Information Cascades with Endogenous Signal Precision

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Abstract

Information cascades rely on individuals' own information being sufficiently imprecise that they ignore it, in favor of information inferred from others' observed behavior. This paper extends the standard information cascade model to allow individuals to choose how precise they want their information to be, when additional precision may be costly. There are two main results. First, free riding is rampant, due to the public–good aspect of information in this environment. Either none or nearly none of the individuals purchase additional information, even if it is almost costless. Second, reducing the cost of additional information may not result in more information obtained; indeed, it may lead to increased prevalence of information cascades and lower efficiency.

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1 Introduction

Information cascades (Banerjee (1992), Bikhchandani, Hirshleifer, Welch (1992)) occur when individuals rationally ignore their own information about the state of the world. (See also Anderson and Holt (1997) for an experimental test of an information cascade model.) In order for such behavior to be rational, it must be that individuals' own signals are imprecise—specifically, less precise than the sum of information available elsewhere, such as that inferred from others' behavior. Indeed, models of information cascades have assumed that the signals individuals receive are only weakly correlated with the actual state of the world, and that there is no opportunity to look for better information.[1] A more realistic environment would allow individuals to obtain better information, albeit possibly at a higher cost. One might wonder whether information cascades would still occur in such an environment.

The purpose of this paper is to examine the effect of endogenizing the choice of information precision on the typical information cascade model. Individuals choose how precise a signal to receive. Low–precision signals are costless, while higher precision can be obtained at a positive cost. Our results are stark.

[1]Some researchers (e.g., Kraemer, Nöth, and Weber (2001), Kraemer and Weber (2001)) have considered models in which information is of varying precision, but individuals were not able to choose the precision of their signals.

Allowing individuals to purchase more precise signals does not eliminate information cascades. There is a pervasive free–rider problem, as is often the case when information is a public good (see, for example, Conlisk (1980) or Bolton and Harris (1999)), so that either no one is willing to purchase any information at a positive cost, or exactly one individual is. If no one chooses to purchase increased precision, the result is similar to the standard information cascade result, in which such signals are not available. When exactly one individual chooses to purchase increased precision, it is the individual moving first who does so, and all others free ride on the information she obtains, so that an information cascade occurs immediately. Finally, lowering the cost of more precise signals may not prevent information cascades; in Section 5 we discuss an example in which it makes cascades more likely and decreases efficiency.

2 The decision problem

The decision problem generalizes the standard information cascade model. A large number of identical, risk–neutral players $i = 1, 2, ...$ sequentially attempts to guess the state of the world, which is unknown to the players but commonly known to be equally likely to be either A or B.[2] A player who correctly guesses the state of the world receives a payoff of 1; a player who guesses incorrectly receives 0. Before making her guess, a player may attempt to gather information about the true state of the world, by receiving a signal. Signals also take on the values A or B; the *precision* of a signal (denoted p) is the probability that its value is the same as the true state of the world. We assume that Player i can buy at most one signal s_i, that $p \in \left[\frac{1}{2}, 1\right]$, and that the cost of a signal with precision p is $c(p)$, where $c : \left[\frac{1}{2}, 1\right] \to [0, \infty]$, c is left–continuous and weakly increasing, and $c(\frac{1}{2}) = 0$. Note that $p = \frac{1}{2}$ represents a completely uninformative signal, and our last assumption states that such a signal is costless. Buying such a signal is therefore equivalent to not buying a signal at all, so we lose no generality in assuming that a player buys exactly one signal. We assume in addition that, before choosing her own signal precision, a player is able to perfectly observe the guesses (though not the signals or their precisions) of the players who have already guessed.

3 Player 1 play

Let $\mathcal{P} = \{p | c(p) = 0\}$, and define $p_0 = sup\ \mathcal{P}$. Since $c(\frac{1}{2}) = 0$, $\mathcal{P} \neq \emptyset$, so that $p_0 \geq \frac{1}{2}$, and since c is left–continuous, we know that $c(p_0) = 0$. Thus, p_0 is the highest costless precision.

Player 1's decision problem is relatively simple. Given her choice of signal precision p_1 (and corresponding cost $c(p_1)$), she can either guess her signal s_1 and earn a payoff of $p_1 - c(p_1)$, or guess the opposite of s_1 and earn $1 - p_1 - c(p_1)$. As $p_1 \geq \frac{1}{2}$, the first expression is always at least as much as the second, so she weakly prefers to guess s_1.[3] Then, the signal precision she chooses will be $p^* = argmax_p[p - c(p)]$.[4] Depending on the form of c, it could be that $p^* = p_0$ or $p^* > p_0$. In the former case, Player 1 chooses

[2] Each player guesses only once, so that "encouragement effects" as found by Bolton and Harris (1999) aren't possible.

[3] In keeping with the standard information cascade model, we will assume that when indifferent, a player will follow her own signal. This has sometimes been justified by allowing players to believe that, with extremely small but positive probability, other players "make mistakes"—guessing the opposite of their own signal. Such beliefs result in players' strictly preferring to follow their own signal in cases where they would have been indifferent, but with no effect otherwise. In the appendix, we examine the effect of allowing players to randomize when indifferent, and find that it doesn't change our main results.

[4] We assume that this p^* is unique. Assuming that c is convex is sufficient, though not necessary, to give a unique p^*.

the maximum precision that is available for free; in the latter, she expends positive resources on still more precise information. These two possibilities will lead to somewhat different results.

4 Player 2 play

Player 1's guess is the same as her signal, so even though succeeding players observe only her guess, not her signal, they can perfectly infer her signal from her guess. The second player can therefore not only choose the precision p_2 of his own signal, but also effectively make use of Player 1's signal s_1 (of precision p^*, which he can also infer) at no cost.

Because he has two pieces of information which might contradict each other, Player 2's decision problem is somewhat more complex than Player 1's. Given s_1, s_2, p_2 and p^*, it is straightforward to show that if $s_1 = s_2$ (his signal is the same as Player 1's guess), which happens with probability $p^*p_2 + (1-p^*)(1-p_2)$, he should guess their common value, and earn an expected payoff of $\frac{p^*p_2}{(1-p^*)(1-p_2)+p^*p_2} - c(p_2)$. If $s_1 \neq s_2$ (which happens with probability $p^*(1-p_2) + (1-p^*)p_2$), he should follow the more precise signal and earn $\frac{max[p^*(1-p_2),(1-p^*)p_2]}{p^*(1-p_2)+(1-p^*)p_2} - c(p_2)$. Then, his expected payoff as a function of p_2 is

$$\begin{aligned}
[p^*p_2 + (1-p^*)(1-p_2)] \left[\tfrac{p^*p_2}{(1-p^*)(1-p_2)+p^*p_2} - c(p_2)\right] + [p^*(1-p_2) + (1-p^*)p_2] \left[\tfrac{max[p^*(1-p_2),(1-p^*)p_2]}{p^*(1-p_2)+(1-p^*)p_2} - c(p_2)\right] \\
= p^*p_2 + max[p^*(1-p_2), (1-p^*)p_2] - c(p_2) \\
= max[p^*, p_2] - c(p_2),
\end{aligned}$$

so his objective is to choose p_2 to maximize $max[p^*, p_2] - c(p_2)$.

Note that for $p_2 \geq p^*$, Player 2's payoff function is identical to Player 1's. Since p^* maximized Player 1's payoff, Player 2's optimal choice over $[p^*, 1]$ must also be p^*. Also, for $p_2 \leq p^*$, his payoff function is $p^* - c(p_2)$, which is maximized at $p_2 = p_0$. Thus, choosing $p_2 = p_0$ dominates any choice in $(p_0, p^*]$, including p^* itself when $p^* > p_0$, which in turn dominates any choice larger than p^*, so that $p_2 = p_0$ is an optimal choice for Player 2.[5]

4.1 Case 1: $p^* = p_0 = \frac{1}{2}$

In this trivial case, Player 1 optimally chooses the only costless precision, $\frac{1}{2}$. Her signal is uninformative, both to her and to all succeeding players, who find themselves in the same situation and therefore also decide to choose $p_i = \frac{1}{2}$. No player receives any information at all, either from their own signals or from the observed actions of others, so there is no information cascade and each player earns an expected payoff of $\frac{1}{2}$, irrespective of their guesses.

4.2 Case 2: $p^* = p_0 > \frac{1}{2}$

In the case where Player 1's optimal signal precision was equal to $p_0 > \frac{1}{2}$, and thus equal to the precision chosen by Player 2, Player 2 is indifferent between following his own signal and following Player 1's. We assume that he follows his own signal. (See, however, Note 3.) Player 3 is then able to infer both Player

[5] Any level of precision in $\left[\frac{1}{2}, p_0\right)$ is also an optimal choice; we show in the appendix that having Player 2 choose one of these other levels doesn't substantially affect our qualitative results.

1's and Player 2's signals from their guesses, and can make use of them, in addition to her own signal, of precision p_3.

If the first two players' guesses (and therefore their signals) were opposite, which happens with probability $2p_0(1-p_0)$, then since their precisions were the same, they together convey no information to Player 3, and she does best by following her own signal. She is thus in the same situation as Player 1 was, and will choose $p_3 = p^* = p_0$. Player 4 will then be in the same situation as Player 2, and will choose $p_4 = p^* = p_0$ and follow his own signal. As long as the numbers of A guesses and B guesses are the same or differ by one, players will continue to choose precision p_0 and to follow their own signals.

If the first two players' guesses were the same, and Player 3's signal matches these guesses, which happens with probability $p_0^2 p_3 + (1-p_0)^2(1-p_3)$, she should naturally follow her signal. If her signal is the opposite of the others' guesses, which happens with probability $(1-p_0)^2 p_3 + p_0^2(1-p_3)$, she should follow her own guess if and only if the conditional probability of her signal being correct and the other two wrong is at least as much as that of her signal being wrong and the other two correct:

$$\frac{(1-p_0)^2 p_3}{(1-p_0)^2 p_3 + p_0^2(1-p_3)} \geq \frac{p_0^2(1-p_3)}{(1-p_0)^2 p_3 + p_0^2(1-p_3)};$$

that is, $(1-p_0)^2 p_3 \geq p_0^2(1-p_3)$. This condition holds when $p_3 \geq \frac{p_0^2}{1-2p_0+2p_0^2}$. It is straightforward to show that when $p_0 \in \left[\frac{1}{2}, 1\right]$, $p_0 \leq \frac{p_0^2}{1-2p_0+2p_0^2} \leq 1$, and both inequalities are strict when $p_0 \in \left(\frac{1}{2}, 1\right)$, which implies that Player 3 is almost always able (though possibly not willing) to buy a signal with more information content than the combination of the first two players' signals.

We now find Player 3's expected payoff (conditional on the first two players' guesses being the same), as a function of p_3:

$$\frac{max[p_0^2(1-p_3), (1-p_0)^2 p_3]}{p_0^2 + (1-p_0)^2} + \frac{p_0^2 p_3}{p_0^2 + (1-p_0)^2} - c(p_3).$$

The first term is the conditional probability that Player 3's guess is correct when her signal is the opposite of the others' guesses; the second is the conditional probability that her guess is correct when her signal is the same as the others' guesses. Her payoff function simplifies to

$$\frac{max\left[p_0^2, (1-2p_0+2p_0^2)p_3\right]}{1-2p_0+2p_0^2} - c(p_3) = max\left[\frac{p_0^2}{1-2p_0+2p_0^2}, p_3\right] - c(p_3).$$

Proposition 1 *Choosing $p_3 = p_0$ maximizes this expression.*

Proof: When $p_3 = p_0$, Player 3's payoff is $\frac{p_0^2}{1-2p_0+2p_0^2}$, since $p_0 \leq \frac{p_0^2}{1-2p_0+2p_0^2}$ and $c(p_0) = 0$. For $p_3 \geq \frac{p_0^2}{1-2p_0+2p_0^2}$, we have

$$\begin{aligned} max\left[\frac{p_0^2}{1-2p_0+2p_0^2}, p_3\right] - c(p_3) &= p_3 - c(p_3) \\ &\leq p_0 \quad \text{(Since } p_0 \text{ maximizes } p - c(p),\ p_0 = p_0 - c(p_0) \geq p_3 - c(p_3)\text{)}. \\ &\leq \frac{p_0^2}{1-2p_0+2p_0^2}. \end{aligned}$$

For $p_3 < \frac{p_0^2}{1-2p_0+2p_0^2}$, we have

$$\begin{aligned} max\left[\frac{p_0^2}{1-2p_0+2p_0^2}, p_3\right] - c(p_3) &= \frac{p_0^2}{1-2p_0+2p_0^2} - c(p_3) \\ &\leq \frac{p_0^2}{1-2p_0+2p_0^2}. \end{aligned}$$

Thus neither $p_3 \geq \frac{p_0^2}{1-2p_0+2p_0^2}$ nor $p_3 < \frac{p_0^2}{1-2p_0+2p_0^2}$ gives Player 3 a higher payoff than choosing $p_3 = p_0$. **QED.**

This proposition implies that when the first two players' guesses are the same, Player 3 is best off choosing $p_3 = p_0$; she then ignores her signal and follows the guesses of Players 1 and 2. We then have an information cascade, just as in the standard model, where Player 3 is forced to choose $p_3 = p_0$. Since Player 3's decisions convey no information to later players, Player 4 and all subsequent players find themselves in the same situation as Player 3, and behave in the same way. A similar argument holds for all players still to move, whenever the difference between the numbers of A and B guesses is two or more.

4.3 Case 3: $p^* > p_0$

In the case where Player 1's optimal signal precision was strictly larger than p_0, Player 2's optimal choice of p_0 implies that he pays nothing for additional information, and since his signal is less precise than Player 1's, he ignores his own signal s_2. Player 2 thus free rides on Player 1's purchase of information. For all subsequent players, Player 2's behavior conveys no information. Player 3's decision problem is therefore exactly the same as Player 2's. So, she will choose to pay nothing for additional information ($p_3 = p_0$), and she will ignore her signal and follow Player 1's guess. All succeeding players will do the same. We therefore have, in this case, an *immediate* information cascade. All players after Player 1 will rationally ignore their own signals and follow Player 1's guess.

5 Discussion

We see from the preceding that making additional information available to players in an information cascade model does not prevent information cascades; if anything, the likelihood of a cascade increases. Leaving aside the trivial case $p^* = p_0 = \frac{1}{2}$, there are two possibilities. First, we have Case 2 above, which happens when there is a positive amount of costless information available, and the first player's optimal choice is to spend nothing on information beyond this amount. In this case, each player chooses a costless signal and follows her own signal, until the difference in guesses between the two states is two or more. Then, there is a standard information cascade: all succeeding players will choose a costless signal and ignore it, following the previous players' guesses instead. With a large number of players, the probability of a cascade eventually occurring approaches one; with an infinite number of players, it is exactly one. Conditional on a cascade occurring, the probability of its being "good" (players' ignoring wrong private signals to make correct guesses) is $\frac{p_0^2}{1-2p_0+2p_0^2}$, and "bad" cascades (players' ignoring correct private signals to make wrong guesses) happen with probability $\frac{(1-p_0)^2}{1-2p_0+2p_0^2}$.

The other possibility, our Case 3 above, is even more stark. When the first player's optimal choice is to spend a positive amount on information, she follows her own signal. All subsequent players choose a costless signal and ignore it, following the first player's guess. Cascades happen immediately, and the probabilities of good and bad cascades are p^* and $1 - p^*$, respectively.

Two noteworthy facts emerge from this analysis. First, in both cases, information is available to the players that is precise enough to be able to break the cascade. A signal with precision greater than $\frac{p_0^2}{1-2p_0+2p_0^2}$ in Case 2, or greater than p^* in Case 3, should be followed, regardless of the guesses of the earlier players. However, players aren't willing to pay for this much precision; they do better by simply following the guesses of these earlier players. Cascades are therefore optimal in a sense—they arise from the refusal of players to pay for the information that would prevent them, rather than from such information's being unavailable.

Second, lowering the cost of additional information makes Case 3 more likely relative to Case 2, and can therefore not only shorten the time needed for a cascade to occur, but also increase the probability of a bad cascade, so that most players become worse off. As a simple example, consider the following functional form for c:

$$c(p) = \begin{cases} 0 & p \in [0.5, 0.75] \\ k & p \in (0.75, 0.85] \\ 1 & p \in (0.85, 1]. \end{cases}$$

Suppose k is relatively large (e.g., 0.2). Then Player 1, and each subsequent player, will choose precision 0.75—the highest costless precision. A cascade will occur as soon as the number of A guesses is two more or two fewer than the number of B guesses. Once the cascade forms, the probability of its being a bad cascade is 0.1. Players' ex ante (before viewing their signals) expected payoffs will be 0.75 before the cascade forms, and 0.9 afterwards. Now, suppose instead that k falls to 0.05. Then Player 1 will choose precision 0.85 and earn payoff 0.8. The cascade happens immediately; all subsequent players will choose precision 0.75. The probability of a bad cascade is now 0.15; players after Player 1 earn expected payoff 0.85. So in this case, lowering the cost of precision makes the cascade immediate, and increases the probability of a bad cascade. Ironically, though lowering the cost of more precise signals results in the other players' free riding on Player 1's information, it is Player 1 who becomes better off, while many of the others become worse off.

Appendix: Relaxing assumptions on player behavior when indifferent

In several places, we have used two assumptions: (1) when a player is indifferent between guessing her signal and guessing the opposite, she follows her own signal, (2) when a player is indifferent between choosing precision p_0 and $p < p_0$, she chooses p_0. It is easy to see that in Case 1 (Section 4.1), (2) doesn't apply and (1) doesn't drive any of the results, and in Case 3 (Section 4.3), (1) doesn't apply and (2) doesn't drive the results. So, in these two cases, removing these assumptions will not affect the results at all. In Case 2 (Section 4.2), however, allowing Player 2 to ignore his own signal with some probability, or to choose a precision less than p_0 (neither of which alters his expected payoff), might affect Player 3's decision problem (and those of later players), so we need to consider the possibility that the results may be affected.

Relaxing either of these assumptions in this case has the same implication: when Player 2's signal is the opposite of Player 1's guess, he might follow Player 1's guess instead of his own signal. Let $1-\alpha$ be the probability that Player 2 chooses a precision less than p_0 (in which case he strictly prefers to follow Player 1's guess), and let $1-\beta$ be the probability that he follows Player 1's guess when his signal has precision p_0. The probability that Player 1 and Player 2 both guess correctly will then be $p_0 - \alpha\beta p_0(1-p_0)$, and the probability they both guess incorrectly $1-p_0-\alpha\beta p_0(1-p_0)$, so the probability that they make the same guess is the sum of these: $1-2\alpha\beta p_0(1-p_0)$. Then, given p_3, the probability that Player 3's signal matches the first two players' correct guesses is $p_3[p_0-\alpha\beta p_0(1-p_0)]$, the probability that her signal is the opposite of the first two players' *correct* guesses is $(1-p_3)[p_0-\alpha\beta p_0(1-p_0)]$, and the probability that her signal is the opposite of the first two players' *incorrect* guesses is $p_3[1-p_0-\alpha\beta p_0(1-p_0)]$.

When Player 3's signal is the opposite of the first two players' guesses, she should guess her signal if this last expression is larger than the previous one, and follow the guesses of Players 1 and 2 if the opposite holds. Her expected payoff, conditional on the first two players' guesses being the same, is then

$$\frac{p_3[p_0-\alpha\beta p_0(1-p_0)]+max\{(1-p_3)[p_0-\alpha\beta p_0(1-p_0)],p_3[1-p_0-\alpha\beta p_0(1-p_0)]\}}{1-2\alpha\beta p_0(1-p_0)} - c(p_3)$$
$$= max\left[\frac{p_0-\alpha\beta p_0(1-p_0)}{1-2\alpha\beta p_0(1-p_0)}, p_3\right] - c(p_3).$$

Proposition 2 *Choosing $p_3 = p_0$ maximizes this expression.*

Proof: The argument is similar to that used in the proof of Proposition 1. As before, we have $p_0 \leq \frac{p_0-\alpha\beta p_0(1-p_0)}{1-2\alpha\beta p_0(1-p_0)}$ for $p_0 \in \left[\frac{1}{2},1\right]$, and the inequality is strict for $p_0 \in \left(\frac{1}{2},1\right)$. So, choosing $p_3 = p_0$ gives payoff $\frac{p_0-\alpha\beta p_0(1-p_0)}{1-2\alpha\beta p_0(1-p_0)}$. For $p_3 \geq \frac{p_0-\alpha\beta p_0(1-p_0)}{1-2\alpha\beta p_0(1-p_0)}$, we have

$$\begin{aligned} max\left[\frac{p_0-\alpha\beta p_0(1-p_0)}{1-2\alpha\beta p_0(1-p_0)}, p_3\right] - c(p_3) &= p_3 - c(p_3) \\ &\leq p_0 \quad \text{(Since } p_0 \text{ maximizes } p-c(p),\ p_0 = p_0 - c(p_0) \geq p_3 - c(p_3)\text{)}. \\ &\leq \frac{p_0-\alpha\beta p_0(1-p_0)}{1-2\alpha\beta p_0(1-p_0)}. \end{aligned}$$

For $p_3 < \frac{p_0-\alpha\beta p_0(1-p_0)}{1-2\alpha\beta p_0(1-p_0)}$, we have

$$\begin{aligned} max\left[\frac{p_0-\alpha\beta p_0(1-p_0)}{1-2\alpha\beta p_0(1-p_0)}, p_3\right] - c(p_3) &= \frac{p_0-\alpha\beta p_0(1-p_0)}{1-2\alpha\beta p_0(1-p_0)} - c(p_3) \\ &\leq \frac{p_0-\alpha\beta p_0(1-p_0)}{1-2\alpha\beta p_0(1-p_0)}. \end{aligned}$$

Thus neither $p_3 \geq \frac{p_0-\alpha\beta p_0(1-p_0)}{1-2\alpha\beta p_0(1-p_0)}$ nor $p_3 < \frac{p_0-\alpha\beta p_0(1-p_0)}{1-2\alpha\beta p_0(1-p_0)}$ gives Player 3 a higher payoff than $p_3 = p_0$. **QED.**

This proposition shows that in Case 2, irrespective of how Player 2 acts when indifferent, Player 3 still optimally chooses precision p_0 and ignores her signal whenever the first two players' guesses are the same. (When either α or β is zero, Player 3 is indifferent between following and ignoring her signal when it is the opposite of the first two players' guesses; she is actually in the same situation as Player 2 when his signal is the opposite of Player 1's guess.) Similar results obtain for Player 4 and subsequent players. Thus, relaxing the two assumptions above doesn't substantially affect the results of this paper.

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