Turn Taking:
Intertemporal Cooperation and Symmetry through Intratemporal Asymmetry *

Sau-Him Paul Lau** and Vai-Lam Mui***

February 2009

Abstract

Turn taking is observed in many field and laboratory settings captured by various widely-studied 2x2 games. This paper develops a repeated game model that allows us to systematically investigate turn-taking behavior in many 2x2 games, including the battle of the sexes, the game of chicken, the game of common-pool-resources assignment, and a particular version of the prisoners' dilemma. We consider the "turn taking with independent randomizations" (TTIR) strategy that achieves three objectives: (a) helping the players reach the turn-taking path, (b) resolving the question of who takes the good turn first, and (c) deterring defection. We determine conditions under which a subgame-perfect equilibrium with TTIR strategy exists and is unique. Regarding testable implications of the model, we show that while in general an increase in the "degree of conflict" of the stage game increases the expected number of periods in reaching the turn-taking path, there exist conditions under which the opposite result occurs.

Key Words: Conflict, Coordination, Turn Taking, Intertemporal Cooperation

JEL Classifications: C70, C72

---

* We are grateful to Murali Agastya, Stephen Chiu, Simon Grant, Yew-Kwang Ng, Xianming Zhou, seminar participants at the Australian National University, City University of Hong Kong, McGill University, Monash University and University of Hong Kong for helpful comments, and to Daniel Kelly and Philip Ng for excellent research assistance. We thank the Research Grants Council of Hong Kong (project HKU7223/04H) for financial support.

** School of Economics and Finance, University of Hong Kong, Pokfulam, Hong Kong. E-mail: laushp@hku.hk. Phone: (852) 2857-8509. Fax: (852) 2548-1152.

*** Corresponding author. Department of Economics, Monash University, Clayton, Victoria 3800, Australia. E-mail: Vai-Lam.Mui@BusEco.Monash.edu.au. Phone: (61) 3-9905-2349. Fax: (61) 3-9905-5476.

1 Introduction

Turn-taking behavior is observed in a variety of field and laboratory settings. An example of turn taking concerns the use of common-pool resources (CPRs) such as fisheries, irrigation systems, and forests. In communities that depend heavily on such resources for their economic livelihood, failure to resolve problems related to the use and preservation of these resources can lead to significant welfare loss and violent conflicts. One illustration of the conflicts studied in the CPR literature is the game of CPR assignment in Ostrom et al. (1994, pp. 58-61). This game captures, in the simplest fashion, a situation in which two fishermen independently decide to go to one of two fishing spots in their community. The good spot has a value of h, and the bad spot has a value of l, where $h > l > 0$. If the two fishermen choose different spots, each will obtain the respective value of the spot. If they choose the same spot, they will split the value of the spot equally.

Both coordination and conflict elements (Friedman, 1994, pp. 7-8; Camerer, 2003, p. 354) are present in this game. To maximize total surplus, the fishermen should choose different fishing spots; however, they may end up at the same spot if there is no coordination. Moreover, a conflict element is present because while both players prefer going to different spots rather than to the same spot, each of them prefers to be the one who goes to the good spot while her opponent choosing the bad spot.[1] If this game is played repeatedly, one might expect that some sort of rotation scheme, in which the fishermen take turns going to the good spot, will eventually develop as a method of achieving intertemproal cooperation in allocating the CPRs. In fact, Berkes (1992) reports that fishermen in Turkey employ a turn-taking scheme to allocate fishing spots. Turn-taking schemes in the use of irrigation system have been adopted in Spain and the Philippines (Ostrom, 1990)

Turn taking is also observed in other settings. For example, faculty members in a department may use turn taking to resolve the question of who will serve as the departmental representative on a university committee. Soldiers in a military operation often take turns to serve as the "point man" in dangerous missions (Bergerud, 1993). These examples possess the feature of the repeated best-shot public good game (Harrison and Hirshleifer, 1989).[2] Turn taking in another game—the repeated battle of the sexes—has also been mentioned in the literature, dating back to Luce and Raiffa (1957).

Researchers have also observed turn-taking behavior in laboratory repeated

---

[1] We use female pronouns throughout this paper; their male counterparts get their turn in a companion paper.

[2] In a best-shot public good game, the socially available amount of public good is the maximum of individual contributions.

games. In an experiment motivated by the importance of turn taking in field examples regarding the management of CPRs discussed in Ostrom (1990), Prisbrey (1992, chapter 1) reports that in a repeated symmetric game with two asymmetric joint-payoff-maximizing outcomes, twenty-one out of twenty-four pairs of subjects succeed in establishing (and subsequently maintaining) turn taking within five periods. Bornstein et al. (1997) study intergroup conflicts that involve different versions of a repeated best-shot public good game.[3] While the exact extent of turn taking in the two-player version of the game was not reported since it was not the main focus of their paper, they reported that "turn taking between the two players was the rule rather than the exception" (Bornstein et al., 1997, p. 399). Helbing et al. (2005) investigate the repeated traffic route choice game in which two players independently choose one of two possible traffic routes each period. This game has a payoff structure similar to the CPR assignment game, and they observe significant incidence of turn-taking behavior (in seventeen out of twenty-four trials). In all three studies, eventual turn taking with initial coordination failure is widely observed.[4] Furthermore, all three studies suggest that when the stage game is a symmetric 2×2 game (i.e., with two players and two actions), turn taking often takes the form of single-period alternation between the players.

The literature reviewed above suggests that turn taking behavior is important in many settings captured by widely studied games with binary choices, and the experimental evidence suggests that delay in reaching successful turn taking is common. This raises the natural question of how differences in the structure of the game under study can affect turn-taking behavior, including the length of delay if the turn-taking path is reached. This paper analyzes turn taking in a repeated symmetric 2×2 game that nests many widely studied games, including those discussed above, as special examples. In the stage

---

[3] Bornstein et al. (1997) call the game used in their experiments a game of chicken, but they mention in their footnote 1 that the payoff structure is qualitatively similar to but slightly different from the traditional conditions defining a game of chicken. As seen from Harrison and Hirshleifer (1989) and Table 2 of this paper, it is more appropriate to call the game studied in Bornstein et al. (1997) a best-shot public good game. Note that the game of chicken has been widely used by scholars to study conflicts between countries or between management and workers (see, for example, Schelling, 1966).

[4] All these laboratory studies concern games with complete information, which is the case considered in our analysis below. Kaplan and Ruffle (2007) study a repeated entry game with incomplete information. In one version of their experiments, each of two players makes a binary choice of entry or exit after observing privately a randomly drawn integer between 101 and 105 each period. Exit yields a payoff of zero, while entry gives the player her drawn number if her opponent exits, but one-third of the drawn number if both players enter. They show that turn taking can be supported as an equilibrium in this repeated game, and find that many subjects take turns to enter to avoid efficiency loss due to simultaneous entry.

game of the repeated game analyzed in this paper, each player chooses between Tough (T) and Soft (S). As seen in the left-hand panel of Table 1, each player obtains payoff t if both choose the same action T, and each obtains payoff s if both choose S. On the other hand, a player obtains payoff h if she chooses T and the other player chooses S. In this case, the other player obtains a lower payoff l. We assume that $h + l > \max\{2s, 2t\}$, so that the two asymmetric outcomes (T, S) and (S, T) maximize total payoffs in this game.

To capture the kind of "trial and error" process that players use to resolve the coordination and conflict problems, as well as the observation that turn taking often takes the form of single-period alternation, we consider a simple strategy—to be called the "turn taking with independent randomizations" (TTIR) strategy—in this paper. Players using this strategy randomize between T and S in the beginning period and also if either of the symmetric outcomes is observed in the preceding period, or rotate between the two asymmetric outcomes if one of these outcomes is reached in the preceding period.

We analyze the subgame-perfect equilibrium when players use the TTIR strategy, and provide economic intuition about the properties of the equilibrium. It turns out that the players' equilibrium probability of randomization (in the randomization phase of the game) and the continuation value of the game have to be determined simultaneously. This implies that the equilibrium randomization probability has to satisfy a "fixed point" requirement. Despite that the mapping determining the fixed point of the randomization probability is not always a contraction mapping, we find that whenever a subgame-perfect equilibrium with TTIR strategy exists, it must be unique and symmetric. Moreover, there are systematic differences between the cases that the two asymmetric outcomes are Nash equilibria in the stage game (to be called the *accommodating* case), and that (T, T) is the unique Nash equilibrium of the stage game (to be called the *mutual-tough* case). We find that the TTIR equilibrium always exists for any discount factor in the accommodating case, but supporting turn taking with the TTIR equilibrium in the mutual-tough case requires that players be sufficiently patient.

Besides the existence and uniqueness results, we also derive testable implications about the equilibrium delay. Stimulated by the results in Lau and Mui (2008) regarding the repeated battle of the sexes, we define the degree of conflict (θ) of the stage game as the ratio of the two players' payoffs at an asymmetric outcome ($\theta = \frac{h}{l}$). It is natural to conjecture that equilibrium delay is increasing in the degree of conflict, since a higher degree of distributional conflict induces the players to behave more aggressively by choosing Tough with a higher probability in the initial periods. We find that this conjecture is correct for the mutual-tough case. However, for the accommodating case, there are games (such as the game of chicken) with the counter-intuitive feature that equilibrium delay is not always increasing in the degree of conflict.

Overall, the results suggest that both the difficulty in sustaining turn-taking behavior, and how the expected length of delay is affected by the primitives of the game, depend crucially on whether the two asymmetric outcomes are Nash equilibria of the stage game.

Obviously, according to the Folk Theorem (for example, Fudenberg and Maskin, 1986), the TTIR equilibrium is one of many subgame-perfect equilibria in the repeated games considered in this paper. Given our objective of understanding how changes in the structure of the game affect turn-taking behavior, we believe that the TTIR equilibrium possesses desirable properties making it an interesting equilibrium to consider.[5] First, our specification of single-period alternation and independent randomizations is consistent with the experimental evidence that turn taking with single-period alternation is usually reached after some initial failure (Prisbrey, 1992; Bornstein et al., 1997; Helbing et al., 2005). Second, we use a general specification allowing for the possibility that players may randomize with different probabilities, and find that whenever the TTIR equilibrium exists, it is unique and symmetric. The uniqueness of the TTIR equilibrium enables us to derive sharp comparative static results. Third, while we mainly focus on the case that punishment against defection during the turn-taking phase takes the form of reversion to the randomization phase, our results are robust to alternative forms of punishment, such as a specification that any defection will trigger the play of the symmetric Nash equilibrium in the stage game forever.

We are not aware of any prior contribution that aims at developing a unified analysis of turn-taking behavior in widely studied 2×2 games. There are, however, other work that feature turn-taking strategies. Our paper is related to several papers analyzing various 2×2 games.[6] Motivated by examples of ro-

---

[5] At first glance, one may think that instead of the TTIR strategy, it is possible to consider a simpler alternative strategy of (a) choosing Tough with probability 0.5 (which maximizes the probability of reaching the efficient asymmetric outcomes) until coordination occurs, and (b) always playing the strategy leading to the symmetric Nash equilibrium of the stage game after any player defects during the turn-taking phase. However, this strategy does not constitute a subgame-perfect equilibrium, and it also has the implausible implication that for all games covered in this paper, changes in the parameter values of the game do not affect the expected length of delay.

[6] Outside the context of 2×2 games, Athey and Bagwell (2001) analyze collusion in an infinitely repeated Bertrand game without side-payments. In their model, prices are publicly observable but each firm receives privately an i.i.d. cost shock each period. A key tension in the model is that production must simultaneously serve an efficiency role (allocating production as efficiently as possible in the current period) and a transfer role (rewarding firms for past truthful revelations of the cost information). They show that when firms are sufficiently patient, these two roles can be achieved in collusive schemes that exhibit turn taking features, in which firms that

tation in presidency between political parties as power sharing arrangements, Ward (1998) studies the conditions under which players will not defect from turn-taking agreements in the repeated game of chicken and a version of the repeated prisoners' dilemma. Ward (1998), however, does not provide an equilibrium analysis of how players resolve the questions of how to get onto a turn-taking path and who has the good turn first. This paper addresses these issues.

Lau and Mui (2008) consider the TTIR equilibrium in the repeated battles of the sexes, and show how a comparison of the "equilibrium" degree of conflict in the repeated battles of the sexes with its (exogenous) degree of conflict in the stage game provides an intuitive way of understanding how turn taking can facilitate intertemporal cooperation. This paper builds on some idea of that paper, but there are also substantial differences. This paper analyzes many widely studied games, and establish the key differences between the accommodating and mutual-tough cases, whereas Lau and Mui (2008) only consider the repeated battles of the sexes which is an example of the accommodating case. This paper allows for the possibility that players may randomize with different probabilities, and show that whenever a TTIR equilibrium exists, it is unique and symmetric. This is more general than the result in Lau and Mui (2008) that whenever a symmetric TTIR equilibrium exists, it is unique. Moreover, this paper performs comparative statics analysis that is not the focus of Lau and Mui (2008).

Our paper is complementary to the work by Bhaskar (2000), who considers the equalitarian convention (which equalizes, as far as possible, the realized payoffs of the two players) in the repeated battle of the sexes and the repeated game of chicken.[7] He finds that the efficient symmetric equilibrium

---

report a high cost in the current period are rewarded with favorable treatments in future periods. Athey and Bagwell (2008) extend the analysis to allow for persistent costs shocks. As pointed out by Athey and Bagwell (2001, 2008), with suitable reinterpretation, their analysis is applicable to collusion in repeated auctions. In an early analysis of bidder cartels, McAfee and McMillan (1992) consider a bid rotation scheme in which the bidders use an exogenous device (such as the phases of the moon) to choose which player's turn it is to bid. Aoyagi (2003) and Skrzypacz and Hopenhayn (2004) extend the analysis of McAfee and McMillan (1992) to consider bid rotation and other collusive schemes in repeated auctions.

[7] The strategy associated with the equalitarian convention considered by Bhaskar (2000) works as follows. Suppose that the realized outcome is (T, S) when an asymmetric outcome occurs for the first time, with player 1's current-period payoff of h exceeding player 2's payoff of l. To implement the egalitarian convention, the players choose (S, T) in succeeding periods until the intertemporal payoff of player 2 exceeds that of player 1. At this point the players switch to playing (T, S) until player 1's intertemporal payoff exceeds player 2's, and so on. This strategy is more complicated than the TTIR strategy with one-period alternation considered in this paper.

can be supported by the equalitarian convention, which involves the use of time-varying strategies and has turn-taking features. Bhaskar's result is an important normative finding. However, experimental studies cited above suggest that successful turn taking in symmetric 2×2 games often takes the form of one-period alternation instead of time-varying sequences. Another implication of the efficient strategies used in Bhaskar (2000) is that changes in the parameter values of the game do not affect the expected length of delay. The positive approach of our work is complementary to the normative approach of Bhaskar (2000), in that the TTIR strategy considered in this paper assumes that turn taking takes the form of one-period alternation and we aim to derive predictions regarding how changes in the parameter values of the game affect the expected length of delay.

This paper is organized as follows. Sections 2 introduces the symmetric repeated game with asymmetric efficient outcomes. Section 3 describes the TTIR strategy and the associated incentive conditions. Section 4 derives the subgame-perfect equilibrium with TTIR strategy for two major cases of the game. Section 5 derives testable comparative static results, and Section 6 concludes. Proofs of major propositions are given in Appendix A, and other derivations are given in Appendix B.

2 The Model

We consider a symmetric infinite-horizon repeated game with discounting. In every period of the game, each of the two players (called 1 and 2) chooses (perhaps randomly) between two actions: playing T or playing S. When making a new decision, say in period n, player i $(i = 1, 2)$ maximizes her intertemporal payoff (which is the discounted sum of the stream of her current and future single-period payoffs):

$$\sum_{m=n}^{\infty} \delta^{m-n} U_i \left(x_{1m}, x_{2m} \right), \tag{1}$$

where $\delta \in (0, 1)$ is the common discount factor, x_{im} $(x_{im} = T$ or $S)$ is the choice of player i at period m, and $U_i \left(x_{1m}, x_{2m} \right)$ is the current-period payoff of player i when player 1 chooses x_{1m} and player 2 chooses x_{2m}. The players'

Such time-varying sequences induce a different continuation value than one-period alternation and hence lead to a different probability of randomization in the beginning of the game. Computational exercises suggest that the equalitarian convention leads to a slightly higher efficiency than the TTIR equilibrium. The relatively small gain, and the complexity involved, may explain why such time-varying pattern of turn taking is not much observed in laboratory studies.

payoffs in the stage game are represented by

$$\begin{aligned} U_i(T,T) &= t, & U_i(S,S) &= s, \\ U_1(T,S) = U_2(S,T) &= h, & U_1(S,T) = U_2(T,S) &= l, \end{aligned} \tag{2}$$

where $i = 1, 2$, and h, l, s, and t are finite real numbers. (Whenever there is no confusion, the time subscript is ignored.) Payoffs are common knowledge. It is assumed that each player observes both players' actions in earlier periods, but not how the other player randomizes.

In this paper we make several assumptions on the payoff parameters in (2). Since interesting turn-taking behavior consists of good and bad turns, the first assumption we make is

$$h > l, \tag{3}$$

which allows for asymmetric outcomes in the stage game.[8] If the two players choose different actions, their payoffs will be different under assumption (3). Without loss of generality, we assume that h ("high") is larger than l ("low"). The specification in (3) implicitly defines the labels T and S for any given game. For example, T represents Good Spot and S represents Bad Spot for the game of CPR assignment.

Stimulated by the various settings in which turn taking has been observed, and by the taxonomies in Rapoport et al. (1976), we classify different symmetric 2×2 games according to twelve possible cases in Table 2.[9] For each of the twelve cases, we look at two aspects: pure-strategy equilibrium (or equilibria) of the stage game and joint-payoff-maximizing outcome(s).

We have two major observations about Table 2. First, it is natural to conjecture that repeated games with restrictions $h > s$ and $l > t$, which correspond to cases 1 to 3 in Table 2, are useful to model turn taking, because (S,T) and (T,S) will then be the pure-strategy equilibria of the stage game. This conjecture turns out to be almost, but not completely, correct. A closer look suggests that players would prefer an outcome with (S,S) in every period rather than an outcome with alternation between (S,T) and (T,S) for the

---

[8] Assumption (3) eliminates the pure coordination game from our analysis, but many widely-studied games (see Table 2) are consistent with this assumption.

[9] As in Rapoport et al. (1976, p. 14), our classification is based on the assumption "that all four payoffs of each player are different, so that each player has a strict preference for the outcomes in accordance with the order of magnitude of the associated payoffs". Since we assume $h > l$, there are only 12 cases (half of $4! = 24$ different ways to represent all possible rankings for the four payoffs) in Table 2. There are many well-known games (such as the battle of the sexes) that involve equality of some payoff parameters, but they can be included as special cases in our classification (see column 4 and note 2 of Table 2).

game of chicken in case 3b (with the additional restriction $2s > h + l$); see also Dixit and Skeath (2004, p. 374, Q. 5). As a result, alternation between (S, T) and (T, S) may not be an interesting equilibrium in this situation. Second, we observe that turn taking may also be plausible in repeated games with restrictions $h > s$ and $t > l$. This can be seen in cases 4a, 5a and 6a, with the common feature among them that while (T, T) is the unique Nash equilibrium in the stage game, the asymmetric outcomes (S, T) and (T, S) are joint-payoff-maximizing.[10]

Combining the above observations, we focus on repeated symmetric games with asymmetric efficient outcomes (i.e., with the stage games represented by cases 1, 2, 3a, 4a, 5a and 6a of Table 2) in analyzing turn taking. Formally, in this paper we assume (3),

$$h + l > 2s, \tag{4}$$

and

$$h + l > 2t. \tag{5}$$

Assumptions (4) and (5) ensure that the sum of the players' payoffs in the two asymmetric outcomes is higher than that in the two symmetric outcomes.

Summing up, our focus is the class of infinitely repeated games with (1) to (5), and we shall hereafter denote a game in this class as G_∞. Examples of this class of games are given in column 4 of Table 2. The analysis in this paper turns out to be partially similar but also partially different for two mutually exclusive possibilities (the accommodating case $t < l$ in cases 1, 2 and 3a, or the mutual-tough case $t > l$ in cases 4a, 5a and 6a), and the different analysis can be traced to the different pure-strategy equilibrium (or equilibria) in the stage game. Before analyzing these two cases, it is helpful to summarize well-known results about the corresponding one-shot games.

When $t < l$, the two asymmetric joint-payoff-maximizing outcomes (T, S) and (S, T) are pure-strategy equilibria of the stage game. (There is also a symmetric mixed-strategy equilibrium.) Key examples for this case include (a) battle of the sexes, (b) game of chicken (the version with $h + l > 2s$), (c) the best-shot public good game, and (d) a particular version of the game of CPR assignment studied by Ostrom et al. (1994) when $h < 2l$, that is, when the bad spot is not too inferior compared to the good spot.

---

[10] Another observation is that turn taking is not likely to be an interesting equilibrium in repeated games for cases 7 to 12 even if the players are patient enough. In these cases, the asymmetric outcomes are not joint-payoff-maximizing. For example, in the assurance game (such as the arms race game in Dixit and Skeath, 1999, Figure 4.10) that can be represented by case 8 of Table 2, the joint-payoff-maximizing outcome in the stage game is (S, S). For the repeated assurance game, the players prefer an outcome with (S, S) in every period instead of rotating between (T, S) and (S, T).

On the other hand, when $t > l$, each player's dominant strategy in the stage game is to choose Tough, and the two asymmetric outcomes are not Nash equilibria. Examples for this case include (a) a particular version of the prisoners' dilemma (see, for example, Dixit and Skeath, 1999, Figure 11.2),[11] (b) another version of the game of CPR assignment studied by Ostrom et al. (1994) with $h > 2l$, that is, when the good spot is "sufficiently more attractive" than the bad spot, and (c) the game of CPR appropriation externality studied by Ostrom et al. (1994, Table 3.4c).

3 TTIR Strategy

In this paper, we consider turn taking in the repeated game G_∞ in which there is *no communication* between the players. Turn taking is observed in experimental and field settings in which communication between the players is not possible, but it is also observed in environments in which the players are able to communicate; for example, Berkes (1992) describes how fishermen draw lots to assign fishing spots, which can be modeled as the use of correlated strategies (Aumann, 1974). In general, coordination and conflict problems can be mitigated by useful precedents when the players interact repeatedly (as in Crawford and Haller, 1990; Bhaskar, 2000; Lau, 2001) and/or by some forms of nonbinding preplay communication (as in Farrell, 1987). We think it is useful to first study equilibrium turn-taking behavior in an environment in which the players cannot communicate or correlate their strategies. In this environment, any benefit accrued to the players in mitigating conflict and enhancing coordination is purely due to turn taking and not to communication.

We consider a simple TTIR strategy in which players use turn taking and independent randomizations to resolve the coordination and conflict problems. The TTIR strategy specifies the following: (a) In the beginning period, the players will independently randomize between T and S. Denote player i's probability of choosing T as p_i ($i = 1, 2$). For meaningful TTIR strategy,

---

[11] Dixit and Skeath (1999, Figures 11.1 and 11.2) consider two games to study the issue of collective action in building an irrigation project. Each of these games satisfies the two usual key properties of the prisoners' dilemma, namely, that Defect is the dominant strategy for both players, and the outcome (Cooperate, Cooperate) Pareto dominates (Defect, Defect). In the first game, (Cooperate, Cooperate) is the joint-payoff-maximizing outcome; see case 4b in Table 2. This is the game that most analyses of the prisoners' dilemma focus on, and we refer to it as the "standard" prisoners' dilemma. In the other game (their Figure 11.2), the two asymmetric outcomes—that is, when one player defects and the other cooperates—maximize the players' total payoff; see case 4a in our Table 2. This non-standard prisoners' dilemma has also been studied by Ward (1998)

p_1 and p_2 are restricted to lie in the open interval $(0, 1)$. (b) As long as the randomization yields the symmetric outcome of either (T, T) or (S, S), the randomization phase will continue. (c) Whenever randomization succeeds in getting the players to the asymmetric outcome of either (T, S) or (S, T), the game will switch to the turn-taking phase in which each player chooses the action her opponent took in the previous period. If no player defects from this strategy, the turn-taking phase will continue. (d) Any defection by a player (or by both players) during the turn-taking phase will trigger a switch back to the randomization phase, and this randomization phase will continue until randomization succeeds in getting the players to the asymmetric outcome of either (T, S) or (S, T) again.[12] (e) Once randomization succeeds in getting the players to either asymmetric outcome, the players will again behave according to steps (c) and (d).

We analyze the subgame-perfect equilibrium when the players use the TTIR strategy. This subgame-perfect equilibrium is referred to as the *TTIR equilibrium*. Whether the TTIR strategy constitutes an equilibrium depends on the incentive conditions in the randomization and turn-taking phases, which we consider in the next two subsections.

3.1 Players' Behavior in the Turn-Taking Phase

Define V^H (*resp.* V^L) as a player's intertemporal payoff at a period in which she plays Tough (*resp.* Soft) and her opponent plays Soft (*resp.* Tough), with the expectation that both players will always choose the equilibrium TTIR strategy. Define V_i^* $(i = 1, 2)$ as player i's intertemporal payoff in the randomization phase (including the beginning period), with the expectation that both players will always choose the equilibrium TTIR strategy.[13]

The two value functions of the turn-taking phase are given by

$$V^H = h + \delta V^L = \frac{h + \delta l}{1 - \delta^2}, \tag{6}$$

and

$$V^L = l + \delta V^H = \frac{l + \delta h}{1 - \delta^2}. \tag{7}$$

---

[12] In our analysis below, we focus on the specification in which defection during the turn-taking phase will trigger a switch back to the randomization phase. As explained in Subsection 4.3, however, our results are robust to other punishment strategies.

[13] Note that this notation implicitly allows for the possibility that the two players' intertemporal payoffs in the randomization phase are different. This can occur if each player uses a different randomization probability in equilibrium.

To ensure that (6) and (7) are well defined, we need to check two no-deviation conditions for each player in the turn-taking phase, one at a player's good turn and the other at her bad turn.

If the actions of players i and j $(i, j = 1, 2$ and $i \neq j)$ in the previous period were S and T, respectively, then, assuming that the equilibrium TTIR strategy will be used by both players in the future (and also by her opponent in the current period), player i will not deviate from the equilibrium strategy in the current period when

$$V^H - (s + \delta V_i^*) = (h - s) + \delta \left(V^L - V_i^* \right) > 0. \tag{8}$$

Similarly, if the actions of players i and j in the previous period were T and S, respectively, then player i will not deviate from the equilibrium strategy in the current period when

$$V^L - (t + \delta V_i^*) = (l - t) + \delta \left(V^H - V_i^* \right) > 0. \tag{9}$$

3.2 Players' Behavior in the Randomization Phase

Next, we examine the randomization phase (including the beginning period). If both players use the TTIR strategy, it is easy to see that the game will remain in the randomization phase in the next period if and only if both players choose the same action in the current period.

For subsequent analysis, it is helpful to define

$$V_i(p_i, p_j) = \frac{p_i p_j t + (1 - p_i)(1 - p_j) s + p_i (1 - p_j) V^H + (1 - p_i) p_j V^L}{1 - \delta \left[p_i p_j + (1 - p_i)(1 - p_j) \right]}, \tag{10}$$

which gives player i's intertemporal payoff at the randomization phase when players i and j choose Tough with probability p_i and p_j respectively, since this phase will continue with probability $p_i p_j + (1 - p_i)(1 - p_j)$ in the next period.

Denote player i's equilibrium probability of choosing T in the randomization phase as p_i^*. In the randomization phase, player i chooses p_i^* to ensure that the other player is indifferent between playing Tough and Soft. It can be deduced from Table 3 that the players' equilibrium randomization probabilities (p_1^* and p_2^*) and equilibrium intertemporal payoffs (V_1^* and V_2^*) are jointly determined by

$$V_i^* = p_j^* (t + \delta V_i^*) + \left(1 - p_j^* \right) V^H = p_j^* V^L + \left(1 - p_j^* \right) (s + \delta V_i^*), \tag{11}$$

where $i, j = 1, 2$ and $i \neq j$. Note that V_i^* in (11) is related to the function

$V_i(p_i, p_j)$ in (10) according to

$$V_i^* = V_i\left(p_i^*, p_j^*\right). \tag{10a}$$

To examine the conditions characterizing the equilibrium randomization probabilities, we combine (10), (10a) and the second equality of (11) to obtain

$$\begin{aligned} p_j^* &= \frac{V^H - s - \delta V_i^*}{(V^H - s - \delta V_i^*) + (V^L - t - \delta V_i^*)} \\ &= \frac{V^H - s - \delta\left\{\frac{p_i^* p_j^* t + \left(1-p_i^*\right)\left(1-p_j^*\right)s + p_i^*\left(1-p_j^*\right)V^H + \left(1-p_i^*\right)p_j^* V^L}{1-\delta\left[p_i^* p_j^* + \left(1-p_i^*\right)\left(1-p_j^*\right)\right]}\right\}}{V^H + V^L - s - t - 2\delta\left\{\frac{p_i^* p_j^* t + \left(1-p_i^*\right)\left(1-p_j^*\right)s + p_i^*\left(1-p_j^*\right)V^H + \left(1-p_i^*\right)p_j^* V^L}{1-\delta\left[p_i^* p_j^* + \left(1-p_i^*\right)\left(1-p_j^*\right)\right]}\right\}}, \end{aligned} \tag{12}$$

where $i, j = 1, 2$ and $i \neq j$, and V^H and V^L depend on parameters δ, h, and l according to (6) and (7). That is, $p_1^* \in (0,1)$ and $p_2^* \in (0,1)$ are the equilibrium randomization probabilities if they satisfy simultaneously the two equations defined by (12).

Condition (12) can be interpreted as follows. If the no-deviation conditions (8) and (9) are satisfied, then player j's equilibrium randomization probability (p_j^*) in the current period is given by the middle term of (12), which involves V_i^*, since the game may remain in the randomization phase in the next period. As given by (10a), player i's continuation payoff V_i^* depends on equilibrium randomization probabilities p_i^* and p_j^* (in the future). Thus, one can think of p_j^* $(j = 1, 2)$ on the left-hand side of (12) as the probability that player j choose T in the current period during the randomization phase, and p_1^* and p_2^* on the right-hand side as the probabilities that the two players choose T in the future (if the game remains in the randomization phase). The two equilibrium conditions in (12) are consistency conditions between current and future randomization probabilities of this infinitely repeated game.

4 TTIR Equilibrium

A TTIR equilibrium of the repeated game G_∞ exists if there exists $p_1^* \in (0,1)$ and $p_2^* \in (0,1)$ that simultaneously satisfy the no-deviation conditions (8) and (9) during the turn-taking phase and the equilibrium randomization conditions (12) during the randomization phase. Moreover, the TTIR equilibrium is unique if there exists only one pair of $p_1^* \in (0,1)$ and $p_2^* \in (0,1)$ that satisfies these conditions.

Our formulation allows for the possibility of asymmetric TTIR strategies. This formulation is more general than, for example, those in Lau and Mui (2008) in which only symmetric TTIR strategies are considered. Using this more general framework, we are able to show that an asymmetric TTIR equilibrium does not exist in the symmetric game G_∞. To simplify exposition, we focus on symmetric TTIR strategies in this section, and leave the discussion of asymmetric TTIR strategies in the Appendix.[14]

Among the three conditions, we expect that the no-deviation condition (8) always holds, since it is related to whether a player will defect when she is supposed to take her good turn. The following Lemma is useful in establishing condition (8).

Lemma 1 *For the repeated game G_∞,*

$$V^H - [s + \delta V_i(p,p)] > 0 \tag{8a}$$

for all $\delta \in (0,1)$ and all $p \in (0,1)$. In particular, (8a) holds for all $\delta \in (0,1)$ at the symmetric TTIR equilibrium (if it exists).

Our remaining tasks are to examine under what circumstances the no-deviation condition (9) at a player's bad turn holds, and to study the equilibrium randomization conditions in (12). The analysis differs for the accommodating and mutual-tough cases, as the underlying structure of the game is different in these two cases.

4.1 The accommodating case ($t < l$)

The analysis of the accommodating case ($t < l$), which includes cases 1, 2 and 3a in Table 2, is quite straightforward. First, we obtain the following Lemma regarding the no-deviation condition (9) when a player is supposed to take her bad turn.

Lemma 2 *For the repeated game G_∞ with $t < l$,*

$$V^L - [t + \delta V_i(p,p)] = (l - t) + \delta \left[V^H - V_i(p,p)\right] > 0 \tag{9a}$$

for all $\delta \in (0,1)$ and all $p \in (0,1)$. In particular, (9a) holds for all $\delta \in (0,1)$ at the symmetric TTIR equilibrium (if it exists).

For the accommodating case, a player will not defect when she is supposed to take her bad turn, because by adhering to the equilibrium strategy, she will have both a current gain of $l - t$ and a future gain of $\delta\left(V^H - V_i^*\right)$. Thus, the

---

[14] In particular, we show in the Appendix that no pair of asymmetric randomization probabilities (p_1, p_2) with $p_1 \neq p_2$ can satisfy the two conditions in (12), even if they satisfy conditions (8) and (9). This rules out asymmetric TTIR equilibrium.

no-deviation condition (9) at a player's bad turn is satisfied for any discount factor in the accommodating case.

With Lemma 1 and Lemma 2, we show in the Appendix that for the accommodating case of game G, the TTIR equilibrium exists, is unique and symmetric for all $\delta \in (0,1)$. We also obtain the closed-form solution for a player's equilibrium randomization probability ($p_1^* = p_2^* = p^*$).

Proposition 1 *For the repeated game G_∞ with $t < l$, there exists a unique TTIR equilibrium for all $\delta \in (0,1)$. Moreover, the TTIR equilibrium is symmetric, with the equilibrium randomization probability of each player given by*

$$p^* = \frac{-c}{b} \tag{13}$$

if $a = 0$, or by

$$p^* = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{14}$$

if $a \neq 0$, where coefficients a, b and c are related to the discount factor and stage-game payoff parameters according to

$$a = \delta\left[(1+\delta)(t-s) - (h-l)\right], \tag{15}$$

$$b = \left(1-\delta^2\right)t + (1+\delta)^2 s - h - (1+2\delta)\,l, \tag{16}$$

and

$$c = h + \delta l - (1+\delta)\,s. \tag{17}$$

4.2 The mutual-tough case ($t > l$)

The derivation of the conditions under which turn taking can be supported as a subgame-perfect equilibrium for the mutual-tough case ($t > l$), which includes cases 4a, 5a and 6a in Table 2, is more difficult but also, arguably, more interesting than the accommodating case.

As the proof for the accommodating case is relatively straightforward, it may appear that it would also be easy to obtain the conditions under which (9) holds for the mutual-tough case by expressing V^L and V_i^* in (9) in terms of the payoff parameters (δ, h, l, s, and t). For example, one may think of using the closed-form solution for p^* similar to (13) and (14) for the accommodating case. However, while (13) and (14) hold for all discount factors $\delta \in (0,1)$ for the accommodating case, they will only hold for sufficiently high discount factors for the mutual-tough case, as we shall show later. Therefore, we need to first determine the range of discount factors in which the TTIR equilibrium exists and (13) and (14) hold for the mutual-tough case.

In the analysis of the mutual-tough case, we proceed as follows. We first conjecture that there exists a critical discount factor $\delta_{TT} \in (0,1)$ such that for all $\delta \in (\delta_{TT}, 1)$, a unique symmetric TTIR equilibrium exists. We then show that if a unique symmetric TTIR equilibrium exists for all $\delta \in (\delta_{TT}, 1)$, the function $p^*(\delta)$ must be strictly decreasing in δ. These results enable us to determine the value of δ_{TT} as a function of the payoff parameters of the stage game. We then close our proof by showing that for all $\delta \in (\delta_{TT}, 1)$, in fact there exists a unique TTIR equilibrium and it is symmetric.

Assuming first that a unique symmetric TTIR equilibrium (with $p_i^* = p_j^* = p^*$ and $V_i^* = V_j^* = V^*$) exists for all $\delta \in (\delta_{TT}, 1)$, we investigate how the equilibrium randomization probability changes when only the discount factor changes. The partial derivative $\frac{\partial p^*}{\partial \delta}$ is given in (19) below. Manipulating various terms in (19) leads to the following Lemma regarding the monotonicity of p^* with respect to δ.[15]

Lemma 3 *For the repeated game G_∞ with $t > l$, if a unique symmetric TTIR equilibrium exists for all $\delta \in (\delta_{TT}, 1)$, then p^* satisfies*

$$0.5 < p^* < 1, \tag{18}$$

and

$$\frac{\partial p^*}{\partial \delta} = \frac{(1-p^*)\left[\frac{\partial V^H}{\partial \delta} - \delta \frac{\partial V^*(\delta,p^*)}{\partial \delta} - V^*\right] - p^*\left[\frac{\partial V^L}{\partial \delta} - \delta \frac{\partial V^*(\delta,p^*)}{\partial \delta} - V^*\right]}{(V^H - s - \delta V^*) + (V^L - t - \delta V^*) + \delta(1-2p^*)\frac{\partial V^*(\delta,p^*)}{\partial p^*}} \tag{19}$$

is negative.

Lemma 3 is a crucial step in obtaining the critical discount factor for the mutual-tough case. According to Lemma 3, the function $p^*(\delta)$ decreases monotonically in $\delta \in (\delta_{TT}, 1)$, provided that a unique symmetric TTIR equilibrium exists. Moreover, it is clear from (12) that $p^*(\delta)$ is a continuous function. Combining these features, we conclude that

$$\lim_{\delta \to \delta_{TT}} p^*(\delta) = 1, \tag{20}$$

since otherwise we could have found a lower discount factor such that $p^*(\delta)$ is still less than 1. Equation (20) says that in the limit when δ tends to the critical discount factor, the punishment of TTIR becomes most severe as $p^*(\delta)$ tends to 1. Figure 1 (with $h = 160$, $l = 40$, $s = 20$ and $t = 80$) illustrates how p^* behaves as a function of δ.

---

[15] On the other hand, for the accommodating case, we have found examples in which the monotonicity property of p^* with respect to δ does not hold.

Furthermore, combining (20) and the second equality of (11), we obtain

$$\lim_{\delta\to\delta_{TT}}\left[V^{L}(\delta)-t-\delta V^{*}(\delta)\right]=0. \tag{21}$$

Equation (21) states that the no-deviation condition at the bad turn must be binding at the critical discount factor δ_{TT}. Using these results, we can determine the critical discount factor as a function of the payoff parameters of the stage game.

Lemma 4 *For the repeated game G_∞ with $t>l$, if a unique symmetric TTIR equilibrium exists for all $\delta\in(\delta_{TT},1)$, then the critical discount factor δ_{TT} depends on the stage-game parameters according to*

$$\delta_{TT}=\frac{t-l}{h-t}. \tag{22}$$

Lemma 4 shows that the critical discount factor for the mutual-tough case depends on the primitives of the stage game in a simple and intuitive way.[16] Its intuition is as follows. When the discount factor δ is sufficiently close to 1 (and future payoffs are important), it is not surprising that the no-deviation condition (9) is non-binding. According to Lemma 3, the endogenously determined p^* of the TTIR strategy is strictly decreasing in δ (between δ_{TT} and 1). As δ decreases (and future payoffs become less important), to ensure that the no-deviation condition (9) holds, p^* must increase to make deviation more costly. However, the maximum possible punishment is when p^* tends to 1. This defines the critical discount factor δ_{TT}. As δ tends to δ_{TT}, $p^*(\delta)$ tends to 1 according to (20). Moreover, the no-deviation condition at the bad turn becomes binding according to (21), and the punishment approaches the Nash punishment (of choosing T with probability 1 at every period), as in Friedman (1971). Thus, as δ approaches δ_{TT}, $V^*(\delta)$ approaches $t+\delta_{TT}t+\delta_{TT}^2t+...=\frac{t}{1-\delta_{TT}}$. Substituting this result and (7) into (21) lead to

$$\frac{l+\delta_{TT}h}{1-\delta_{TT}^2}=\lim_{\delta\to\delta_{TT}}V^{L}(\delta)=\lim_{\delta\to\delta_{TT}}\left[t+\delta V^{*}(\delta)\right]=t+\delta_{TT}\left(\frac{t}{1-\delta_{TT}}\right). \tag{21a}$$

Simplifying (21a), we can determine the critical discount factor as in (22).

Having determined δ_{TT} as a function of the payoff parameters according to (22), we now show that for the mutual-tough case, there exists a unique TTIR

---

[16] Our analysis makes clear that the critical discount factor for the mutual-tough case is determined by the limiting case of V^* tending to the intertemporal payoff associated with the strategy of always choosing Tough. When a player considers whether to deviate or not at her bad turn of the turn-taking path in this limiting case, payoff s does not appear in V^* since the outcome (S,S) will never be reached in the future. Consequently, the critical discount factor does not depend on s, but depends only on the other three payoff parameters.

equilibrium for all $\delta \in (\delta_{TT}, 1)$ and it is symmetric. This is given in Proposition 2.

Proposition 2 *For the repeated game* G_∞ *with* $t > l$*, there exists a unique TTIR equilibrium for all* $\delta \in (\delta_{TT}, 1)$ *where* δ_{TT} *is given by (22). Moreover, the TTIR equilibrium is symmetric, with each player's equilibrium randomization probability given by (13) if* $a = 0$ *in (15) or by (14) if* $a \neq 0$ *in (15).*

Proposition 2 shows that for the mutual-tough case, the discount factor must be larger than $\delta_{TT} = \frac{t-l}{h-t}$ for the TTIR strategy to constitute a subgame-perfect equilibrium. This is in sharp contrast to the accommodating case, in which the TTIR strategy constitutes a subgame-perfect equilibrium for any discount factor. According to (9), a player who is supposed to take the bad turn according to the TTIR strategy will not deviate if the current gain from deviating, $t - l$, is smaller than the future loss from deviating, $\delta \left(V^H - V^*\right)$. When $t < l$ (i.e., the accommodating case), this condition is satisfied for any discount factor because deviation actually yields a current loss, not a gain. When $t > l$ (i.e., the mutual-tough case), however, deviation yields a current gain. If the discount factor is too low, then (9) cannot be satisfied no matter how the randomization probability (which affects a player's intertemporal payoff) is chosen. There is a trade-off between current gain and future loss if a player deviates from the equilibrium strategy. It is this trade-off that distinguishes the analysis and results of the mutual-tough case from those of the accommodating case.

4.3 Alternative punishment strategies

In the above analyzes, we assume that any defection during the turn-taking phase will trigger a switch back to the randomization phase, and this randomization phase will continue until randomization succeeds in getting the players to an asymmetric outcome again. This is in the spirit of the Markov strategy, as it is assumed a player takes the same action (of randomization) when she faces the same environment (of observing a symmetric outcome in the preceding period, whether it is in the initial randomization phase or in the punishment phase). It is interesting to investigate whether our results continue to hold under alternative punishment strategies.

Consider a different TTIR strategy with the following punishment strategy: each player uses the strategy that leads to the symmetric equilibrium of the stage game (i.e., symmetric mixed-strategy equilibrium for the accommodating case, or (T, T) for the mutual-tough case); see, for example, Lau and Mui (2008). Another possibility is to punish the player that deviates by restarting the turn-taking path at her bad turn with the other player starting at her good turn. This punishment strategy does not treat players symmetrically, but the punishment path is efficient. It can be shown that with either one of these punishment strategies, the no-deviation conditions (8) and (9) during the turn-taking phase are satisfied for all $\delta \in (0,1)$ in the accommodating case, or for $\delta \in (\delta_{TT}, 1)$ in the mutual-tough case. More generally, for any credible punishment that deters defection during the turn-taking phase, the analysis in the randomization phase is the same as above. In particular, p^* and V^*, which are related by (11) with symmetric TTIR strategies, are the same as above. Consequently, we obtain the same behavioral implications whenever an alternative punishment strategy prevents players from deviating from equilibrium behavior.

5 Testable Implications

In this section we consider two sets of comparative static results, regarding the time in reaching the turn-taking path (for both accommodating and mutual-tough cases) and the critical discount factor (for the mutual-tough case).

5.1 Delay in Reaching the Turn-Taking Path

One characteristic of the TTIR equilibrium is that the equilibrium randomization probability varies systematically with the primitives of the game, according to (13) or (14). This characteristic allows us to derive comparative static results.

In the TTIR equilibrium, each player chooses T with probability p^* in the randomization phase. Therefore, the probability that the players succeed in reaching an asymmetric outcome in a particular period during this phase is given by $2p^*(1-p^*)$. Define the number of time periods taken by the players using the TTIR strategy to reach the turn-taking path as the delay (D). It is easy to see that the delay is a geometric random variable and its expected value is given by

$$E(D) = \frac{1}{2p^*(1-p^*)}. \tag{23}$$

In Lau and Mui (2008), we examine the welfare properties of turn taking in a specific game (the repeated battle of the sexes), and find that the degree of conflict of the stage game is a crucial factor. Stimulated by the analysis in that paper, we focus here on comparative static results about the delay with respect to this factor.

We define two concepts related to payoffs h and l in the stage game of G. In the stage game, the maximum and minimum amounts of the players' total gain attained as a result of reaching an asymmetric outcome are $h+l-\min\{2s,2t\}$ and $h+l-\max\{2s,2t\}$, respectively. Holding the value of s and t constant, a rise in $h+l$ increases both the maximum and the minimum gains that the players attain when they reach an asymmetric outcome. Therefore, we define

$$\lambda = h + l \tag{24}$$

as the efficiency gain parameter when achieving either one of the two asymmetric outcomes. We also define

$$\theta = \frac{h}{l} \tag{25}$$

as the conflict of interest parameter. Without loss of generality, we can normalize the payoffs so that $l > 0$. As a result, $\lambda > 0$ and $\theta > 1$ according to (3), (24) and (25).

From (24) and (25), we can obtain $h = \frac{\theta\lambda}{1+\theta}$ and $l = \frac{\lambda}{1+\theta}$. Therefore, while the original specification of the stage game uses the four parameters s, t, h, and l as primitives, the game can also be expressed in terms of parameters s, t, λ, and θ. This is illustrated in the right-hand panel of Table 1. This re-specification is useful for some systematic changes in parameters h and l, as will be shown in Proposition 3. When the efficiency parameter λ, as well as s and t, are held constant, an increase in θ implies that there is a higher degree of conflict in the stage game. This can be illustrated in Table 4 involving two games of chicken. The left panel corresponds to the case when $s = 90$, $t = 10$, $\lambda = 200$, and $\theta = 4$. The game in the right panel is obtained from increasing the degree of conflict to $\theta = 9$, with the values of the other three parameters held constant.

It is natural to conjecture that an increase in the degree of distributional conflict will induce the players to behave more aggressively in choosing Tough with a higher probability at the randomization phase. As a result, the expected value of delay will be increased. It turns out that this conjecture is correct for most but not all games. The following Proposition shows that an increase in the degree of conflict (θ) will increase expected delay in the mutual-tough case, but may decrease equilibrium delay in a range of θ for a number of games (including game of chicken) in the accommodating case.

Proposition 3 *(a) At the TTIR equilibrium of repeated game G_∞ with $t > l$ and $\delta \in (\delta_{TT}, 1)$, the expected value of delay, given by (23), is always increasing in the degree of conflict (θ) of the stage game.*

(b) At the TTIR equilibrium of repeated game G_∞ with $t < l$, expected delay is always increasing in θ if $t \geq s$. However, if $s > t$, there exist games in

which expected delay is not always increasing in θ.

The intuition of Proposition 3 is as follows. Since a change in the degree of conflict affects expected delay through its effect on the equilibrium randomization probability, we have $\frac{\partial E(D)}{\partial \theta} = \frac{\partial E(D)}{\partial p^*} \times \frac{\partial p^*}{\partial \theta}$. We know from (23) that $E(D)$ reaches its minimum at $p^* = 0.5$. Furthermore, $E(D)$ is decreasing in p^* when $0 < p^* < 0.5$, but is increasing in p^* when $0.5 < p^* < 1$. Thus, $\frac{\partial E(D)}{\partial p^*} > 0$ if and only if $0.5 < p^* < 1$. On the other hand, $\frac{\partial p^*}{\partial \theta}$ is given by (A6). As shown in the Appendix, the numerator of (A6) is positive. For the denominator, two of the three terms are positive because the no-deviation conditions hold. The third term, $\delta(1-2p^*)\frac{\partial V^*}{\partial p^*}$, is also positive if $0.5 < p^* < 1$. Thus, $0.5 < p^* < 1$ is a sufficient condition for $\frac{\partial p^*}{\partial \theta} > 0$. Combining these results, we conclude that $0.5 < p^* < 1$ is a sufficient condition for $\frac{\partial E(D)}{\partial \theta} > 0$.

For the mutual-tough case (i.e., cases 4a, 5a and 6a in Table 2) and the accommodating case with $t \geq s$ (i.e., cases 2 and 2' in Table 2, including the battle of the sexes game and the game of CPR assignment with $h < 2l$), $0.5 < p^* < 1$ is satisfied, according to the proof of Proposition 3. In these games, the payoff parameters are such that each player chooses Tough with an equilibrium probability higher than 0.5 in the randomization phase to ensure that her opponent is indifferent in choosing Tough or Soft. When θ increases, the intertemporal efficiency gain from turn taking is more unevenly distributed, and it becomes more attractive to be the first player to take the good turn. Thus, each player behaves more aggressively in choosing Tough with a higher probability. Since the increase in equilibrium randomization probability occurring in the range $(0.5, 1)$ means that each player's probability of playing Tough is further away from 0.5, the probability of reaching the turn-taking path in a particular period, given by $2p^*(1-p^*)$, is reduced. As a result, expected delay increases because of the players' more aggressive behavior.

The condition $0.5 < p^* < 1$ is not always satisfied in the accommodating case when $s > t$ (i.e., cases 1 and 3a in Table 2). We have performed computational analysis for these games and found counter-intuitive pattern that $E(D)$ is not always increasing in θ.[17] We present one such counter-example as an illustration. We consider the games of chicken with $s = 90$, $t = 10$, $\lambda = 200$, $\delta = 0.6$ and θ ranging from 2 to 10. In all these games, $h > s > l > t$. Figure 2 shows that when the degree of conflict is relatively small, the equilibrium randomization probability is less than 0.5. This is because the payoff at the outcome (Tough, Tough) is relatively unattractive in these games of chicken, and each player chooses Tough with a probability less than 0.5 to ensure that her opponent is willing to randomize between Tough and Soft. In this example,

---

[17] Besides the non-monotonic relationship between $E(D)$ and θ that we present here, there are other patterns, including $E(D)$ is always increasing in θ, and $E(D)$ is always decreasing in θ.

when θ increases and being the first player to take the good turn becomes more attractive, the equilibrium value p^* increases. However, since $0 < p^* < 0.5$ when θ is relatively small, an increase in p^* means that it is getting closer to 0.5 and thus, expected delay will decrease. This explains the counter-intuitive result. On the other hand, when θ is relatively large and the efficiency gain of turn taking is more unevenly distributed, even though a player's current-period payoff at the outcome (Tough, Tough) is still unattractive, the higher level of θ will tilt the trade-off of choosing Tough versus Soft in favor of the former. As a result, $0.5 < p^* < 1$. In this range of θ, the behavior of $E(D)$ as a function of θ is qualitatively the same as the mutual-tough case. Combining the above results, we observe the U-shaped pattern between $E(D)$ and θ for the games of chicken in Figure 3.[18]

5.2 Critical Discount Factor (for the Mutual-Tough Case)

The critical discount factor for the mutual-tough case is given in Lemma 4. We first examine how the critical discount factor is affected by a change in parameter t. Note that the left-hand term of (21a) is a player's intertemporal payoff of adhering to the equilibrium strategy when her bad turn comes up, whereas the right-hand term is the intertemporal payoff of defecting. An increase in t (at an unchanged δ_{TT}) will increase the current and future payoffs of defecting. To restore the equilibrium condition (21a), the critical discount factor has to increase. Formally, differentiating δ_{TT} with respect to t gives

$$\frac{\partial \delta_{TT}}{\partial t} = \frac{h-l}{(h-t)^2} > 0, \tag{26}$$

because of (3) and (5). Holding the value of h and l constant, an increase in t (up to $\frac{h+l}{2}$) makes it harder for the players to use TTIR to support turn taking as an equilibrium outcome.

We also study how changes in the degree of conflict and the efficiency gain affect the critical discount factor for the mutual-tough case. The critical discount

---

[18] Note that the variation of $E(D)$ with respect to θ in Figure 3 is quite small. This is because we choose the parameters of the games of chicken to illustrate the non-monotonic relationship of $E(D)$ with respect to θ, and thus p^* moves from below 0.5 to above 0.5. Since the values of p^* in Figure 3 are close to 0.5, the corresponding values of $E(D)$ are close to 2 and do not vary much. We have also found other games in which the changes in expected delay with respect to θ are more substantial and are monotonic. Those games would be more useful for designing experiments to examine the validity of the theoretical results in this paper.

factor in (22) can be expressed as a function of t, θ and λ as follows:

$$\delta_{TT} = \frac{t-l}{h-t} = \frac{(1+\theta)\,t - \lambda}{\theta\lambda - (1+\theta)\,t}. \tag{22a}$$

Because $t > l > 0$ and $\lambda = h + l > 2t$, we have

$$\frac{\partial \delta_{TT}}{\partial \theta} = \frac{\lambda\,(\lambda - 2t)}{\left[\theta\lambda - (1+\theta)\,t\right]^2} > 0, \tag{27}$$

and

$$\frac{\partial \delta_{TT}}{\partial \lambda} = \frac{-t\left(\theta^2 - 1\right)}{\left[\theta\lambda - (1+\theta)\,t\right]^2} < 0. \tag{28}$$

Equation (27) says that δ_{TT} is increasing in θ, which means that an increase in the degree of conflict makes it harder for the players to use TTIR to support turn taking as an equilibrium intertemporal cooperation mechanism. An increase in distributional conflict (when other parameters are unchanged) implies that the player's payoff at the bad turn along the equilibrium turn-taking path is relatively unattractive. As a result, the players have a greater incentive to defect (at a given discount factor), and the TTIR equilibrium can only be supported above a higher critical discount factor.

Equation (28) says that δ_{TT} is decreasing in λ, which means that a decrease in the efficiency gain from achieving either of the asymmetric outcomes (provided that (4) and (5) still hold) leads to an increase in the critical discount factor above which turn taking can be supported as an equilibrium outcome.

We summarize the above results about the critical discount factor in the mutual-tough case in the following Proposition.

Proposition 4 *For repeated game G_∞ with $t > l$, a unique TTIR equilibrium exists for all $\delta \in (\delta_{TT}, 1)$. The critical discount factor δ_{TT} is increasing in t, increasing in θ, and decreasing in λ.*

6 Concluding Remarks

Motivated by the importance of turn-taking behavior in many field and experimental settings, this paper develops an analysis of turn taking in a repeated symmetric 2×2 game that nests many widely studied games as special examples. Our focus on the delay in reaching the turn-taking path, and our specification of one-period alternation in the TTIR strategies, are inspired by experimental findings. The analysis generates novel testable implications, which provides guidance for new laboratory experiments examining how differences in the structure of the strategic environment affect the effectiveness of turn taking as a mechanism for intertemporal cooperation. For example, our analysis predicts that the expected delay in reaching the turn-taking path is always increasing in the degree of conflict of the stage game for the mutual-tough case. This prediction and related ones for the accommodating case can be tested in controlled laboratory experiments that systematically vary the degree of conflict parameter.

In this paper we analyze turn taking in a benchmark model that does not allow for communication or correlated randomization. However, the results reported in Ostrom et al. (1994) suggest that nonbinding communication can be efficiency-enhancing in the laboratory repeated games, a result that is broadly consistent with findings that cheap talk can be efficiency-enhancing in static mixed-interest games such as the battle of the sexes (Cooper et al., 1989). On the other hand, the results reported in Prisbrey (1992) suggest that asymmetric turn-taking schemes—for example, one in which a player is supposed to take the good turn for two periods and then take the bad turn for one period, with her opponent doing the opposite—are more difficult to sustain. In the future, we plan to investigate whether extending the model to incorporate different kinds of asymmetry and/or communication will make it easier or more difficult for the (potentially more sophisticated) turn-taking strategies to achieve intertemporal cooperation.

Finally, the model considered in this paper rules out the possibility that a player may attempt to "modify the game" to her advantage. However, in environments related to deciding whether turn taking can be used to determine, for example, who will be the chairperson of a department or which nation will chair an important international committee, players may be reluctant to take the bad turn because they are concerned that the player who gets to take the good turn in a particular period may attempt to alter the game to her advantage. A possible direction for future research is to investigate, in such an environment, when and how some kind of turn-taking strategies may still be able to mitigate the coordination and conflict problems that are more difficult than the one considered in our benchmark model.

7 Appendix A: Proofs of Major Propositions

Proof of Proposition 1. We are going to prove that, first, the solution to the two conditions in (12) exists and, second, it is unique. We then show that this solution satisfies (8) and (9).

Existence. It suffices to prove the existence of symmetric TTIR strategies with

$p_1 = p_2 = p$ that satisfy (12). To apply well-known mathematical results, we extend the domain of p from $(0,1)$ to $[0,1]$. We define the continuous function

$$f(p) = \frac{V^H - s - \delta V(p)}{[V^H - s - \delta V(p)] + [V^L - t - \delta V(p)]} \tag{A1}$$

over $p \in [0,1]$, where $V(p) = V_i(p,p)$ is defined according to (10). It is easy to observe that the symmetric solution to (12) is a fixed point of the function $f(.)$ in (A1).

According to Lemma 1 and Lemma 2, when $t < l$, (8a) and (9a) hold for any $p \in (0,1)$. We conclude from (A1) that $0 < f(p) < 1$. Moreover, it is easy to show that $0 < f(0) < 1$ and $0 < f(1) < 1$. Therefore, $f(.)$ is a continuous function from the compact set $[0,1]$ to itself. Applying the Brouwer's Fixed Point Theorem, we conclude that this function has a fixed point. That is, there exists a $p \in [0,1]$ such that $f(p) = p$. Moreover, since $f(0) > 0$ and $f(1) < 1$, we conclude that $f(p) = p$ does not hold at $p = 0$ or $p = 1$, and the solution to (12) exists in the open interval $(0,1)$.

Uniqueness. We now establish that only one pair of (p_1^*, p_2^*) satisfies the conditions in (12). Note that (11) leads to

$$V_i^* = \frac{p_j^* t + \left(1 - p_j^*\right) V^H}{1 - \delta p_j^*} = \frac{p_j^* V^L + \left(1 - p_j^*\right) s}{1 - \delta \left(1 - p_j^*\right)}. \tag{A2}$$

Substituting (6) and (7) into the second equality of (A2) and simplifying give

$$a \left(p_j^*\right)^2 + b p_j^* + c = 0, \tag{A3}$$

where $j = 1, 2$ and a, b and c are given in (15) to (17).[19] From (15) to (17), we know that $c > 0$ but that a and b can be either positive, negative or zero.

Since (A3) is a quadratic equation in p_j^*, there are at most two real roots. Together with the existence result above, there must be either one or two p_j^* in the interval $(0,1)$.[20] From standard results for quadratic equations, we

---

[19] A common approach to show the uniqueness of equilibrium is to use the Contraction Mapping Theorem. We have, however, found some counterexamples for the mutual-tough case (such as $h = 160$, $l = 40$, $t = 80$, $s = 20$ and $\delta = 0.75$) that $f(p)$ in (A1) is not a contraction mapping. Hence, we use a different approach. One advantage of using the formulas for quadratic equations to prove the uniqueness of equilibrium is that we also obtain the closed-form solution for the equilibrium randomization probability. The closed-form solution, given by (13) or (14), forms the basis for quantitative welfare analysis related to game G_∞, such as those performed in Lau and Mui (2008).

[20] Note that the existence result above is an integral part of the proof of uniqueness,

know that if $a = 0$, then there is just one p_j^* and it is given by (13).[21] If $a \neq 0$, the two roots to (A3) are given by (14) and

$$p_j^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a}. \tag{A4}$$

Since at least one p_j^* lies in $(0, 1)$, either (14) or (A4), but not both, may lie outside the interval $(0, 1)$.

We can further show that (a) if $a < 0$, then p_j^* in (A4) is negative; (b) if $a > 0$ and $b > 0$, then p_j^* in (A4) is negative; (c) if $a > 0$ and $b < 0$, then p_j^* in (A4) is larger than 1 for all $\delta \in (0, 1)$; and (d) $b = 0$ is inconsistent with $a > 0$ and $b^2 - 4ac \geq 0$. Therefore, whether a in (15) is positive or negative, p_j^* in (A4) does not lie in the interval $(0, 1)$.

Consequently, there is only one solution to (12) that satisfies $0 < p_j^* < 1$, and the solution is given by (13) if $a = 0$ or (14) if $a \neq 0$. Moreover, $p_1^* = p_2^* = p^*$ since both of them are defined according to (13) or (14).

Finally, applying Lemma 1 and Lemma 2, it is easy to see that the unique solution to (12) satisfies the no-deviation conditions (8) and (9). We conclude that there is a unique TTIR equilibrium for G_∞ with $t < l$ at all $\delta \in (0, 1)$ and the TTIR equilibrium is symmetric. ■

Proof of Proposition 3. At the symmetric TTIR equilibrium, the second equality of (11) can be rewritten as

$$p^*(\theta)\left[V^L(\theta) - t - \delta V^*(p^*(\theta))\right] = [1 - p^*(\theta)]\left[V^H(\theta) - s - \delta V^*(p^*(\theta))\right], \tag{A5}$$

where the dependence of V^H on θ, V^L on θ, p^* on θ, and V^* on $p^*(\theta)$ are explicitly stated. It is clear from (6), (7), (10) and (24) that V^* depends on θ via $p^*(\theta)$ only.

---

since by showing that at least one pair of TTIR strategies satisfies (12), we conclude that at least one p_j^*, defined by (A3), is a real root and the term $(b^2 - 4ac)$ is non-negative. Note also that if there are two distinct p_j^* $(j = 1, 2)$ in the interval $(0, 1)$ satisfying (A3), say, p_j^h and p_j^l with $p_j^h > p_j^l$, then there exist four TTIR equilibria. Two of these equilibria, (p_1^h, p_2^h) and (p_1^l, p_2^l), are symmetric. The other two equilibria, (p_1^h, p_2^l) and (p_1^l, p_2^h), are asymmetric. We now show that for $j = 1, 2$, there exists only one p_j^* in the interval $(0, 1)$ that satisfies (A3), and that $p_1^* = p_2^*$.

[21] The pure coordination game in Crawford and Haller (1990) can be represented as $s = t$ and $h = l$ (and, thus, $t < l$, because of (5)), using the notation in Table 1. In this case, the no-deviation conditions (8) and (9) become the same, and it can be shown that our results (which holds for $h > l$) will also be applicable when $h = l$. It can further be shown that a in (15) is 0 for this game. Thus, for all $\delta \in (0, 1)$, $p^* = 0.5$ according to (13).

Differentiating (A5) with respect to θ, and simplifying, we obtain

$$\frac{\partial p^*}{\partial \theta} = \frac{\frac{\partial V^H}{\partial \theta} - p^* \left[\frac{\partial \left(V^H + V^L\right)}{\partial \theta}\right]}{\left(V^H - s - \delta V^*\right) + \left(V^L - t - \delta V^*\right) + \delta \left(1 - 2p^*\right) \frac{\partial V^*}{\partial p^*}}. \tag{A6}$$

The sign of various components of (A6) are obtained as follows. We know from (3), (5), (6), (7), (24), (25), and the first equality of (A2) when $p_1^* = p_2^* = p^*$ and $V_1^* = V_2^* = V^*$ that

$$\frac{\partial V^*}{\partial p^*} = \frac{(1+\delta)\, t - (h + \delta l)}{\left(1 - \delta p^*\right)^2 (1+\delta)} = \frac{(1+\delta)\, t - \left[(1+\delta)\left(\frac{h+l}{2}\right) + (1-\delta)\left(\frac{h-l}{2}\right)\right]}{\left(1 - \delta p^*\right)^2 (1+\delta)} < 0, \tag{A7}$$

$$\frac{\partial V^H}{\partial \theta} = \frac{\lambda}{(1+\delta)(1+\theta)^2} > 0, \tag{A8}$$

and

$$\frac{\partial \left(V^H + V^L\right)}{\partial \theta} = 0. \tag{A9}$$

Moreover, (8) and (9) hold for $t < l$ and $\delta \in (0, 1)$ or for $t > l$ and $\delta \in (\delta_{TT}, 1)$. When $0.5 < p^* < 1$, both the denominator and numerator of (A6) are positive. Thus, $0.5 < p^* < 1$ is a sufficient condition for

$$\frac{\partial p^*}{\partial \theta} > 0. \tag{A10}$$

Moreover, since $2p^* \left(1 - p^*\right)$ is decreasing in p^* when $0.5 < p^* < 1$, we conclude from (23) and (A10) that $0.5 < p^* < 1$ is a sufficient condition for

$$\frac{\partial E(D)}{\partial \theta} = \frac{\partial E(D)}{\partial p^*} \times \frac{\partial p^*}{\partial \theta} \tag{A11}$$

to be positive.

Since $0.5 < p^* < 1$ is satisfied for game G_∞ with $t > l$ and $\delta \in (\delta_{TT}, 1)$ according to Lemma 3, this proves part (a).

Since $V^H > V^L$, it is easy to see from (A5) that $V^H - s - \delta V^* > V^L - t - \delta V^*$ when $t \geq s$. Consequently, $p^* > 1 - p^*$ and thus, $0.5 < p^* < 1$. This proves part (b). ■

8 Appendix B: Other Proofs

The following result is useful in subsequent analysis. From (3) and $\delta \in (0,1)$, we have

$$\begin{aligned} h+\delta l &= (1+\delta)\left(\frac{h+l}{2}\right)+(1-\delta)\left(\frac{h-l}{2}\right) \\ &> (1+\delta)\left(\frac{h+l}{2}\right). \end{aligned} \tag{B1}$$

Proof of Lemma 1. We obtain three preliminary results useful for the proof of Lemma 1. First, define

$$U(p)=p^2(t)+(1-p)^2(s)+p(1-p)(h+l) \tag{B2}$$

as a player's current-period payoff when both players choose Tough with probability p. Using (B2), it is straightforward to obtain

$$\begin{aligned} \frac{h+l}{2}-U(p) &= \left[p^2+(1-p)^2+2p(1-p)\right]\left(\frac{h+l}{2}\right)-U(p) \\ &= p^2\left(\frac{h+l}{2}-t\right)+(1-p)^2\left(\frac{h+l}{2}-s\right) \\ &> 0. \end{aligned} \tag{B3}$$

Second, from Table 3 and (B2), we have

$$\begin{aligned} &V_i(p,p) \\ &= p^2\left[t+\delta V_i(p,p)\right]+(1-p)^2\left[s+\delta V_i(p,p)\right]+p(1-p)\left(h+\delta V^L\right)+(1-p)p\left(l+\delta V^H\right) \\ &= U(p)+\delta\left\{\left[p^2+(1-p)^2\right]V_i(p,p)+p(1-p)\left(V^H+V^L\right)\right\}. \end{aligned} \tag{B4}$$

Therefore, we obtain

$$V_i(p,p)=\frac{U(p)+\delta p(1-p)\left(V^H+V^L\right)}{1-\delta\left[p^2+(1-p)^2\right]}. \tag{B5}$$

From (6), (7), (B3) and (B5), we have

$$\begin{aligned} \frac{V^H+V^L}{2}-V_i(p,p) &= \frac{\left\{1-\delta\left[p^2+(1-p)^2\right]\right\}\left(\frac{V^H+V^L}{2}\right)-\left[U(p)+\delta p(1-p)\left(V^H+V^L\right)\right]}{1-\delta\left[p^2+(1-p)^2\right]} \\ &= \frac{\left\{1-\delta\left[1-2p(1-p)\right]\right\}\left(\frac{V^H+V^L}{2}\right)-\left[U(p)+\delta p(1-p)\left(V^H+V^L\right)\right]}{1-\delta\left[p^2+(1-p)^2\right]} \end{aligned}$$

$$= \frac{(1-\delta)\left(\frac{V^H+V^L}{2}\right)+2\delta p(1-p)\left(\frac{V^H+V^L}{2}\right)-\left[U(p)+\delta p(1-p)\left(V^H+V^L\right)\right]}{1-\delta\left[p^2+(1-p)^2\right]}$$

$$= \frac{\frac{h+l}{2}-U(p)}{1-\delta\left[p^2+(1-p)^2\right]}$$

$$> 0. \tag{B6}$$

Third, $h > l$ implies $V^H > V^L$. This inequality, together with (B6), leads to

$$V^H > \frac{V^H+V^L}{2} > V_i(p,p). \tag{B7}$$

We now prove Lemma 1. Using (6) and (7), we have

$$V^H = h + \delta l + \delta^2 V^H. \tag{B8}$$

From (B4), we have

$$s+\delta V_i(p,p) = s+\delta U(p)+\delta^2\left\{\left[p^2+(1-p)^2\right]V_i(p,p)+p(1-p)\left(V^H+V^L\right)\right\}. \tag{B9}$$

We prove Lemma 1 by showing that (a) the coefficient of δ^2 in (B8) is larger than that in (B9), and (b) $h+\delta l$ in (B8) is larger than $s+\delta U(p)$ in (B9). First, (B7) implies that

$$V^H = \left[p^2+(1-p)^2+2p(1-p)\right]V^H$$

$$> \left[p^2+(1-p)^2\right]V_i(p,p)+p(1-p)\left(V^H+V^L\right).$$

Second, (4), (B1) and (B3) imply that

$$h+\delta l-[s+\delta U(p)] > (1+\delta)\left(\frac{h+l}{2}\right)-[s+\delta U(p)]$$

$$= \left(\frac{h+l}{2}-s\right)+\delta\left[\frac{h+l}{2}-U(p)\right]$$

$$> 0.$$

This proves Lemma 1. ■

Proof of Lemma 2. For the accommodating case, $l > t$. According to (B7), $V^H > V_i(p,p)$. Combining these results, we obtain (9a). ■

Proof of Lemma 3. The following analysis applies to the symmetric TTIR equilibrium with $p_1^* = p_2^* = p^*$ and $V_1^* = V_2^* = V^*$. From (8), (9), and the first equality of (12), we have

$$1-p^* = \frac{V^L-t-\delta V^*}{(V^H-s-\delta V^*)+(V^L-t-\delta V^*)} > 0. \tag{B10}$$

Also, (6), (7) and the first equality of (12) imply

$$p^* - 0.5 = \frac{h - l - (1+\delta)(s-t)}{2(1+\delta)\left[\left(V^H - s - \delta V^*\right) + \left(V^L - t - \delta V^*\right)\right]}. \tag{B11}$$

It is easy to see that the denominator of (B11) is positive. For the numerator, either $t - s \geq 0$ or $t - s < 0$ can be consistent with $t > l$. If $t - s \geq 0$, it is easy to conclude from (3) that the numerator of (B11) is positive. If $t - s < 0$, then (4), $t > l$ and $0 < \delta < 1$ imply

$$\begin{aligned} h - l - (1+\delta)(s-t) &> h - l - 2(s-t) \\ &= 2\left[\left(\frac{h+l}{2} - s\right) + t - l\right] > 0. \end{aligned} \tag{B12}$$

Combining (B10), (B11), and (B12), we have (18).

From the second equality of (11), we have

$$p^*(\delta)\left[V^L(\delta) - t - \delta V^*(\delta, p^*(\delta))\right] = [1 - p^*(\delta)]\left[V^H(\delta) - s - \delta V^*(\delta, p^*(\delta))\right],$$

where the dependence of p^* on δ, and V^* on δ and $p^*(\delta)$ are written explicitly. Differentiating the above expression with respect to δ, and rearranging, gives (19).

Consider the denominator of (19). Combining (18) and (A7), the third term in the denominator of (19) is positive. The first two terms in the denominator of (19) are positive when (8) and (9) hold. Therefore, we have

$$\left(V^H - s - \delta V^*\right) + \left(V^L - t - \delta V^*\right) + \delta(1 - 2p^*)\frac{\partial V^*}{\partial p^*} > 0. \tag{B13}$$

Consider the numerator of (19). First, (18) implies

$$p^* > 1 - p^* > 0. \tag{B14}$$

Second, using (6) and (7), we have

$$\begin{aligned} &\left(\frac{\partial V^L}{\partial \delta} - \delta\frac{\partial V^*}{\partial \delta} - V^*\right) - \left(\frac{\partial V^H}{\partial \delta} - \delta\frac{\partial V^*}{\partial \delta} - V^*\right) \\ &= \frac{\partial V^L}{\partial \delta} - \frac{\partial V^H}{\partial \delta} \\ &= \frac{-\partial\left(V^H - V^L\right)}{\partial \delta} \\ &= \frac{h - l}{(1+\delta)^2} > 0. \end{aligned} \tag{B15}$$

Third, using (B6), we have

$$\frac{\partial}{\partial\delta}\left\{\delta\left[\frac{V^H+V^L}{2}-V^*\left(\delta,p^*\left(\delta\right)\right)\right]\right\}=\frac{\partial}{\partial\delta}\left\{\frac{\delta\left[\frac{h+l}{2}-U\left(p^*\right)\right]}{1-\delta\left[\left(p^*\right)^2+\left(1-p^*\right)^2\right]}\right\}$$

$$=\frac{\left[\frac{h+l}{2}-U\left(p^*\right)\right]}{\left\{1-\delta\left[\left(p^*\right)^2+\left(1-p^*\right)^2\right]\right\}^2}>0. \tag{B16}$$

Fourth, (7) implies $\frac{\partial V^L}{\partial\delta}=\frac{\partial\left(\delta V^H\right)}{\partial\delta}=\delta\frac{\partial V^H}{\partial\delta}+V^H$, and (6) implies $\frac{\partial V^H}{\partial\delta}=\frac{\partial\left(\delta V^L\right)}{\partial\delta}=\delta\frac{\partial V^L}{\partial\delta}+V^L$. Using these relationships, (B15) and (B16), we have

$$\begin{aligned}\frac{\partial V^L}{\partial\delta}-\delta\frac{\partial V^*}{\partial\delta}-V^*&=\frac{\partial}{\partial\delta}\left[\delta\left(V^H-V^*\right)\right]\\&=\frac{\partial}{\partial\delta}\left[\delta\left(\frac{V^H+V^L}{2}-V^*\right)\right]+\frac{\partial}{\partial\delta}\left[\delta\left(\frac{V^H-V^L}{2}\right)\right]\\&>\frac{\partial}{\partial\delta}\left[\delta\left(\frac{V^H-V^L}{2}\right)\right]\\&=\frac{1}{2}\left[\frac{\partial\left(\delta V^H\right)}{\partial\delta}-\frac{\partial\left(\delta V^L\right)}{\partial\delta}\right]\\&=\frac{1}{2}\left(\frac{\partial V^L}{\partial\delta}-\frac{\partial V^H}{\partial\delta}\right)>0.\end{aligned}\tag{B17}$$

Therefore, (B14), (B15), and (B17) imply that

$$\begin{aligned}&p^*\left(\frac{\partial V^L}{\partial\delta}-\delta\frac{\partial V^*}{\partial\delta}-V^*\right)-\left(1-p^*\right)\left(\frac{\partial V^H}{\partial\delta}-\delta\frac{\partial V^*}{\partial\delta}-V^*\right)\\&>\left(1-p^*\right)\left(\frac{\partial V^L}{\partial\delta}-\delta\frac{\partial V^*}{\partial\delta}-V^*\right)-\left(1-p^*\right)\left(\frac{\partial V^H}{\partial\delta}-\delta\frac{\partial V^*}{\partial\delta}-V^*\right)>0.\end{aligned}\tag{B18}$$

That is, the numerator of (19) is negative. Combining (B13) and (B18) gives Lemma 3. ■

Proof of Proposition 2.[22] To show the existence of $p^*\in(0,1)$ satisfying (12) for the mutual-tough case, we define the following continuous function

$$g\left(p\right)=\frac{V^H-s-\delta V\left(p\right)}{\left[V^H-s-\delta V\left(p\right)\right]+\left[V^L-t-\delta V\left(p\right)\right]}-p \tag{B19}$$

---

[22] An important feature that distinguishes the proof of Proposition 2 from that of Proposition 1 is that condition (9a) does not hold for all $p\in(0,1)$ for the mutual-tough case, but it does hold for $p=p^*$ when $\delta>\delta_{TT}$.

over $p \in [0,1]$, where $V(p) = V_i(p,p)$ according to (10). It is easy to observe that $g(.)$ is a well-defined function for every $p \in [0,1]$, and the solution to (12) is defined by $g(p) = 0$.

Using (B5) and (4) to (7), we can show that for all $\delta \in (\delta_{TT}, 1)$,

$$g(0) = \frac{V^H - s - \delta V(0)}{[V^H - s - \delta V(0)] + [V^L - t - \delta V(0)]} - 0$$

$$= \frac{V^H - s - \delta\left(\frac{s}{1-\delta}\right)}{V^H + V^L - s - t - 2\delta\left(\frac{s}{1-\delta}\right)} > 0, \tag{B19a}$$

and

$$g(1) = \frac{V^H - s - \delta V(1)}{[V^H - s - \delta V(1)] + [V^L - t - \delta V(1)]} - 1$$

$$= \frac{-\left[V^L - t - \delta\left(\frac{t}{1-\delta}\right)\right]}{V^H + V^L - s - t - 2\delta\left(\frac{t}{1-\delta}\right)} < 0. \tag{B19b}$$

Applying the Intermediate Value Theorem, we know that there exists a $p \in [0,1]$ such that $g(p) = 0$.[23] Moreover, $g(p) = 0$ does not hold at $p = 0$ or $p = 1$, as observed in (B19a) and (B19b). Therefore, we conclude that the solution to (12) exists in the interval $(0,1)$.

The proof of the uniqueness of the solution to (12) for the mutual-tough case is also similar to that of Proposition 1, except that the analysis holds only for $\delta \in (\delta_{TT}, 1)$ when $t > l$, whereas it holds for all $\delta \in (0,1)$ when $t < l$. ■

References

[1] Aoyagi, M. (2003), "Bid Rotation and Collusion in Repeated Auctions," *Journal of Economic Theory*, 112, 79-105.

[2] Athey, S. and K. Bagwell (2001), "Optimal Collusion with Private Information," *Rand Journal of Economics*, 32, 428-465.

[3] Athey, S. and K. Bagwell (2008), "Collusion with Persistent Cost Shocks," *Econometrica*, 76, 493-540.

[4] Aumann, R. J. (1974), "Subjectivity and Correlation in Randomized Strategies," *Journal of Mathematical Economics*, 1, 67-96.

---

[23] We use the Intermediate Value Theorem instead of the Fixed Point Theorem because $f(p)$, as defined in (A1), is not necessarily a mapping from $[0,1]$ to $[0,1]$ for the mutual-tough case.

[5] Bergerud, E. M. (1993), *Red Thunder, Tropic Lightning: The World of a Combat Division in Vietnam.* Boulder: Westview Press, Inc.

[6] Berkes, F. (1992), "Success and Failure in Marine Coastal Fisheries of Turkey," in D. W. Bromley (ed.), *Making the Commons Work: Theory, Practice, and Policy*, 161-182. San Francisco: Institute for Contemporary Studies.

[7] Bhaskar, V. (2000), "Egalitarianism and Efficiency in Repeated Symmetric Games," *Games and Economic Behavior*, 32, 247-262.

[8] Bornstein, G., D. Budescu and S. Zamir (1997), "Cooperation in Intergroup, N-person, and Two-Person Games of Chicken," *Journal of Conflict Resolution*, 41, 384-406.

[9] Camerer, C. (2003), *Behavioral Game Theory: Experiments in Strategic Interaction.* Princeton: Princeton University Press.

[10] Cooper, R., D. V. DeJong, R. Forsythe and T. W. Ross (1989), "Communication in the Battle of the Sexes Game: Some Experimental Results," *Rand Journal of Economics*, 20, 568-587.

[11] Crawford, V. P. and H. Haller (1990), "Learning How to Cooperate: Optimal Play in Repeated Coordination Games," *Econometrica*, 58, 571-595.

[12] Dixit, A. and S. Skeath (1999), *Games of Strategy*, New York: W. W. Norton & Company.

[13] Farrell, J. (1987), "Cheap Talk, Coordination, and Entry," *Rand Journal of Economics*, 18, 34-39.

[14] Friedman, J. W. (1971), "A Non-Cooperative Equilibrium for Supergames," *Review of Economic Studies*, 38, 1-12.

[15] Friedman, J. W. (1994), "Introduction and Overview," in J. W. Friedman (ed.), *Problems of Coordination in Economic Activity*, 3-15. Boston: Kluwer Academic Publishers.

[16] Fudenberg, D. and E. Maskin (1986), "The Folk Theorem in Repeated Games with Discounting or with Incomplete Information," *Econometrica*, 54, 533-554.

[17] Harrison G. and J. Hirshleifer (1989), "An Experimental Evaluation of Weakest Link/Best Shot Models of Public Goods," *Journal of Political Economy*, 97, 201-225.

[18] Helbing, D., M. Schönhof, H-U. Stark and J. A. Holyst (2005), "How Individuals Leanr to Take Turns: Emergence of Alternating Cooperation in a Congestion Game and the Prisoner's Dilemma," *Advances in Complex Systems*, 8, 87-116.

[19] Kaplan, T. R. and B. J. Ruffle (2007), "Which Way to Cooperate?" Working paper, University of Exeter and Ben-Gurion University.

[20] Lau, S.-H. P. (2001), "Aggregate Pattern of Time-Dependent Adjustment Rules, II: Strategic Complementarity and Endogenous Nonsynchronization," *Journal of Economic Theory*, 98, 199-231.

[21] Lau, S.-H. P. and V.-L. Mui (2008), "Using Turn Taking to Mitigate Coordination and Conflict Problems in the Repeated Battle of the Sexes Game," *Theory and Decision,* 65, 153-183.

[22] Luce, R. D. and H. Raiffa (1957), *Games and Decisions: Introduction and Critical Survey*, New York: Wiley.

[23] McAfee, R. P. and J. McMillan (1992), "Bidding Rings," *American Economic Review*, 82, 579-599.

[24] Ostrom, E. (1990), *Governing the Commons: The Evolution of Institutions for Collective Action*, New York: Cambridge University Press.

[25] Ostrom, E., R. Gardner and J. Walker (with A. Agrawal, W. Blomquist, E. Schlager, and S. Y. Tang), (1994), *Rules, Games, and Common-Pool Resources*, Ann Arbor: University of Michigan Press.

[26] Prisbrey, J., (1992), "Cooperation in Reciprocity Games and in the Voluntary Contributions Mechanism," Ph.D. Dissertation, California Institute of Technology.

[27] Rapoport, A., M. J. Guyer and D. G. Gordon (1976), *The 2×2 Game*. Ann Arbor: University of Michigan Press.

[28] Schelling, T. C. (1966), *Arms and Influence*, New Haven: Yale University Press.

[29] Skrzypacz, A. and H. Hopenhayn (2004), "Tacit Collusion in Repeated Auctions," *Journal of Economic Theory*, 114, 153-169.

[30] Ward, H. (1998), "A Game Theoretic Analysis of the Politics of Taking It in Turns," *British Journal of Political Science*, 28, 355-387.

Table 1: Strategies and Payoffs of the Stage Game

(a) The h and l Specification

1\2	Tough	Soft
Tough	(t,t)	(h,l)
Soft	(l,h)	(s,s)

(b) The λ and θ Specification

1\2	Tough	Soft
Tough	(t,t)	$\left(\frac{\theta\lambda}{1+\theta},\frac{\lambda}{1+\theta}\right)$
Soft	$\left(\frac{\lambda}{1+\theta},\frac{\theta\lambda}{1+\theta}\right)$	(s,s)

Table 2: Symmetric Two-By-Two Games

Parameter Restrictions	Pure-Strategy Equilibrium or Equilibria	Joint-Payoff-Maximizing Outcome(s)	Examples of Symmetric Games with Asymmetric Joint-Payoff-Maximizing Outcomes
$h > l > s > t$ (case 1)	(S,T); (T,S)	(S,T); (T,S)	1: market entry game (Farrell, 1987).
$h > l > t > s$ (case 2)	(S,T); (T,S)	(S,T); (T,S)	2: game of CPR assignment with $h < 2l$ (Ostrom et al., 1994, Figure 3.5b). 2' ($h > l > t = s$): battle of the sexes (Cooper at al., 1989, Figure 1).
$h > s > l > t$ (case 3)			
3a: With $h + l > 2s$	(S,T); (T,S)	(S,T); (T,S)	3a: game of chicken (Dixit and Skeath, 2004, p. 374, Q. 5 with $k > 1$). 3a' ($h > s = l > t$): best-shot public goods game (Harrison and Hirshleifer, 1989; Bornstein et al., 1997).
3b: With $h + l < 2s$	(S,T); (T,S)	(S,S)	
$h > s > t > l$ (case 4)			
4a: With $h + l > 2s$	(T,T)	(S,T); (T,S)	4a: non-standard prisoners' dilemma (Dixit and Skeath, 2004, Figure 12.2).
4b: With $h + l < 2s$	(T,T)	(S,S)	
$h > t > l > s$ (case 5)			
5a: With $h + l > 2t$	(T,T)	(S,T); (T,S)	5a: game of CPR assignment with $h > 2l$ (Ostrom et al., 1994, Figure 3.5d); traffic route choice game (Helbing et al., 2005). 5a' ($h > t > l = s$): experimental game used in Prisbrey (1992).
5b: With $h + l < 2t$	(T,T)	(T,T)	
$h > t > s > l$ (case 6)			
6a: With $h + l > 2t$	(T,T)	(S,T); (T,S)	6a' ($h > t > s = l$): CPR appropriation externality game (Ostrom et al., 1994, Figure 3.4c).
6b: With $h + l < 2t$	(T,T)	(T,T)	
$s > h > l > t$ (case 7)	(S,S)	(S,S)	
$s > h > t > l$ (case 8)	(S,S); (T,T)	(S,S)	
$s > t > h > l$ (case 9)	(S,S); (T,T)	(S,S)	
$t > h > l > s$ (case 10)	(T,T)	(T,T)	
$t > h > s > l$ (case 11)	(T,T)	(T,T)	
$t > s > h > l$ (case 12)	(S,S); (T,T)	(T,T)	

Notes:

(1) In all 12 cases, $h > l$.

(2) Games with equality of some payoff parameters can be included as special cases in the above classification, if they also satisfy the conditions in columns 2 and 3. These special cases are denoted by a prime affixed to the case under consideration. For example, 1' is a special case of 1.

Table 3: Strategies and Intertemporal Payoffs at the Beginning of the Repeated Game

1\2	**Playing Tough at Period 0**	**Playing Soft at Period 0**
Playing Tough at Period 0	$\left(t+\delta V_1^*, t+\delta V_2^*\right)$	$\left(V^H, V^L\right)$
Playing Soft at Period 0	$\left(V^L, V^H\right)$	$\left(s+\delta V_1^*, s+\delta V_2^*\right)$

Table 4: Two Games of Chicken which Only Differ in the Value of θ

(a) A game of chicken with $s=90, t=10, \lambda=200$, and $\theta=4$

1\2	**Tough**	**Soft**
Tough	$(10,10)$	$(160,40)$
Soft	$(40,160)$	$(90,90)$

(b) A game of chicken with $s=90, t=10, \lambda=200$, and $\theta=9$

1\2	**Tough**	**Soft**
Tough	$(10,10)$	$(180,20)$
Soft	$(20,180)$	$(90,90)$

Figure 1: p* as a function of δ (for the mutual−tough case)

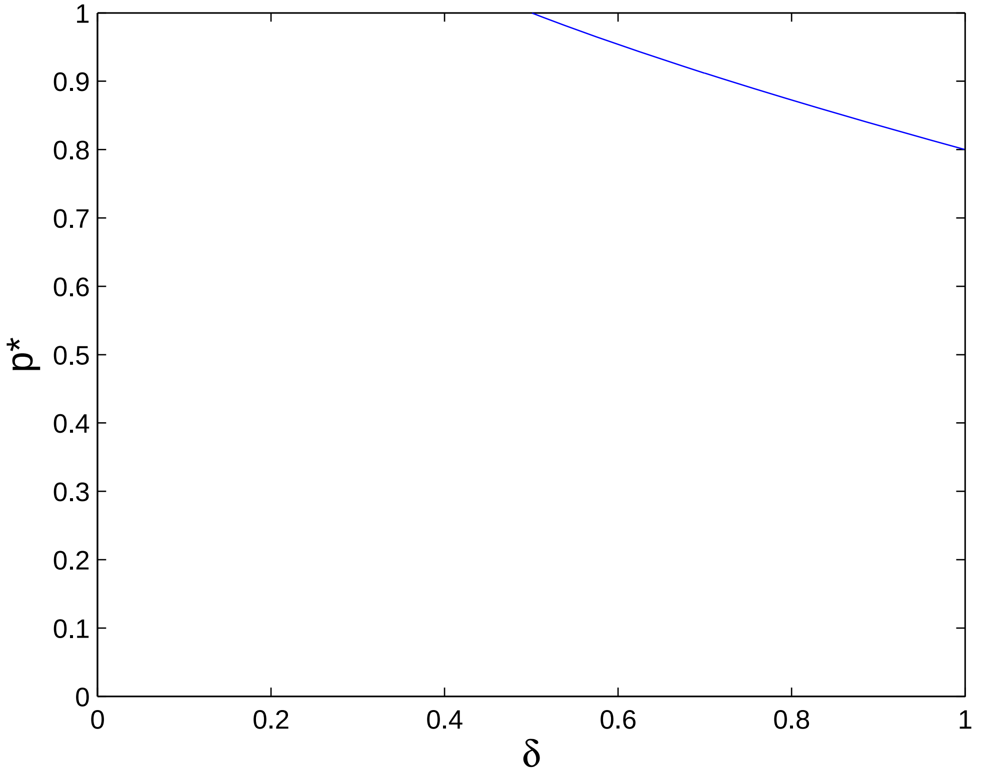

Figure 2: p* as a function of θ (for games of chicken)

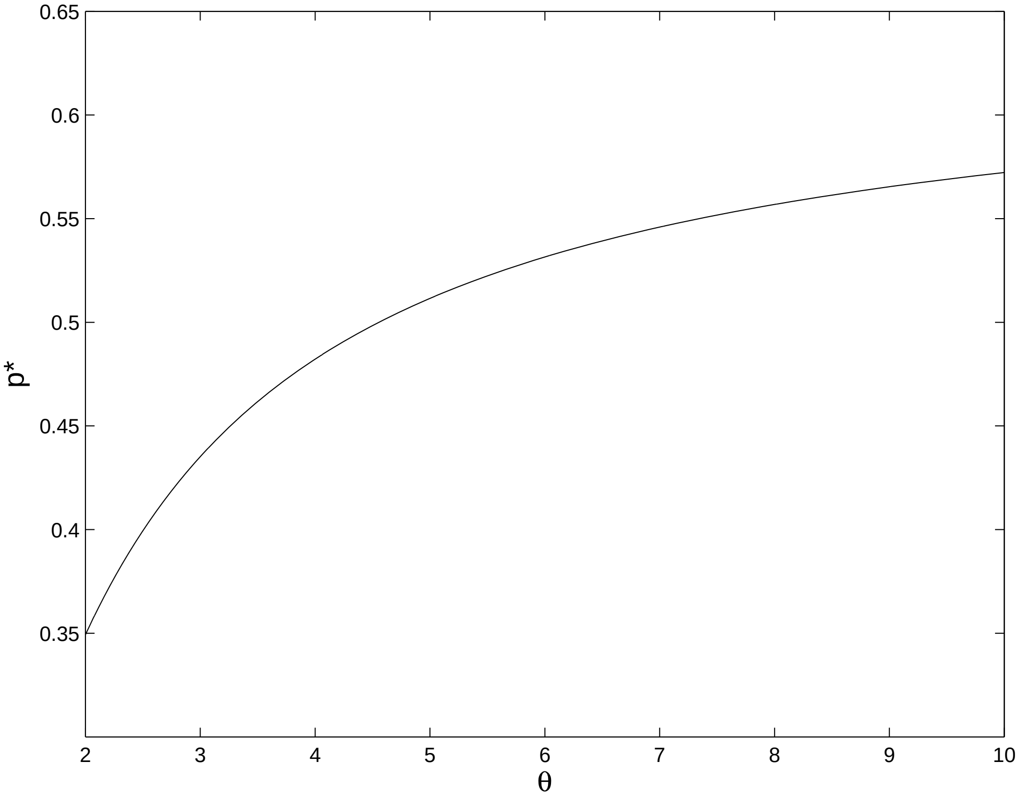

Figure 3: Expected value of delay as a function of θ (for games of chicken)