapirism, the inclusions could start to sink in relation to an external marker. Given enough time the diapir could become inclusion free. On the other hand, in outcropping diapirs, dissolution of salt may be faster than the sinking of inclusions, leading to a gradual accumulation of inclusions at the surface. Whether or not the inclusions are raised with the diapir, the relative sinking of inclusions in relation to the rising salt could cause downward-drag of internal markers/layering, perhaps forming structures like the domes-in-domes described by Richter-Bernburg (1987, his fig. 10).

Potentially, the centre of a diapir rises fastest and so is able to carry the largest inclusions. However, the original stratigraphy of the source of diapirs is a more important control in the final distribution of inclusions than velocity variations inside the diapir. For example, in the Great Kavir, the clean Old Salt occupies the centre of most diapirs, whereas the Younger Salt, with its many inclusions occupies the external parts. That inclusions in the diapirs in the Zagros Mountains are larger than those in the Great Kavir may indicate either faster velocity or greater viscosity of the salt in the Zagros, or simply differences in the original stratigraphy, and the size and distribution of inclusions at the source.

Urai et al. (1986) suggested that natural deformation of salt probably occurs close to the transition region between dislocation creep and solution-precipitation creep. Van Keken et al. (1993) arrived at a similar conclusion when modelling numerically the diapiric ascent of salt and the dependence of the deformation mechanism on strain rate. Gansser (1992) reported highly deformed rims of mobile salt often associated with peripheral brines, around the more competent interiors of the diapirs in the Iranian domes and in Texas. On the one hand salt extruding at the surface in Iran has been shown to flow up to 0.5 m/day after rain (Talbot and Jarvis, 1984), and on the other hand, salt is capable of supporting inclusions of mafic volcanic rocks on top of hills (Fig. 4; e.g., Gansser, 1960; Jackson et al., 1990), indicating that salt may either have a yield strength at surface conditions or that salt dissolves faster than the inclusion sinks. If however salt is still ductile at surface conditions, the large inclusions have to be supported by upward movement of the salt, and it is shown here that even the largest inclusions could be dynamically sustained by most known extruding salt diapirs rising at normal rates.

**Conclusion**

Consider thick layers of denser sediments or igneous rocks interlayered within a salt sequence. The salt starts to move slowly sideways and upwards to form a pillow. Competent layers break into blocks of different sizes that start to sink. Simultaneously, the pillow grows and the rise rate of salt accelerates exponentially, dragging increasingly larger blocks of dense material upwards. The size of inclusions lifted by each diapir depends on such initial processes as breaking, sinking and exponential acceleration, and on the steady-state vertical velocity achieved by the diapir at a later stage. If the diapir decelerates at its level of neutral buoyancy or beneath a competent layer, larger inclusions may sink through the salt back to the source and after sufficient time the salt may be cleared of large inclusions. The only evidence of the passage of sinking inclusions may be the presence of folded salt layers. The main conclusion of this study is that even the largest inclusion known would be lifted by most salt rising at typical velocities.

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