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# ATMOSPHERIC SCIENCE AND GEOPHYSICAL FLUID DYNAMICS RESEARCH GROUP

## The bathtub vortex

**Supervisor:** Michael Page

**Level:** Honours

**Background:** It is a common myth that the direction of rotation of bath water as it drains down a plughole is determined by the rotation of the Earth – in particular the hemisphere in which it is performed. Careful experiments have been performed to confirm that the rotation of the Earth has only a miniscule effect on the motion (Shapiro, 1962; Trefethen *et al*, 1965) and it can also be demonstrated theoretically using simple scaling arguments. More recently, Tyvand & Haugen (2005) published a computational study of the problem and also demonstrated that the influence of the rotation of the Earth is negligible. But why does the water often flow down the plughole in a consistent direction, and why is that direction opposite in the Australia to what is commonly observed in the United Kingdom, for example?

**Objectives:** The project will review previous analytical, experimental and computational work on this problem for simplified geometries. Among other things, this will help identify the principal determinants of the direction of motion of the fluid at the outlet and assist in identifying the key physical principles based on a mathematical analysis of the equations of motion.

**Expectations:**

- To undertake a thorough review of existing primary literature sources on this problem.
- To identify the dominant forces that affect the fluid motion, derive the corresponding governing equations and assess the relative size of the relevant nondimensional parameters.
- To simplify and solve the equations in appropriate cases, consider whether simplified models can adequately represent the phenomenon and perhaps undertake simple computational experiments.

**Assumed knowledge:**

- MTH3011 Partial differential equations; and
- MTH2051/3051 Introduction to computational mathematics; and
- MTH3360 Fluid dynamics.

**Reading:**


- Andersen A. *et al*, Anatomy of a bathtub vortex, *Phys. Review Letters* **91**, 104501, 2003.
- Shapiro, A.H., Bath-tub vortex, *Nature* **196**, pp1080-1081, 1962.
- Thefethen, L.M. *et al*, The bath-tub vortex in the southern hemisphere, *Nature* **207**, pp1084-1085, 1965.
- Tyvand, P.A. and Haugen, K.B., An impulsive bathtub vortex, *Phys. Fluids* **17**, 062105, 2005.
- Yukimoto, S. *et al*, Structure of a bathtub vortex: importance of the bottom boundary layer, *Theor. Comput. Fluid Dyn.* **24**, pp323-327, 2010.


**Last reviewed:** 10 September 2012

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# APPLIED AND COMPUTATIONAL MATHEMATICS

## On the roll-up of a 'vortex sheet'

**Supervisor:** Michael Page

**Level:** Honours

**Background:** The position of a thin interface between two fluids that are moving at different speeds is known to be unstable in an inviscid fluid. This is known as Kelvin-Helmholtz instability, and typically it leads to counter-rotating vortices on the interface. Such vortices are often observed between atmospheric layers, and also in the wake of moving vehicles.

In this project the Kelvin-Helmholtz instability is considered for an idealised problem of a thin straight interface between two infinite fluids that have different constant velocities parallel to the interface. For a two-dimensional inviscid fluid this problem can be posed in the complex plane and determined computationally using complex-valued functions (see for example Krasny 1986 and Baker & Pham 2006). It can also be shown that the subsequent roll-up of the vortex develops as the singularity of one of the functions approaches the real axis, where the interface is initially located.

**Objectives:** The project will review and compare some of the existing primary literature sources on vortex roll-up and the desingularisation of the equations, including both the approach of Krasny (1986) and the introduction of additional factors, for example including viscosity. Krasny's results will be reproduced and then extended to other initial conditions. Key features in the roll-up will be identified beyond the critical time at which it 'overturns' and begins to form a spiral shape.

**Expectations:**

- A review will be undertaken of some of the existing primary literature sources on this problem in order to identify the current state of knowledge.
- The equations that determine the motion of the interface will be derived and the corresponding complex-plane formulation justified.
- MATLAB (for example) will be used to recalculate and extend the results of the periodic problem in Krasny (1986), where a small parameter $\delta$ was introduced in order to 'desingularise' the interface.
- Krasny's approach will be extended to the simpler problem of the roll-up due to an isolated disturbance, for example using recent work by Baker & Pham (2006).

**Assumed knowledge:**

- MTH3011 Partial differential equations (required); and
- MTH3020 Complex analysis and integral transforms (required); and
- MTH3360 Fluid dynamics (preferred).

**Reading:**


- Baker, G.R. and Pham L.D., A comparison of blob methods for vortex sheet roll-up, *J. Fluid Mech.* **547**, pp297-316, 2006.
- Krasny, R., Desingularisation of periodic vortex sheet roll-up, *J. Comp. Phys.* **65**, pp292-313, 1986.


**Last reviewed:** 10 September 2012

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# The Immersed Interface Method for solving elliptic PDEs

**Supervisor:** Michael Page

**Level:** Honours

**Background:** Most of the simple approaches to solving elliptic partial differential equations numerically using finite-difference techniques rely upon the solution being smooth and continuous everywhere in the domain. In addition, the domain must have a simple, regular geometry – such as a rectangle or a circle. Over the last decade or so, a couple of approaches have been developed which allow discontinuities of the solution and/or consider domains with an irregular shape. In the latter case, these methods can be a simpler alternative to using finite-element methods.

The two most common approaches used for these problems are the Immersed Boundary Method, which was first developed by Charles Peskin in the 1970s, and the Immersed Interface Method. Some aspects of these techniques are similar, but there are important differences. This project examines the basis of the Immersed Interface Method and uses it to examine the accuracy of the approach for some simple test problems with exact solutions.

**Objectives:** The project will review and compare some of the existing primary literature sources on both the Immersed Interface Method and the Immersed Boundary Method. Some trials of the Immersed Interface Method will be undertaken, initially based on the test problems in Li and Ito (2006) but then extended to a broader range of configurations, with the intention of identifying the advantages and limitations of the method.

**Expectations:**

- Primary literature sources on the Immersed Interface Method, including LeVeque and Li (1994), and the Immersed Boundary Method will be reviewed, and the two approaches compared in detail.
- The material covered in the first three chapters of Li and Ito (2006) will be examined in detail and all numerical results checked using MATLAB. Some additional simple one and two-dimensional test problems involving elliptic PDEs will also be solved using a similar approach in order to test the capabilities of the method.
- The approach will be applied to some simple 'embedded' boundary-value problems in two-dimensional domains with internal boundaries.

**Assumed knowledge:**

- MTH3011 Partial differential equations (required); and
- MTH2051/3051 Introduction to computational mathematics (preferred).

**Reading:**


- LeVeque, R.J. and Li, Z., The immersed interface method for elliptic equations with discontinuous coefficients and singular sources, *SIAM J. Numer. Anal.* **31**, pp1019-1044, 1994.
- Li, Z. and Ito, K., The Immersed Interface Method, *SIAM*, 2006.


**Last reviewed:** 10 September 2012

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# Applications of pseudospectral numerical methods

**Supervisor:** Michael Page

**Level:** Honours

**Background:** The pseudo-spectral method (see Fornberg, 1998) is a numerical technique for the solution of ordinary and partial differential equations based on high-accuracy formulae for the derivatives in terms of the unknown values of the solution on a uniformly-spaced grid.

**Objectives:** This project will involve analysing and using the 'pseudospectral method' for the solution of some ordinary and partial differential equations in simple geometries. The intention is first to understand the properties, accuracy and efficiency of the method, for example in comparison with the simple finite-difference methods seen in MTH2032 and MTH3011. A literature survey will be used to identify the types of problems that have been solved successfully using the method, and its advantages and disadvantages.

A variety of simple test ODE problems with known exact solutions will then be used to examine its advantages and disadvantages in more detail, including some with varying coefficients and others with thin 'boundary-layer' type solutions. The method will then be extended to some simple elliptic and parabolic PDEs with known exact solutions. The final objective is to obtain reliable and accurate solutions to some simple two-dimensional test problems in fluid dynamics, initially for steady (linear) viscous flow and then for unsteady and/or the nonlinear cases. These results will be compared with some previously published calculations based on second-order methods.

**Assumed knowledge:**

- MTH3011 Partial differential equations (required); and
- MTH2051/3051 Introduction to computational mathematics (preferred).

Detailed numerical computations will be need to undertaken as part of the project, for example using MATLAB.

**Reading:**

- Fornberg, B., A practical guide to Pseudospectral Methods', *Cambridge*, 1998.

**Last reviewed:** 14 September 2012