Macroscopic Fundamental Diagrams of arterial road networks governed by adaptive traffic signal systems

Tim Garoni

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A comparative study of Macroscopic Fundamental Diagrams of arterial road networks governed by adaptive traffic signal systems

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Fundamental Diagram

- Consider a one-dimensional flow (vehicles along a freeway)
- The functional relationship between flow and density is the **fundamental diagram** (Greenshields, 1935)
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![Graph of fundamental diagram](image)

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- What should happen in a network?
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- Intuitively makes sense to have a unimodal FD in one dimension
- What should happen in a network?
- How should one even define network flow? (No prescribed direction)
Macroscopic Fundamental Diagrams

- Simplest idea: relate arithmetic means of link density and flow
- If network has link set $\Lambda$:
  \[ \rho = \frac{1}{|\Lambda|} \sum_{\lambda \in \Lambda} \rho_\lambda, \quad J = \frac{1}{|\Lambda|} \sum_{\lambda \in \Lambda} J_\lambda \]
- $\rho_\lambda$ is density of link $\lambda$ and $J_\lambda$ is its flow
Two Extreme Cases

- Existence of MFDs is trivial:
  - If all links have the same FD
  - and if the distribution of congestion is always perfectly uniform
  - then network MFD coincides with common link FD

- Existence of MFDs is impossible:
  - If one has a network and is free to vary the demand on each link in any way imaginable, then no MFD can exist
  - e.g. half the links have $\rho/\lambda = 1$ and other half have $\rho/\lambda = 0$, then $\rho = 1/2$ and $J = 0$
  - e.g. all links have $\rho/\lambda = 1/2$, then $\rho = 1/2$ but $J > 0$ (could even have $J = J_{\text{max}}$)

- Existence of MFDs clearly not independent of demand

- MFDs are interesting because there is something in between

- In practice, on many networks the demand will rise and fall in a fairly constrained way during a typical day
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- In practice, on many networks the demand will rise and fall in a fairly constrained way during a typical day
What are MFDs?

Consider a fixed network with link set $\Lambda$

First of all, one needs to agree on what $\rho$ and $J$ mean.

- $\rho_\lambda(t)$ and $J_\lambda(t)$ are stochastic processes
- Aggregate variables

$$
\rho(t) = \frac{1}{|\Lambda|} \sum_{\lambda \in \Lambda} \rho_\lambda(t) \quad J(t) = \frac{1}{|\Lambda|} \sum_{\lambda \in \Lambda} J_\lambda(t)
$$

- MFD is the relationship between $\mathbb{E} J(t)$ and $\mathbb{E} \rho(t)$
- Can be interested in instantaneous or stationary MFDs

"Heterogeneity" is also important

Helbing 2009; Mazloumian, Geroliminis & Helbing 2010; Geroliminis & Sun 2011; de Gier, G & Zhang 2013

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$$h(t) = \sqrt{\frac{1}{|\Lambda|} \sum_{\lambda \in \Lambda} [\rho_\lambda(t) - \rho(t)]^2}$$

Helbing 2009; Mazloumian, Geroliminis & Helbing 2010; Geroliminis & Sun 2011; de Gier, G & Zhang 2013

- $J$, $\rho$, $h$ all stochastic processes
- In time dependent context, heterogeneity can explain hysteresis
Asymmetric Simple Exclusion Process (ASEP)

“Everything should be made as simple as possible, but not simpler”

(Albert Einstein)

- Want an Ising model of traffic flow
- One-dimensional stochastic cellular automata very popular in statistical mechanics starting in 1990s
  - Such models do a reasonable job of explaining qualitative behaviour of freeway traffic
  - “Phantom” jams emerge as consequence of collective behaviour
- Cellular automata are discrete dynamical systems
- Space, time, and state variables are discrete
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ASEP with open boundaries:
- If \( x_1(t) = 0 \), then with probability \( \alpha \), \( x_1(t + 1) = 1 \)
- For each cell \( i = 1, \ldots, L \) with \( x_i(t) = 1 \)
  - If \( x_{i+1}(t) = 0 \) then with probability \( p \), \( x_i(t + 1) = 0 \) and \( x_{i+1}(t + 1) = 1 \)
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- If \( x_L(t) = 1 \), then with probability \( \beta \), \( x_L(t + 1) = 0 \)
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![Cellular Automata Diagram]

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Nagel-Schreckenberg process

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  - Vehicles can have different speeds $0, 1, \ldots, v_{\text{max}}$
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Let $x_n$ and $v_n$ denote the position & speed of the $n$th vehicle
Nagel-Schreckenberg process

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  ![Diagram](image)

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![Diagram showing vehicle positions and speeds]

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\begin{array}{cccccccc}
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NetNaSch model

Goal: Minimal stat-mech model that can mimic realistic traffic signals
  ▶ Take multiple NaSch models and glue them together
    \[\alpha_1, \beta_1 \quad \alpha_2, \beta_2 \quad \alpha_3, \beta_3 \quad \alpha_4, \beta_4 \quad \alpha_5, \beta_5 \quad \alpha_6, \beta_6 \quad \alpha_7, \beta_7 \quad \alpha_8, \beta_8\]

  ▶ Need to include:
    ▶ Multiple lanes with lane changing
    ▶ Turning decisions (random)
    ▶ Input and output (endogenous/exogenous)
    ▶ Appropriate rules for how vehicles traverse intersections

Varying all the \(\alpha, \beta, \gamma, \delta, p_n\ldots\) cannot give an MFD
Varying a lower-dimensional space of parameters can
Static demand – Approach to Stationarity

Generate MFD by setting $\alpha_\lambda = \alpha$, $\beta_\lambda = \beta$, $\gamma_\lambda = \delta_\lambda = 0$ for all $\lambda \in \Lambda$.

- Intersections governed by model of SCATS with adaptive linking
- Instantaneous MFD converges to stationary curve
- Although there is uniform boundary demand, the density distribution in the network is not homogeneous
Static demand – Stationary MFDs

- Use MFDs to quantify performance of signal systems

Isotropic and time independent rates ($\gamma=0, \delta=0$), $p_T = 0.1$ at 6 hr

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Isotropic boundary demand

Higher demand on west side
Static demand – Stationary MFDs

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Isotropic and time independent rates ($\gamma=0$, $\delta=0$), $p_T = 0.1$ at 6 hr

SOTL
SCATS−L
SCATS−F

Higher demand on west side
- Anisotropic demand can still produce well-defined MFD
Self-organizing traffic lights

- SOTL is a toy model of a highly adaptive acyclic signal system
- Always gives green to phase with the highest demand

SCATS–L: isotropic and time independent rates ($\gamma=0$, $\delta=0$), $p_T = 0.1$

SOTL: isotropic and time independent rates ($\gamma=0$, $\delta=0$), $p_T = 0.1$

- SOTL has lower heterogeneity than SCATS
- Accounts for its better MFD
**Time-dependent demand**

- Vary $\alpha, \beta$ over 24 hours to mimic am/pm peaks
- Hysteresis observed - clockwise and anticlockwise

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Buisson & Ladier 2009
Empirical data from Toulouse

Zhang, G & de Gier 2013
Simulated data
Time-dependent demand

▸ Hysteresis in MFD consequence of heterogeneity

SOTL: time dependent rates, $p_T = 0.1(\gamma = 0, \delta = 0)$

SOTL: time dependent rates ($\gamma=0, \delta=0$), $p_T = 0.1$

Zhang, G & de Gier 2013
Simulated data
Two-bin model

- Consider two adjacent networks (bins) exchanging vehicles
- Each bin has same well-defined MFD $J(\rho)$

\[
\frac{d\rho_1}{dt} = \frac{a_1 - b_1 J(\rho_1) + p_2 J(\rho_2) - p_1 J(\rho_1)}{L_1}
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- Let bin 1 be boundary layer, bin 2 the interior

![Loading](image1)

![Recovery](image2)
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Loading

Recovery

Instantaneous MFD
Open Problems

- Can we observe anticlockwise hysteresis empirically?
- Can we understand cross-correlations between flow, density and density heterogeneity?
- How does driver adaptivity affect the shape of MFDs?
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- Can we observe anticlockwise hysteresis empirically?
- Can we understand cross-correlations between flow, density and density heterogeneity?
- How does driver adaptivity affect the shape of MFDs?
- How should one partition networks in order to produce well-defined MFDs?
- Several groups are attempting to use MFDs as a basis for perimeter control?
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Paths

Consider a particular node $n$ in a traffic network.

Definition

A path $P$ is an ordered pair of lanes $(\lambda, \lambda')$ with $\lambda \in mn$ and $\lambda' \in nm'$. Vehicles can only move from one link to another along paths. Ignore the actual dynamics through the intersection. No cells in the intersection – we use paths to glue the CA on adjacent links together.
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Details of the model

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Allow only left→right (right→left) at odd (even) time steps.
Details of the model

Lane changing (topological)

- Red car: not needed
- Blue car: not needed, is allowed
- Green car: needed

Each vehicle wants to be in a lane for which there exists a path consistent with its desired turn.

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- Turning decisions affect lane changing dynamics.
Mark paths

Consider each lane $\lambda$ of each link $l$
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