

Check the metric and the connection

Here we substitute the computed expressions for $g_{ab}(x)$, $g_{ab,c}(x)$ and $\Gamma_{bc}^a(x)$ into $g_{ab;c}$. We should get zero, to within the truncation order of the expansions.

And yes, we do!

```
::KeepHistory(false).  
::PostDefaultRules( @@collect_terms!(%), @@sumflatten!(%) ).
```

```
{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u#,v#}::Indices.  
{x^{a},R_{a b c d},\nabla_{e}{R_{a b c d}}>::SortOrder.
```

```
\nabla_{#}::PartialDerivative.
```

```
g_{a b}::Metric.  
R_{a b c d}::RiemannTensor.  
R^{a}_{b c d}::RiemannTensor.
```

```
# --- imported from metric.lib -----
```

```
metric:="import metric.lib metric":  
@run(metric){"/Users/leo/local/sh/cdbfile"}:
```

```
dmatrix:="import metric.lib dmetric":  
@run(dmetric){"/Users/leo/local/sh/cdbfile"}:
```

```
# --- imported from connection.lib -----
```

```
Gamma01:="import connection.lib Gamma01":  
@run(Gamma01){"/Users/leo/local/sh/cdbfile"}:
```

```
Gamma02:="import connection.lib Gamma02":  
@run(Gamma02){"/Users/leo/local/sh/cdbfile"}:
```

```
Gamma03:="import connection.lib Gamma03":  
@run(Gamma03){"/Users/leo/local/sh/cdbfile"}:
```

```
Gamma04:="import connection.lib Gamma04":  
@run(Gamma04){"/Users/leo/local/sh/cdbfile"}:
```

```
# --- build the connection -----
```

```

gamma:= \Gamma^{a}_{b c d} x^d
      + \Gamma^{a}_{b c d e} x^d x^e
      + \Gamma^{a}_{b c d e f} x^d x^e x^f
      + \Gamma^{a}_{b c d e f g} x^d x^e x^f x^g:

@substitute!(%)(\Gamma^{a}_{b c d} -> @(Gamma01),
               \Gamma^{a}_{b c d e} -> @(Gamma02),
               \Gamma^{a}_{b c d e f} -> @(Gamma03),
               \Gamma^{a}_{b c d e f g} -> @(Gamma04)):

# --- build  $g_{ab,c} - g_{ad}\Gamma_{bc}^d - g_{db}\Gamma_{ac}^d$  -----
tst:=dmet_{a b c} - met_{a d} gamma^{d}_{b c} - met_{d b} gamma^{d}_{a c}:

@substitute!(tst)(met_{a b} -> @(metric),
                 dmet_{a b c} -> @(dmetric),
                 gamma^{a}_{b c} -> @(gamma)):

@distributed!(tst): @eliminate_metric!(tst):

# --- since  $g_{ab}(x)$  and  $\Gamma_{bc}^a(x)$  are accurate to terms including  $\mathcal{O}(\epsilon^5)$ ,
#      then errors in tst must be  $\mathcal{O}(\epsilon^6)$ 

poly:=@(tst):

x^{a}::Weight(label=xterms,value=1).

term00:=@(poly): @keep_weight!(term00){xterms}{0}:
term01:=@(poly): @keep_weight!(term01){xterms}{1}:
term02:=@(poly): @keep_weight!(term02){xterms}{2}:
term03:=@(poly): @keep_weight!(term03){xterms}{3}:
term04:=@(poly): @keep_weight!(term04){xterms}{4}:
term05:=@(poly): @keep_weight!(term05){xterms}{5}:

# -----

tst:=@(term00) + @(term01) + @(term02) + @(term03) + @(term04)+ @(term05):

@prodsort!(%): @rename_dummies!(%): @canonicalise!(%);

```

--- if all is well, we should see tst being of order $\mathcal{O}(\epsilon^6)$ -----

$$\begin{aligned}
tst := & -\frac{2}{135} x^d x^e x^f x^g x^h R_{adei} R_{bjcf} R^g{}_{ihj} - \frac{2}{135} x^d x^e x^f x^g x^h R_{adei} R_{bfcj} R^g{}_{ihj} + \frac{2}{135} x^d x^e x^f x^g x^h R_{adei} R_{bifj} R_{cghj} + \frac{1}{45} x^d x^e x^f x^g x^h R_{adei} R_{bfgj} R_{cihj} \\
& + \frac{1}{135} x^d x^e x^f x^g x^h R_{adei} R_{bjfi} R_{cghj} + \frac{1}{45} x^d x^e x^f x^g x^h R_{adei} R_{bfgj} R_{cjhj} - \frac{1}{60} x^d x^e x^f x^g x^h R_{adei} \nabla_{bf} R_{cghi} - \frac{1}{60} x^d x^e x^f x^g x^h R_{adei} \nabla_{cf} R_{bgjh} \\
& - \frac{1}{60} x^d x^e x^f x^g x^h R_{adei} \nabla_{fg} R_{bich} - \frac{1}{60} x^d x^e x^f x^g x^h R_{adei} \nabla_{fg} R_{bhci} + \frac{1}{135} x^d x^e x^f x^g x^h R_{adei} R_{bfij} R_{cghj} - \frac{1}{60} x^d x^e x^f x^g x^h R_{adei} \nabla^f{}_i R_{bgch} \\
& - \frac{1}{72} x^d x^e x^f x^g x^h \nabla_b R_{cdei} \nabla_f R_{aghi} - \frac{1}{72} x^d x^e x^f x^g x^h \nabla_c R_{bdei} \nabla_f R_{aghi} - \frac{1}{36} x^d x^e x^f x^g x^h \nabla_d R_{aefi} \nabla_g R_{bich} - \frac{1}{36} x^d x^e x^f x^g x^h \nabla_d R_{aefi} \nabla_g R_{bhci} \\
& - \frac{1}{72} x^d x^e x^f x^g x^h \nabla^d R_{aefi} \nabla_i R_{bgch} - \frac{1}{60} x^d x^e x^f x^g x^h \nabla_{de} R_{afgi} R_{bich} - \frac{1}{60} x^d x^e x^f x^g x^h \nabla_{de} R_{afgi} R_{bhci} - \frac{2}{135} x^d x^e x^f x^g x^h R_{aicd} R_{befj} R^g{}_{ihj} \\
& - \frac{2}{135} x^d x^e x^f x^g x^h R_{adci} R_{befj} R^g{}_{ihj} + \frac{2}{135} x^d x^e x^f x^g x^h R_{aidj} R_{befi} R_{cghj} + \frac{1}{135} x^d x^e x^f x^g x^h R_{aidj} R_{befj} R_{cghj} - \frac{1}{60} x^d x^e x^f x^g x^h R_{bdei} \nabla_{af} R_{cghj} \\
& - \frac{1}{60} x^d x^e x^f x^g x^h R_{bdei} \nabla_{cf} R_{aghi} - \frac{1}{60} x^d x^e x^f x^g x^h R_{bdei} \nabla_{fg} R_{aich} - \frac{1}{60} x^d x^e x^f x^g x^h R_{bdei} \nabla_{fg} R_{ahci} + \frac{1}{135} x^d x^e x^f x^g x^h R_{adij} R_{befi} R_{cghj} \\
& - \frac{1}{60} x^d x^e x^f x^g x^h R_{bdei} \nabla^f{}_i R_{agch} - \frac{1}{72} x^d x^e x^f x^g x^h \nabla_a R_{cdei} \nabla_f R_{bgjh} - \frac{1}{72} x^d x^e x^f x^g x^h \nabla_c R_{adei} \nabla_f R_{bgjh} - \frac{1}{36} x^d x^e x^f x^g x^h \nabla_d R_{aice} \nabla_f R_{bgjh} \\
& - \frac{1}{36} x^d x^e x^f x^g x^h \nabla_d R_{aeci} \nabla_f R_{bgjh} - \frac{1}{72} x^d x^e x^f x^g x^h \nabla^i R_{adce} \nabla_f R_{bgjh} - \frac{1}{60} x^d x^e x^f x^g x^h \nabla_{de} R_{bfgi} R_{aich} - \frac{1}{60} x^d x^e x^f x^g x^h \nabla_{de} R_{bfgi} R_{ahci}
\end{aligned}$$