Introduction

**Effective rates Present value**

**Equations of value**

**Agenda**

1. Effective rate of interest;
2. Present value;
3. Equations of value;
   1. Under simple interest;
   2. Under compound interest;
Suppose we want to compare interest rates quoted in non-yearly periods versus compound rates on a standard yearly schedule. It would be useful to bring them to **the same playing field**.
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1. This is just the same as saying, suppose I start with $1, with nominal rate 0.08 and compounding at 4 times per year. Then the value at the end of the year is,

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S = (1) \left(1 + \frac{0.08}{4}\right)^{1\times4} = $1.0824
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2. So the effective rate of interest I received over the year (at yearly compounding) is just,

\[ r_e = \frac{\text{amount I gained over year}}{\text{amount I invested}} - 1 \]

\[ = \frac{1.0824}{1} - 1 \]

\[ = 0.0824 \]
Which leads to the following definition,

**Definition: Effective (equivalent) rates**

The **effective rate** $r_e$ (at once-yearly compounding) that is equivalent to a nominal rate $r$ compounded $n$ times a year is,

$$r_e = \left(1 + \frac{r}{n}\right)^n - 1$$ (1)
Example

A bank is offering two different types of savings accounts – the first has a nominal rate of 5.15% compounded quarterly, whilst the other just has a single yearly compounding at a rate of 5.35%. Which would you choose?
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Checking (with $1):

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S = P\left(1 + \frac{r}{n}\right)^{nt}
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= (1)(1 + \frac{0.0515}{4})^{4\times10}
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\]

or using \( r_e \),

\[
S = P(1 + r_e)^t \\
= (1)(1 + 0.0525)^{10} \\
= \$1.67
\]
Scenario

The tale of the generous parents

Your parents have been discussing your finances with you. They have (very generously) suggested that in 20 years’ time, they are willing to give you $100,000 to help you out. Clearly this is very nice of them. However, you can’t keep it out of your head to wonder, just how generous are they being? What would this kind of money mean today if they were just to give it to you?
Previously, we have been interested to know what the **future value** (normally represented by $S'$) of a certain sum (the principle, $P$) would be in a number of years time,
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\begin{array}{c}
P \\
1 & 2 & 3 & 4 \\
\end{array}
\longrightarrow
\begin{array}{c}
? \\
\end{array} \\
t
$$

However, in our current scenario, we want to know what the **present value** of a future amount is, given that it represents the end-points of some value-adding process, that is,

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Clearly, the way the money is being changed through time will matter. We’ll consider a few.
Example (Approach 1: just inflation)

Suppose that (as with inflation) the money is not invested, and its value change is therefore the product of simple inflationary changes. Assume therefore, a compound interest scenario with yearly period and (average) inflation of 3% per annum.
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\[ S = P(1 + r)^n \]

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substituting,

\[ P = 100,000(1 + 0.03)^{-20} = $55,368 \text{ (still a lot!)} \]
Example (Approach 2: their super fund)

Now suppose they are going to use money from their superannuation fund (which they fortuitously can't touch for 20 years anyhow). Suppose that it receives no more outside money between now and 20 years into the future, just the interest it yields, which is calculated semi-annually at around 11% per annum (it’s a good fund). How much would need to be in there now?
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\[ = \$11,746 \]

!
Example (Approach 3: the ‘Endowment Trust’ fund)

Suppose instead, that the money will come from a dedicated fund for this purpose offered by their bank. This account uses continuous compounding at a nominal rate of 11%. How much would need to be invested now?
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Example (Dummy check case study: A can of Coke)

If a can of (regular) Coca-cola costs $1.80 today, what would it have cost me when I was a kid (20 years ago)? (Assume 3% inflation through these years).
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\[ \approx $1.00 \]
Definition: *Present Value (periodic)*

To obtain a **compound amount** of value $S$ which has been maturing at the periodic rate of $r$ for $n$ periods, one needs to invest the starting amount, or principle,

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Definition: *Present Value (continuous)*

To obtain a compound amount of value $S$ which has been maturing continuously at the nominal rate of $r$ for $t$ years, one needs to invest the starting amount, or principle,

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otherwise called the **present value** of $S$. 
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- We have an inherent preference for things *now* – we are impatient;
Why? Why is money **today** apparently ‘more valuable’ than the same amount tomorrow?:

A topic of lots of economic discussion, but at this stage, we consider a relatively simple explanation:

1. We have an inherent preference for things **now** – we are impatient;
2. Tomorrow is uncertain;
Scenario II

Actually, you owe your parents some money. At present, you owe them $500 to be paid in 6 months and $350 in 9 months. Unfortunately, you don’t want to do installments, you’d prefer to pay $100 now and the rest in 12 months time. In negotiations, they agree to consider either simple or compounded (quarterly) interest. What will you do?
Solution technique:

1. Work out the timings;
2. Using the focal date bring all the payments and debts to it;
3. Set up the equation of value;
4. Solve.

Debts: $500 $350
Payments: $100

$t$ (months)
Solution technique:

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Debts: $100

Payments: $500 $350

focal date
Solution technique:

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focal date
Example

Using a focal date of **now** or in **12 months**, and simple interest at the nominal value of 7%, what would be the single sum you owe?
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Let $x$ be the payment now, then,

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\text{value of repayment} = \text{value of debts}
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\therefore x + 100(1 + 0.07 \frac{0}{12}) = 500(1 + 0.07 \frac{6}{12})^{-1} + 350(1 + 0.07 \frac{9}{12})^{-1}
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\therefore \quad x + 100 = 483.09 + 332.54
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\therefore \quad x = \$715.63
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Let \( y \) be the payment in 12 months, then,
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Let \( x \) be the payment now, then,

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\therefore x + 100(1 + 0.07 \cdot \frac{0}{12}) = 500(1 + 0.07 \cdot \frac{6}{12})^{-1} + 350(1 + 0.07 \cdot \frac{9}{12})^{-1}
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\therefore y + 100(1 + 0.07 \cdot \frac{12}{12}) = 500(1 + 0.07 \cdot \frac{6}{12}) + 350(1 + 0.07 \cdot \frac{3}{12})
\]

\[
\therefore y + 107 = \$517.5 + 356.13
\]

\[
\therefore y = \$766.63
\]
Checking the two payments (now = $715.63, 12 months = $766.63), the value of the 12 month payment is,

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\[ = \$716.48 \]

!!!
Checking the two payments (now = $715.63, 12 months = $766.63), the value of the 12 month payment is,

\[ P = 766.63(1 + 0.07)^{-1} \]
\[ = 716.48 \]

Caution!
When using **simple interest** in equations of value, the **focal date** must be agreed before hand, since it will affect the total value exchanged.
Try the compound interest version yourself;
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2. Check if the value of the future (12 month) payment is the same as the current payment.
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2. Check if the value of the future (12 month) payment is the same as the current payment.
3. Which repayment method would you pick?