Monash University
Semester One 2008 Demonstration Exam
Faculty of Business and Economics

EXAM CODES:	ECC5650
TITLE OF PAPER:	MICRO-ECONOMIC THEORY
EXAM DURATION:	3 hours
READING TIME:	15 minutes

THIS PAPER IS FOR STUDENTS STUDYING AT: Clayton

Conditions of this Examination

- During an examination, you must not have in your possession: a book, notes, paper, calculater, pencil case, mobile phone or other material/item which has not been authorised for the examination or specifically permitted as noted below;
- Any matieral or item on your desk, chair or person will be deemed to be in your possession;
- You are reminded that possession of unauthorised materials in an exam is a discipline offence under Monash Statute 4.1.

Authorised Materials: NONE

CALCULATOR	NONE
OPEN BOOKS	NONE
OTHER SPECIFICALLY PERMITTED ITEMS	NONE

Instructions to Candidates

- There are three sections in this examination: A, B and C;
- Each section is not worth the same amount of marks;
- Marks for each section are given in the paper;
- You must answer each section;
- In section A you must answer all questions;
- In section B and C you do not have to answer all questions;
- Indicate the question you are attempting in the answer booklet. Your working should be consice, logical and clear.

Section A

You must answer all questions in this section. This section is worth 15 marks.

1. In standard Consumer Theory, state and explain the meaning of the following axioms:
   (a) Completeness;
   (b) Local nonsatiation.
2. Sketch the following situations to illustrate that the conditions of Brouwer's Fixed-Point theorem are sufficient, but not necessary.
   (a) S is compact, S is convex, f is *not* continuous, and a fixed point of f exists.
   (b) S is compact, S is *not* convex, f is continuous, and a fixed point of f exists.
3. *Without* appealing to the method of Calculus, provide a proof for the claim:

   "The function $f : \mathbb{R} \to \mathbb{R}$, such that, $f(x) = \log x^{\beta}$ is a concave function."

Section B

Answer 3 out of the following 4 questions. Each question is worth equal value. This section is worth 30 marks.

1. In the Theory of Consumption under Uncertainty, state and explain the meaning of the following axioms:
   (a) Completeness;
   (b) Continuity;
   (c) Substition;
2. Consider the utility maximization problem:

$$\max_{x_1, x_2} u(x_1, x_2) = x_1 + x_2 \qquad \text{s.t.} \quad x_1^{\alpha} + x_2^{\alpha} = b \quad .$$

   (a) Appeal to the Envelope Theorem to find $\frac{\delta v(x_1, x_2)}{\delta b}$.
   (b) Obtain an expression for $v(x_1, x_2)$ and so verify your answer given in (a).
3. For the following minimization problem, you may assume (i.e. without proof) that the optimal solution is $x_1^* = 28/13$ and $x_2^* = 36/13$. Show that the optimal

solution does indeed satisfy the Kuhn-Tucker conditions for a minimum.

$$\begin{array}{rl} \min\limits_{x_1,x_2} & C = \sum\limits_{i=1}^{2}(x_i-4)^2 \\ \text{subject to} & 2x_1+3x_2 \geq 6 \\ & -3x_1-2x_2 \geq -12 \\ \text{and} & x_i > 0\,, \quad i=1,2 \end{array}$$

4. Consider the compensated, or *Hicksian* demand function $x_i^h = x_i(\mathbf{p}, e(\mathbf{p}, \bar{u}))$.

   (a) Show that the price effect on compensated demand is given by,

   $$\frac{\delta e}{\delta p_i}(\mathbf{p}, \bar{u}) = x_i^*(\mathbf{p}, \bar{u})\ ,$$

   (b) And hence, provide a proof for the *Slutsky Equation*;

   (c) Write brief notes on the two components of the *Slutsky Equation.*

Section C

Answer 1 out of the following 2 questions. Each question is worth equal value. This section is worth 15 marks.

1. Suppose that a preference relation on $X \subset \mathbb{R}_+^m$ is complete, reflexive, transitive, continuous and strongly monotonic. Assume also that a utility function $u : \mathbb{R}_+^m \to \mathbb{R}$ has been shown to uniquely exist. Complete the existence proof for the function $u(x)$ by showing:

   (a) That $u(x)$ actually represents the preference relation; and

   (b) That $u(x)$ is a continuous function.

2. Consider the *Stone-Geary* Utility function,

   $$u(\mathbf{x}) = \prod_{i=1}^{n}(x_i - a_i)^{b_i}$$

   where $b_i \geq 0$ and $\sum_{i=1}^{n} b_i = 1$, and each $a_i \geq 0$ can be interpreted as the 'subsistence' level of consumption of the commodities in $\mathbf{x}$.

   Derive:

   (a) The *Marshallian demand functions*, and so

   (b) The *indirect utility function*, and hence,

   (c) Show that $v(x)$ is proportional to $y - \sum_{i=1}^{n} a_i p_i$ (the level of 'discretionary income').

Equation Sheet

Roy's Identity $x_i^* = -\frac{\delta v}{\delta p_i} \bigg/ \frac{\delta v}{\delta y}$

Shephard's Lemma $x_i^h(\mathbf{p}, \bar{u}) = \frac{\delta e(\mathbf{p},\bar{u})}{\delta p_i}$

Slutsky Equation $\frac{\delta x_i(\mathbf{p},y)}{\delta p_j} = \frac{\delta x_i^h(\mathbf{p},\bar{u})}{\delta p_j} - \frac{\delta x_i(\mathbf{p},y)}{\delta y} x_j(\mathbf{p}, \bar{u})$

Elasticity Relations

$$\eta_i = \frac{\delta x_i(\mathbf{p},y)}{\delta y} \frac{y}{x_i(\mathbf{p},y)}$$

$$\epsilon_{ij} = \frac{\delta x_i(\mathbf{p},y)}{\delta p_j} \frac{p_j}{x_i(\mathbf{p},y)}$$

$$s_i = \frac{p_i x_i(\mathbf{p},y)}{y}$$