Monash University
Semester One 2008 Demonstration Exam
Faculty of Business and Economics

EXAM CODES:	ECC5650
TITLE OF PAPER:	MICRO-ECONOMIC THEORY
EXAM DURATION:	3 hours
READING TIME:	15 minutes

THIS PAPER IS FOR STUDENTS STUDYING AT: Clayton

Conditions of this Examination

- During an examination, you must not have in your possession: a book, notes, paper, calculater, pencil case, mobile phone or other material/item which has not been authorised for the examination or specifically permitted as noted below;
- Any matieral or item on your desk, chair or person will be deemed to be in your possession;
- You are reminded that possession of unauthorised materials in an exam is a discipline offence under Monash Statute 4.1.

Authorised Materials: NONE

CALCULATOR	NONE
OPEN BOOKS	NONE
OTHER SPECIFICALLY PERMITTED ITEMS	NONE

Instructions to Candidates

- There are three sections in this examination: A, B and C;
- Each section is not worth the same amount of marks;
- Marks for each section are given in the paper;
- You must answer each section;
- In section A you must answer all questions;
- In section B and C you do not have to answer all questions;
- Indicate the question you are attempting in the answer booklet. Your working should be consice, logical and clear.

Section A

You must answer all questions in this section. This section is worth 15 marks.

1. In standard Consumer Theory, state and explain the meaning of the following axioms:

   (a) Completeness;

   > For $x, y \in X$ either xRy or yRx. That is, for any pair of bundles in X a preference comparison can be made.

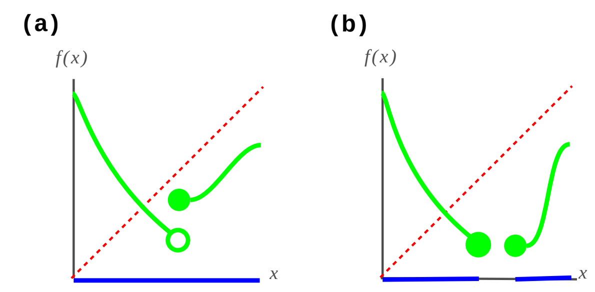
   (b) Local nonsatiation.

   > Given $x \in X$ and $\epsilon > 0$, then $\exists y : |x - y| < \epsilon$ such that $y \succ x$. That is, for any bundle x there is always a strictly preferred bundle near to x.

2. Sketch the following situations to illustrate that the conditions of Brouwer's Fixed-Point theorem are sufficient, but not necessary.

   (a) S is compact, S is convex, f is *not* continuous, and a fixed point of f exists.

   (b) S is compact, S is *not* convex, f is continuous, and a fixed point of f exists.



3. *Without* appealing to the method of Calculus, provide a proof for the claim:

   "The function $f : \mathbb{R} \to \mathbb{R}$, such that, $f(x) = \log x^{\beta}$ is a concave function."

For concavity: $f(x^t) \geq tf(x_0) + (1-t)f(x_1)$, where $t \in [0,1]$ and $x_1 \geq x_0$, $x^t = tx_0 + (1-t)x_1$. Now, LHS $f(x^t) = \log[tx_0 + (1-t)x_1]^\beta$ and RHS .. $\log(x_0^t x_1^{1-t})^\beta$ leading to:

$$\begin{aligned} tx_o + (1-t)x_1 &\geq x_0^t x_1^{1-t} \\ t + (1-t)\frac{x_1}{x_0} &\geq \left(\frac{x_1}{x_0}\right)^{1-t} \end{aligned}$$

But since $x_1 \geq x_0$ we have $\left(\frac{x_1}{x_0}\right)^{1-t} \geq 1$ so,

$$\frac{x_1}{x_0} \geq 1 \quad .$$

Section B

Answer 3 out of the following 4 questions. Each question is worth equal value. This section is worth 30 marks.

1. In the Theory of Consumption under Uncertainty, state and explain the meaning of the following axioms:

   (a) Completeness;

   For any $g, g' \in \mathcal{G}$, either gRg' or $g'Rg$. That is, any two gambles in $\mathcal{G}$ can be ordered.

   (b) Continuity;

   For any gamble $g \in \mathcal{G}$, there is some probability, $\alpha \in [0,1]$, such that $g \sim (\alpha \cdot a_1, (1-\alpha) \cdot a_n)$. That is, any gamble in $\mathcal{G}$ can be represented by an alternate, two event gamble with some unique value of α mixing the events.

   (c) Substition;

   If $h = (p_1 \cdot h^1, \ldots, p_k \cdot h^k)$, and $h = (p_1 \cdot h^1, \ldots, p_k \cdot h^k)$ are in $\mathcal{G}$, and if $h^i \sim g^i$ for every i, then $h \sim g$. That is, one can substitute any element of a gamble's outcome vector for another and retain the same preference over the gamble, so long as the agent is indifferent over the old and substituted event.

2. Consider the utility maximization problem:

$$\max_{x_1, x_2} u(x_1, x_2) = x_1 + x_2 \qquad \text{s.t.} \quad x_1^\alpha + x_2^\alpha = b \quad .$$

   (a) Appeal to the Envelope Theorem to find $\frac{\delta v(x_1, x_2)}{\delta b}$.

$$\begin{aligned} x^* &= \left(\frac{b}{2}\right)^{\frac{1}{\alpha}} \\ \frac{\delta v(x_1, x_2)}{\delta b} &= \lambda^* = \frac{1}{\alpha}\left(\frac{b}{2}\right)^{\frac{1-\alpha}{\alpha}} \end{aligned}$$

(b) Obtain an expression for $v(x_1, x_2)$ and so verify your answer given in (a).

$$v(x_1, x_2) = 2\left(\frac{b}{2}\right)^{\frac{1}{\alpha}}$$

3. For the following minimization problem, you may assume (i.e. without proof) that the optimal solution is $x_1^* = 28/13$ and $x_2^* = 36/13$. Show that the optimal solution does indeed satisfy the Kuhn-Tucker conditions for a minimum.

$$\begin{aligned} \min_{x_1, x_2} \quad & C = \sum_{i=1}^{2} (x_i - 4)^2 \\ \text{subject to} \quad & 2x_1 + 3x_2 \geq 6 \\ & -3x_1 - 2x_2 \geq -12 \\ \text{and} \quad & x_i > 0\,, \quad i = 1, 2 \end{aligned}$$

The Lagrange function is:

$$L = \sum_{i=1}^{2} (x_i - 4)^2 + \lambda_1[6 - 2x_1 - 3x_2] + \lambda_2[-12 + 3x_1 + 2x_2]$$

and the Kuhn-Tucker conditions (for minimization) will be:

$$\begin{aligned} L_{x_1} &= 2(x_1 - 4) - 2\lambda_1 + 3\lambda_2 \geq 0 \\ L_{x_2} &= 2(x_2 - 4) - 3\lambda_1 + 2\lambda_2 \geq 0 \\ L_{\lambda_1} &= 6 - 2x_1 - 3x_2 \leq 0 \\ L_{\lambda_2} &= -12 + 3x_1 + 2x_2 \leq 0 \\ x_i L_{x_i} &= 0\,, \quad i = 1, 2 \\ x_i, \lambda_i &\geq 0\,, \quad i = 1, 2 \end{aligned}$$

And, since we have been told that $x_i^* > 0$ for $i = 1, 2$ we can conclude (by the penultimate condition) that $L_{x_1} = L_{x_2} = 0$. Substituting and solving for λ_1 and λ_2 for the first two conditions yields, $\lambda_1^* = 0$ and $\lambda_2^* = 16/13$ which are both nonnegative as required by the last condition. Similarly, checking conditions 3 and 4 we find that $L_{\lambda_1} = -6\frac{8}{13}$ and $L_{\lambda_2} = 0$ which are both nonpositive as required. Hence all Kuhn-Tucker conditions are satisfied at the given point.

4. Consider the compensated, or *Hicksian* demand function $x_i^h = x_i(\mathbf{p}, e(\mathbf{p}, \bar{u}))$.

   (a) Show that the price effect on compensated demand is given by,

$$\frac{\delta e}{\delta p_i}(\mathbf{p}, \bar{u}) = x_i^*(\mathbf{p}, \bar{u}) \ ,$$

We recall first the standard consumer minimization problem (written in maximization form),

$$-e(\mathbf{p}, \bar{u}) = \max_x \{-\mathbf{p} \cdot x | u(x) - \bar{u} \geq 0\}$$

and appealing to the Envelope Theorem, we conclude that:

$$\frac{\delta e}{\delta p_i}(\mathbf{p}, \bar{u}) = \left.\frac{\delta L}{\delta p_i}\right|_{x^*, \lambda^*}$$

where L is the Langrangian for the above problem,

$$-L = -\mathbf{p} \cdot x - \lambda[\bar{u} - u(x)]$$

and so, we condclude that

$$-\left.\frac{\delta L}{\delta p_i}\right|_{x^*, \lambda^*} = -x_i^*$$

implying the result to be shown.

   (b) And hence, provide a proof for the *Slutsky Equation*;

We can get at the Slutsky Equation by taking the price derivative of the Hicksian demand function given at the top of the problem, $x_i^h = x_i(\mathbf{p}, e(\mathbf{p}, \bar{u}))$:

$$\frac{\delta x_i^h}{\delta p_j}(\mathbf{p}, \bar{u}) = \frac{\delta x_i(\mathbf{p}, y)}{\delta p_j} + \frac{\delta x_i(\mathbf{p}, y)}{\delta y} \frac{\delta e}{\delta p_j}(\mathbf{p}, \bar{u})$$

but from our work above, we can re-write and re-arrange to obtain:

$$\frac{\delta x_i(\mathbf{p}, y)}{\delta p_j} = \frac{\delta x_i^h(\mathbf{p}, \bar{u})}{\delta p_j} - \frac{\delta x_i(\mathbf{p}, y)}{\delta y} x_j(\mathbf{p}, \bar{u})$$

as required.

   (c) Write brief notes on the two components of the *Slutsky Equation*.

The Slutsky Equation decomposes the effect of a change in price of another good j in the consumer's consumption bundle on their demand for good i. The two effects are taken up in the first and second term.

i. The substitution effect. This effect captures the fact that the consumer will change his consumption bundle demands to attain the same effective level of utility than before the change. Hence, he will attempt to find substitutes for the utility that was obtained from good j before the price increase.

ii. The income effect. However, given that the price of good j has gone down (say), this leaves the consumer with an apparent relative increase in income to spend on other goods, that is, to re-optimize. Hence, they will likely move from their original indifference curve to a new indifference curve after the change.

A diagram could be helpful.

Section C

Answer 1 out of the following 2 questions. Each question is worth equal value. This section is worth 15 marks.

1. Suppose that a preference relation on $X \subset \mathbb{R}_+^m$ is complete, reflexive, transitive, continuous and strongly monotonic. Assume also that a utility function $u : \mathbb{R}_+^m \to \mathbb{R}$ has been shown to uniquely exist. Complete the existence proof for the function $u(x)$ by showing:

   (a) That $u(x)$ actually represents the preference relation; and

   Now consider two bundles x and y. By the previous work (assumed) we can write their corresponding utility levels (as Real numbers) $u(x)$ and $u(y)$, and furthermore, by using the vector of 1s $e \in \mathbb{R}_+^m$ we can say that $u(x)e \sim x$ and $u(y)e \sim y$.
   By use of transitivity and monotonicity (assumed) we can write the following sequence:

$$\begin{aligned} & xRy \\ \Leftrightarrow \quad & \Big(u(x)e \sim x\Big) R \Big(y \sim u(y)e\Big) \\ \Leftrightarrow \quad & u(x)eRu(y)e \\ \Leftrightarrow \quad & u(x)Ru(y) \end{aligned}$$

   which implies that the utility function $u(\cdot)$ preserves the preference relation R into the real-valued function.

   (b) That $u(x)$ is a continuous function.

Now suppose that we have a sequence of bundles x^i such that at the limit, the sequence approaches a specific bundle $x^i \to x$. To prove continuity, we need to show that the outputs of our real-valued function will do the same, that is, that $u(x^i) \to u(x)$. We prove by contradiction. Suppose that the above statement, $u(x^i) \to u(x)$ is not true. This implies that there exists some $\epsilon > 0$ and an infinite set of i's such that $u(x^i) > u(x) + \epsilon$, or alternatively that $u(x^i) < u(x) - \epsilon$. That is, that the limiting value of the transformed output from our utility function is strictly larger than, or smaller than the actual value of utilitly associated with the bundle x.
Assuming the first case (strictly larger than), we can further write:

$$x^i \sim u(x^i)e \succsim u(x)e + \epsilon e \sim x + \epsilon e$$

which implies (by transitivity) that

$$x^i \succsim x + \epsilon e \quad .$$

However, since $x^i \to x$, for very large values of i in the infinite set we must have that $x + \epsilon e > x^i$ which implies that $x + \epsilon e \succ x^i$ (by strong monotonicity). We have found a contradiction.
Hence, we conclude that $u(x)$ is continuous.

2. Consider the *Stone-Geary* Utility function,

$$u(\mathbf{x}) = \prod_{i=1}^{n} (x_i - a_i)^{b_i}$$

where $b_i \geq 0$ and $\sum_{i=1}^{n} b_i = 1$, and each $a_i \geq 0$ can be interpreted as the 'subsistence' level of consumption of the commodities in $\mathbf{x}$.

Derive:

(a) The *Marshallian demand functions*, and so

$$x_i^* = \frac{b_i}{p_i}\left(y - \sum_{i=1}^{n} a_i p_i\right) + a_i$$

(b) The *indirect utility function*, and hence,

$$v(x) = \prod_{i=1}^{n} \left[\frac{b_i}{p_i}(y - \sum_{i=1}^{n} a_i p_i)\right]^{b_i}$$

(c) Show that $v(x)$ is proportional to $y - \sum_{i=1}^{n} a_i p_i$ (the level of 'discretionary income').

$$
\begin{aligned}
v(x) &= \prod_{i=1}^{n}\left[\frac{b_i}{p_i}\left(y-\sum_{i=1}^{n}a_ip_i\right)\right]^{b_i} \\
&= \left(y-\sum_{i=1}^{n}a_ip_i\right)\prod_{i=1}^{n}\left(\frac{b_i}{p_i}\right)^{b_i} \qquad \text{Recall: } \quad \sum b_i = 1
\end{aligned}
$$

Equation Sheet

Roy's Identity $\qquad x_i^* = -\frac{\delta v}{\delta p_i} \bigg/ \frac{\delta v}{\delta y}$

Shephard's Lemma $\qquad x_i^h(\mathbf{p}, \bar{u}) = \frac{\delta e(\mathbf{p},\bar{u})}{\delta p_i}$

Slutsky Equation $\qquad \frac{\delta x_i(\mathbf{p},y)}{\delta p_j} = \frac{\delta x_i^h(\mathbf{p},\bar{u})}{\delta p_j} - \frac{\delta x_i(\mathbf{p},y)}{\delta y} x_j(\mathbf{p}, \bar{u})$

Elasticity Relations

$$\eta_i = \frac{\delta x_i(\mathbf{p},y)}{\delta y} \frac{y}{x_i(\mathbf{p},y)}$$

$$\epsilon_{ij} = \frac{\delta x_i(\mathbf{p},y)}{\delta p_j} \frac{p_j}{x_i(\mathbf{p},y)}$$

$$s_i = \frac{p_i x_i(\mathbf{p},y)}{y}$$