THE DYNAMICS OF PIZZA TOSSING

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Summary We investigate a variation of the classic bouncing ball problem where the ball is replaced with a disc (pizza dough) and the platform with which the disc interacts (pizza maker's hand) undergoes a combined angular and linear oscillation along the vertical axis (pizza tossing action). In addition to the act of pizza tossing, the vibro-impact system described above also applies to a class of standing-wave ultrasonic motors, which operates by tossing the rotor at rates above 20 kHz. Through numerical simulations, we investigate the performance of different pizza tossing techniques by varying the amplitude ratio (L) and the phase lag (θ) between the vertical and rotational component of the oscillating platform. Our results show that the energy efficiency and the rotational speed are maximized when $L \simeq 1$, and $\theta = 0$.

INTRODUCTION

The nonlinear dynamical system of a ball bouncing on a vibrating platform is one that is simple to describe, yet difficult to analyze. The system displays a range of interesting behaviours, including the period doubling route to chaos, and eventually periodic orbits known variously as the 'sticking solution', 'complete chattering' or 'locking' [1, 3].

Although the system is worthy of investigation in itself for the intriguing behaviour it exhibits, we study a variation of the bouncing ball problem that has direct real-world applications: a bouncing disk on a vibrating platform with combined angular and linear oscillation described by

$$b(t) = \Phi \sin(\omega t + \theta) \qquad \text{and} \qquad s(t) = A \sin(\omega t), \tag{1}$$

where b and s are respectively, the angular and linear displacement of the platform. The angular oscillation of the platform imparts rotary motion to the bouncing disk and gives rise to effects such as impulsive frictional torque at each collision, and the possibility of stick-slip rotation while the disk and platform are in contact. The system corresponds to the stator-rotor interaction in a class of standing-wave ultrasonic motor [4], and a technique used by pizza makers to flatten the pizza dough into the desired shape – pizza tossing.

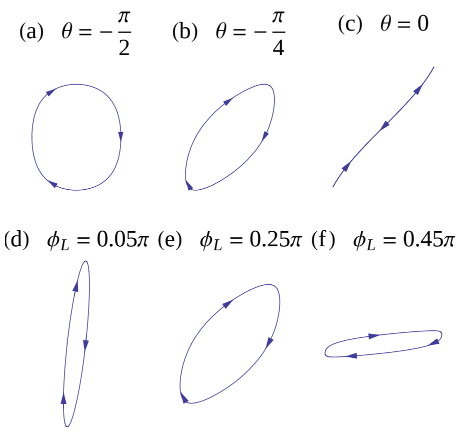

Figure 1. Pizza tossing techniques: the trajectory traced by a point on the oscillating platform. (a – c) the effect varying phase lag θ with $\phi_L = \pi/4$), (d – f) the effect of varying the amplitude ratio $L = \tan(\phi_L)$ with $\theta = -\pi/4$.

There are many interesting and non-trivial questions related to pizza tossing: what kind of thickness distribution will the tossed pizza dough form and how should the material property of the dough be modelled? The question that we try to answer in this paper – relevant to both the pizza tossing and the ultrasonic motor – is this, what is the optimal pizza tossing technique?

It is often assumed by researchers of standing-wave ultrasonic motors that the optimal stator motion to spin the rotor is an elliptical motion (Fig 1.a); however, pizza makers would use a line type motion (Fig 1.c) for pizza tossing. Furthermore, it is unclear how the performance change when the amplitude ratio ($L = a_e\Phi/A$) is varied (a_e is the radius at which the pizza is held).

THE BOUNCING DISK MODEL

To answer the above questions about the best pizza tossing technique and stator trajectory in ultrasonic motors, a general model for the bouncing disk problem was formed using the following assumptions: 1.) collisions have zero duration and is treated using speed independent coefficient of restitution α, 2.) contact pressure is uniform, and 3.) torque is transmitted to the disk by Coulomb friction with cofficient μ and an effective radius of a_e.

Due to the discontinuities introduced by separations, impulsive forces, and friction forces, the motion of the disk can be divided into four phases: 1.) parabolic flight, 2.) impact, 3.) sticking contact, and 4.) sliding contact. Analytical solution for each phase can be obtained separately, thus the main challenge in setting up a computational model is in determining the correct sequence and duration of the phases. The flow chart shown in Fig 2 describes how the four phases are to be sequenced.

RESULTS AND DISCUSSION

Three performance measures are used to compare the pizza tossing techniques: 1.) the final rotational speed reached by the pizza dough $\dot{r}_f$, 2.) the speed ratio ν – defined as final speed $\dot{r}_f$ over maximum platform speed $\omega\Phi$, and 3.) the energy efficiency η – defined as the ratio of the rotational kinetic energy gained by the disk ΔE_r over the total energy input from the platform E_{in}.

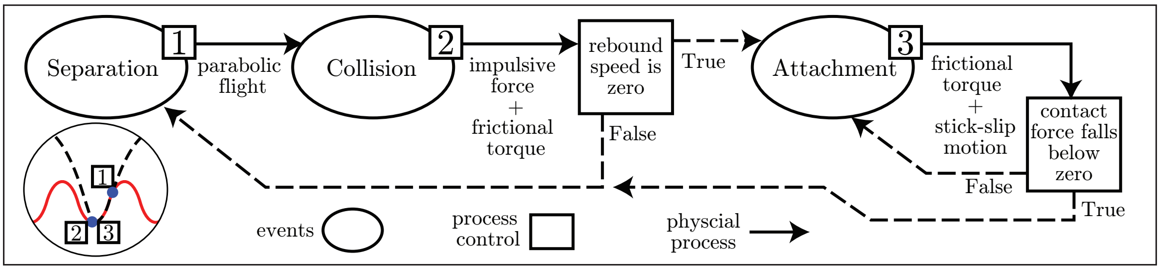

Figure 2. Model simulation flow chart. The inset shows a possible trajectory of x (dashed black) and s (solid red) for one cycle of simulation starting with a collision and ending with separation.

While varying the amplitude ratio L, $c = \sqrt{A^2 + (a_e\Phi)^2}$ is kept constant, thus by defining $L = \tan(\phi_L)$, we have $(a_e\Phi, A) = c(\sin(\phi_L), \cos(\phi_L))$, and by varying ϕ_L between 0 and $\pi/2$ all possible L from 0 to ∞ are considered. The parameters chosen for the present investigation are $c = 25$ cm – $2c$ is approximately the arc length traced out by the pizza tossing action, $\omega = 6\pi$ rad/s $= 3$ Hz – chosen to so that the vertical speed of the toss in the order of 5 m/s, $d = 30$ cm – the diameter of a typical pizza and is used to determine the radius of friction and gyration ($a_e = 0.4d$ and $a_g = d/2\sqrt{2}$), $\mu = 0.6$ – roughly the coefficient of friction between skin and paper, $g = 10$ m/s^2, $\alpha = 0$ – assuming perfectly plastic collision. The initial conditions are set with a pizza dough at rest with $t_0 = 3\pi/2\omega$, corresponding to the lowest point of the tossing motion. Finally, the simulation is run for a single cycle because pizza makers let the dough come to a stop before the next toss.

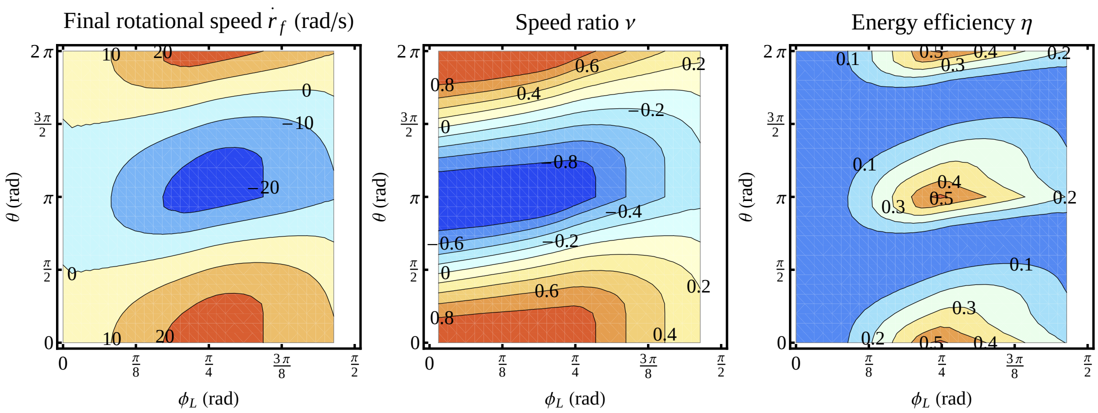

Figure 3. The performance of different pizza tossing technique when phase lag θ and amplitude ratio ϕ_L are varied

The contour plots of the three performance measures in Fig 3 show that the the optimal pizza tossing technique predicted by our bouncing disk model is very similar to the actual motion employed by pizza makers. The maximum final speed, speed ratio and energy efficiency all have θ at close to 0 orπ, and ϕ_L close to $\pi/4$ ($L \simeq 1$). The results for the θ runs counter to the common assumption that $\theta = \pi/2$ is the optimal stator motion for standing-wave ultrasonic motors.

One possible explanation for the descrepancy is that the rotor in standing-wave ultrasonic motors are commonly spring-loaded, while the pizza is only under the effect of gravity, that is, the optimal phase lag is related to the part of the cycle that as higher contact force)

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