The Econometric Specification of Input Demand Systems Implied by Cost Function Representations

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Summary: In the case of input demand systems based on specification of technology by a Normalized Quadratic cost function, it is common to estimate either a system of $n-1$ demand equations alone, or to supplement them by the cost function. In this paper it is argued that, in either case, because of an adding up condition, the implied error structure is inadmissible and estimation of the model is not invariant to the deleted equation. Alternatively, if the technology is specified by a Translog cost function, it is common to estimate either a system of share equations alone, or to supplement them by the cost function. By adding up, one of the share equations is excluded, and the corresponding system of n-1 share equations is essentially incomplete, whereas if the n-1 share equations are supplemented by the cost function the implied error structure of the cost function is likely to be empirically inconsistent with the errors of the share equations. For these two functional forms, alternative estimation strategies are suggested that are complete, admissible and invariant to the deleted equation. The issues are illustrated by comparison of estimated elasticities for various specifications using a pseudo-micro dataset for Australian broadacre agricultural production.

Keywords: Cost Function; Input demands; Share equations; Translog; Normalized Quadratic; Error specification.

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1. Introduction

In many areas of applied econometrics such as for example production theory, energy economics and agricultural economics, it is common to specify the technology by a cost function, and by far the majority of empirical applications employ either a Translog cost function or a Normalized Quadratic cost function. Estimation by maximum likelihood or SUR methods is often justified by an appeal to the analogous results in consumer demand estimation. In the case of consumer demand systems with exogenous total expenditure, it is well known that estimation of a system of *n* expenditure equations or *n* share equations leads to a complete, but degenerate, system specification, and deleting an arbitrary equation leads to no loss of information. Parameter estimates are invariant to the deleted equation. See, for example, Barten (1969), Powell (1969), Bewley (1986) and McLaren (1990).

In the case of input demand systems based on specification of technology by a Normalized Quadratic cost function, it is common to estimate either a system of *n*-1 demand equations alone, or to supplement them by the cost function, either in log or level form. In either of these cases it is shown in this paper that the implied error structure is inadmissible and thus is inconsistent with the implicit assumptions of the estimation method and lacks invariance. In the case of input demand systems based on specification of technology by a Translog cost function, it is common to estimate either a system of share equations alone, or to supplement them by the cost function, typically in log form. By adding up, one of the share equations is excluded. However, because the level of cost is endogenous rather than exogenous, the analogy with consumer demand systems is misleading. Translation to shares actually induces a spurious adding up condition. It is argued in this paper that a system of *n*-1 share equations is essentially incomplete, whereas if the *n*-1 share equations are supplemented by the cost function the implied error structure of the cost function is empirically inconsistent with the error structure of the shares.

Alternative econometric specifications for the two functional forms are suggested that are complete, admissible and invariant to the deleted equation. More generally, the Translog and Normalized Quadratic are just two typical applications, and the results extend to cost function specification in general. Similar issues would also arise in technology specified by profit functions or revenue functions. The various systems of models associated with alternative specifications are estimated using a pseudo-micro dataset for Australian broadacre agricultural production, and the resulting estimates for economic measures of interest are reported.

2. The Typical Translog and Normalized Quadratic Specifications

To introduce ideas, consider the case of production theory where technology is modelled by the cost function. Define n inputs $x=\left[x_1, x_2, \ldots, x_n\right]'$ with prices $w=\left[w_1, w_2, \ldots, w_n\right]'$, m outputs $y=\left[y_1, y_2, \ldots, y_m\right]'$ and v fixed factors $z=\left[z_1, z_2, \ldots, z_v\right]'$. Then the cost function is defined as:

$$C(w, y, z)=\underset{x}{\mathrm{Min}}\left\{\sum_{i=1}^{n} w_i x_i : x, z \text{ produce } y\right\}=\sum_{i=1}^{n} w_i X_i(w, y, z) \qquad (2.1)$$

where the $X_i(w,y,z)$ denote the cost minimizing input demand equations[1]. The cost function inherits the standard regularity properties: nonnegativity; homogeneity; monotonicity and concavity (see Chambers (1988). The two prime examples of functional forms used are the Translog (TL) cost function, and the Normalized Quadratic (NQ) cost function. The typical procedures used to derive estimating equations are as follows.

For the TL the cost function may be specified as

$$\begin{aligned} \ln C(w,y,z) = {} & \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln w_i + \sum_{k=1}^{m} \beta_k \ln y_k \\ & + \tfrac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} \ln w_i \ln w_j + \tfrac{1}{2} \sum_{k=1}^{m} \sum_{l=1}^{m} \beta_{kl} \ln y_k \ln y_l \\ & + \sum_{i=1}^{n} \sum_{k=1}^{m} \delta_{ik} \ln w_i \ln y_k + \sum_{g=1}^{v} \psi_g \ln z_g + \sum_{i=1}^{n} \sum_{g=1}^{v} \rho_{ig} \ln w_i \ln z_g \\ & + \sum_{k=1}^{m} \sum_{g=1}^{v} \varphi_{kg} \ln y_k \ln z_g + \tfrac{1}{2} \sum_{g=1}^{v} \sum_{h=1}^{v} \psi_{gh} \ln z_g \ln z_h . \end{aligned} \tag{2.2}$$

Applying (the logarithmic form of) Shephard's Lemma gives the system of input demand equations in share form

$$S_i(w,y,z) = \alpha_i + \sum_{j=1}^{n} \alpha_{ij} \ln w_j + \sum_{k=1}^{m} \delta_{ik} \ln y_k + \sum_{g=1}^{v} \rho_{ig} \ln z_g \qquad i = 1,\ldots,n \tag{2.3}$$

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[1] The notation is that lower case letters denote variables, while uppercase letters represent the functions representing the decision variables as functions of the givens, the exogenous variables. Greek letters are reserved for parameters.

where the cost shares are defined by $S_i(w,y,z)=\frac{w_i X_i(w,y,z)}{C(w,y,z)}$. Regularity conditions on the cost function (2.2) lead to the following common restrictions on the system (2.3):

Symmetry[2]: $\alpha_{ij}=\alpha_{ji}$

Homogeneity: $\sum_{j=1}^{n}\alpha_{ij}=0; i=1,\ldots,n$

Adding up:

$$\sum_{j=1}^{n}\alpha_i=1;\sum_{i=1}^{n}\alpha_{ij}=0,\ j=1,\ldots,n;\sum_{i=1}^{n}\delta_{ik}=0,k=1,\ldots,m;\sum_{i=1}^{n}\rho_{ig}=0,g=1,\ldots,v.$$

In fact these symmetry restrictions follow from the twice continuous differentiability of the cost function (2.2), so a complete set of parameter restrictions would add the further symmetry restrictions $\beta_{kl}=\beta_{lk};\psi_{gh}=\psi_{hg}$. Both the homogeneity and the adding up restrictions on system (2.3) follow from the homogeneity of degree one in *w* of the cost function. Restrictions imposed by monotonicity of the cost function (that the $X_i(w,y,z)$ be nonnegative, which also implies that the cost function is nonnegative) and the concavity of the cost function (that the Hessian matrix of the cost function i.e. the matrix $\frac{\partial X_i(w,y,z)}{\partial w_j}$ be negative semidefinite) are usually not

[2] Strictly speaking, symmetry only applies to the coefficients in the share equations. However, since nonsymmetric parameters are not identified in the cost function, introduction of notation for asymmetric parameters seems unnecessary.

imposed a priori, but may be checked for a particular sample ex post. In typical empirical applications, a subset of *n*-1 shares from the system (2.3) is estimated. Sometimes this system is augmented by the cost function (2.2), in which case the additional symmetry restrictions become relevant. Applications of the Translog function are too numerous to list. Examples of papers that estimate the share system alone are Binswanger (1974) and Fuss (1977), while an examples of share systems complemented by the cost function are Christensen and Greene (1976) and Sickles and Streitwieser (1998).

In the case of the NQ, define normalized cost and normalized input prices as the total variable cost and a subvector of input prices, both normalized by the price of the n^{th} input:

$$\begin{aligned}&\frac{C(w,y,z)}{w_n} = C(\tilde{w},1,y,z) \equiv \tilde{C}(\tilde{w},y,z) \qquad \text{(using HoD1 of } C\text{)}\\ &\tilde{w} = \left[\tilde{w}_1,\tilde{w}_2,\ldots\tilde{w}_{n-1}\right]' = \left[\frac{w_1}{w_n},\ldots,\frac{w_{n-1}}{w_n}\right]'.\end{aligned} \tag{2.4}$$

The cost function in NQ form typically has a representation like the following expression (see, for example, Shumway (1983) and Moschini (1988))[3]:

[3] The symbol α_n has been used in place of the more common α_0, anticipating its role in the equation for the n^{th} demand.

$$\tilde{C}(\tilde{w},y,z)=\alpha_n+\sum_{i=1}^{n-1}\alpha_i\tilde{w}_i+\sum_{k=1}^{m}\beta_k y_k+\tfrac{1}{2}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\alpha_{ij}\tilde{w}_i\tilde{w}_j$$
$$+\tfrac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m}\beta_{kl}y_k y_l+\sum_{i=1}^{n-1}\sum_{k=1}^{m}\delta_{ik}\tilde{w}_i y_k+\sum_{g=1}^{v}\psi_g z_g \quad (2.5)$$
$$+\sum_{i=1}^{n-1}\sum_{g=1}^{v}\rho_{ig}\tilde{w}_i z_g+\sum_{k=1}^{m}\sum_{g=1}^{v}\varphi_{kg}y_k z_g+\tfrac{1}{2}\sum_{g=1}^{v}\sum_{h=1}^{v}\psi_{gh}z_g z_h$$

where for simplicity the notation for parameters replicates the notation in the TL specification as closely as possible, but of course the individual parameters here have a quite different interpretation. A system of cost-minimizing input quantity equations can be derived by applying Shephard's lemma:

$$X_i\left(w,y,z\right)=\frac{\partial C}{\partial w_i}=\frac{\partial C}{\partial\tilde{C}}\frac{\partial\tilde{C}}{\partial\tilde{w}_i}\frac{\partial\tilde{w}_i}{\partial w_i}=w_n\frac{\partial\tilde{C}}{\partial\tilde{w}_i}\frac{1}{w_n}=\frac{\partial\tilde{C}}{\tilde{w}_i}$$
$$=\alpha_i+\sum_{j=1}^{n-1}\alpha_{ij}\tilde{w}_j+\sum_{k=1}^{m}\delta_{ik}y_k+\sum_{g=1}^{v}\rho_{ig}z_g \quad (2.6)$$
$$i=1,\ldots,n-1.$$

(At this point, the reader might expect to see the notation $\tilde{X}_i\ \left(\tilde{w},y,z\right)=\dfrac{\partial\tilde{C}}{\tilde{w}_i}$ But since demands are HoD0, $X_i\left(w,y,z\right)=X_i\left(\tilde{w},1,y,z\right)=\tilde{X}_i\ \left(\tilde{w},y,z\right)$, and hence an alternative notation is not necessary.) The homogeneity in prices condition is maintained by the normalization process, the symmetry of price effects is satisfied by the restrictions $\alpha_{ij}=\alpha_{ji}$ and the global concavity in prices of this cost function is equivalent to the restriction that the matrix of parameters $\left[\alpha_{ij}\right]_{n-1\times n-1}$ be negative semi-definite, (which, not being a function of variables, can be imposed in estimation by means of the Cholesky decomposition). Monotonicity of the cost function in input prices, which corresponds to nonnegativity of the input demands, is usually not imposed a priori, but may be checked for a particular sample ex post. Again, note that no other symmetry type restrictions occur in the equation system (2.6), because none of the parameters β_{kl}, ψ_{gh} appears in these equations, but the cost function parameters would also satisfy $\beta_{kl} = \beta_{lk}$ and $\psi_{gh} = \psi_{hg}$.

In most empirical applications, the system of *n*-1 demands (specifically excluding the equation for the numeraire) of the system (2.6) is estimated. Sometimes this system is also augmented by the cost function (2.5), but rarely by the demand function for the numeraire input *n*.

The implied equation for the numeraire commodity, $X_n(w, y, z)$ can be derived as follows. By definition,

$$C(w, y, z) \equiv \sum_{i=1}^{n} w_i X_i(w, y, z)$$

which implies that

$$\tilde{C} = \frac{C}{w_n} \equiv \sum_{i=1}^{n} \tilde{w}_i X_i(w, y, z)$$

and hence

$$X_n(w, y, z) \equiv \tilde{C}(\tilde{w}, y, z) - \sum_{i=1}^{n-1} \tilde{w}_{\boldsymbol{i}} X_i(w, y, z)$$

where the dependence on w, y, z is to remind us that this is a relation among functions, as well as among variables. The implications of this inherent asymmetry of the NQ function are probably easier to see in terms of the implied cost function, rather than the normalized cost function. The implied structure of the corresponding (un-normalized) cost function is

$$C(w,y,z)=\alpha_n w_n+\sum_{i=1}^{n-1}\alpha_i w_i+\sum_{k=1}^{m}\beta_k y_k w_n+\tfrac{1}{2}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\alpha_{ij}w_i\frac{w_j}{w_n}$$
$$+\tfrac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m}\beta_{kl}y_k y_l w_n+\sum_{i=1}^{n-1}\sum_{k=1}^{m}\delta_{ik}w_i y_k+\sum_{i=1}^{n-1}\sum_{g=1}^{v}\rho_{ig}w_i z_g \tag{2.7}$$
$$+\sum_{g=1}^{v}\psi_g z_g w_n+\sum_{k=1}^{m}\sum_{g=1}^{v}\varphi_{kg}y_k z_g w_n+\tfrac{1}{2}\sum_{g=1}^{v}\sum_{h=1}^{v}\psi_{gh}z_g z_h w_n$$

which emphasises the extremely asymmetric treatment of the n^{th} input. Applying Shephard's Lemma then gives the *n* input demand equations directly:

$$X_i(w,y,z)=\frac{\partial C(w,y,z)}{\partial w_i}=\alpha_i+\sum_{j=1}^{n-1}\alpha_{ij}\frac{w_j}{w_n}+\sum_{k=1}^{m}\delta_{ik}y_k+\sum_{g=1}^{v}\rho_{ig}z_g \qquad i=1,\ldots,n-1$$
$$X_n(w,y,z)=\frac{\partial C(w,y,z)}{\partial w_n}=\alpha_n+\sum_{k=1}^{m}\beta_k y_k-\tfrac{1}{2}\sum_{i=1}^{n-1}\sum_{j=1}^{n-1}\alpha_{ij}\frac{w_i}{w_n}\frac{w_j}{w_n} \tag{2.8}$$
$$+\tfrac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m}\beta_{kl}y_k y_l+\sum_{k=1}^{m}\sum_{g=1}^{v}\varphi_{kg}y_k z_g+\sum_{g=1}^{v}\psi_g z_g+\tfrac{1}{2}\sum_{g=1}^{v}\sum_{k=1}^{v}\psi_{gh}z_g z_h$$

which further emphasises the asymmetric treatment of input *n*.

3. Estimation Issues

These two alternative "standard" specifications, which account for the majority of empirical applications in areas as diverse as agricultural economics and energy demand, raise a number of questions. The two particular functional forms, TL and NQ, are merely two common alternative specifications of the technology as characterized by the cost function. So apart from mathematical simplicity, why carry out estimation based on shares in one case, and quantities demanded in another? The appropriate choice of transformation of endogenous variables for estimation purposes should be based on the implied statistical properties of the error terms, which are usually introduced after this specification stage. Is it even appropriate to estimate a set of *demand* equations such as (2.6)? Is it logical, or even necessary, to delete the *n*th equation of such a system? Can, or should, the cost function be appended to either of the systems (2.3) or (2.6)? And if so, should cost be represented in level or log form?

To begin to answer these questions, abstract from the issues raised by specific choices of functional forms. The starting point is the theory of production and the application of duality. Given a primal technology characterised by $V(y,z)$, where $x \in V(y,z)$ indicates that, given the vector of fixed factors *z*, the output vector *y* can be produced by the input vector *x*, then the behaviour of a cost minimizing firm can be characterised by the cost function $C(w,y,z)$ which is defined as the minimum cost of producing a given vector of outputs subject to the production technology:

$$C(w,y,z) = \underset{x}{\text{Min}} \left\{ \sum_{i=1}^{n} w_i x_i : x \in V(y,z) \right\} = \sum_{i=1}^{n} w_i X_i(w,y,z) . \qquad (3.1)$$

Then $C(w,y,z)$ satisfies the standard regularity conditions of a cost function: non-negative; concave in *w*; non-decreasing in *w*; homogeneous of degree 1 in *w*; and increasing in *y*. $C(w,y,z)$ is said to be dual to the specification of technology *V*. The structure of *C* contains both the structure of technology and the results of optimization, in this case cost minimization. For more details see Chambers (1988). The resulting demand equations are often called Hicksian demands, in order to reinforce the analogy with consumer demand, where they would correspond to income compensated (utility constant) demands. Application of the Envelope Theorem to (3.1) gives Shephard's Lemma:

$$X_i(w,y,z) = C_{w_i}(w,y,z) = \frac{\partial C(w,y,z)}{\partial w_i}, i = 1,\ldots,n, \qquad (3.2)$$

the input demand equations. These equations are analogous to the Hicksian demand equations of consumer demand theory (though in this case the output vector y is observable, whereas in Hicksian demands the level of utility is unobservable). Provided the cost function is twice continuously differentiable in prices then Young's Theorem implies

$$\frac{\partial X_i}{\partial w_j} = C_{w_i w_j} = C_{w_j w_i} = \frac{\partial X_j}{\partial w_i}, \quad i, j = 1, \ldots, n \tag{3.3}$$

which is the equivalent of Slutsky symmetry in consumer demand, and here implies the symmetry of the cross price responses of input demand equations.

By the property of homogeneity of degree one of the cost function, Euler's Theorem implies the fundamental identity that

$$C(w, y, z) \equiv \sum_{i=1}^{n} w_i C_{w_i}(w, y, z) \equiv \sum_{i=1}^{n} w_i X_i(w, y, z) . \tag{3.4}$$

In the notation implicit in the above, and that has been used so far, the exogenous variables, the variables taken as given in the optimization problem (3.1), have been denoted by lower case letters w, y, z and can be identified directly with the data on these variables. The solutions for the decision variables in the optimization problem (3.1) have been denoted by capital letters, $C(w, y, z),\ X_i(w, y, z),$ $S_i(w, y, z),\ i = 1, \ldots, n$ indicating that they are derived functions of the exogenous variables. The endogenous variables will be identified with the observed data corresponding to these decision variables, and will be denoted by the corresponding lower case letters $c, x_i, s_i, i = 1, \ldots, n$. In practice, of course, the model does not fit the data exactly, and this leads to the specification of an appropriate statistical model to complement the economic model. The economic model corresponds to a parameterized form of the cost function, say $C(w, y, z; \theta)$ (where at this point θ simply represents all possible parameters characterising the cost function), which implies the specification of the conditional means. Adding a set of errors then leads to a set of stochastic equations which might be written most simply as follows:

$$\begin{aligned} c_t &= C(w_t, y_t, z_t; \theta) + v_t \\ x_{it} &= X_i(w_t, y_t, z_t; \theta) + u_{it}, \quad i = 1, \ldots, n \\ t &= 1, \ldots, T \end{aligned} \tag{3.5}$$

where the subscript t denotes variation over a sample.

The role of the errors in the following models is what could be described as standard in the applied literature. Theory is used to establish the functional form of the conditional mean of the data, say the $X_i(w_t, y_t, z_t; \theta)$, and the errors take the role of the "unexplained" variation. This is in contrast to a stream of literature that attempts to model the errors explicitly based on a random cost function, to be discussed further in the conclusion. However, the structure of these additive errors is still quite constrained because of the theory. The accounting definition of variable cost, and the corresponding implied properties of the cost function, imply the two identities:

$$\begin{aligned} c &\equiv \sum_{i=1}^{n} w_i x_i \\ C(w, y, z) &\equiv \sum_{i=1}^{n} w_i X_i(w, y, z) \end{aligned} \tag{3.6}$$

which impose structure on the implied error terms. Typical notation misses the distinction between the two related, but separate, restrictions in (3.6).

Returning to the basic system (3.5), SUR or ML type estimation of this system would typically (implicitly) assume that the n+1 vector of errors $\left(u_{1t}, u_{2t}, \ldots, u_{nt}, v_t\right)$ be independently and identically distributed with constant (parameterized) variances and covariances. But this would lead to a logical inconsistency. The data will satisfy $c_t \equiv \sum_{i=1}^{n} w_{it} x_{it}$ by construction, and this identity together with the functional identity (3.4) produces an identity involving the error terms and exogenous variables:

$$v_t \equiv \sum_{i=1}^{n} w_{it} u_{it} . \tag{3.7}$$

In other words, the errors in the stochastic form in (3.5) do not satisfy the "usual" adding up condition. Since the exogenous variables are arbitrary and time varying, identity (3.7) means that, if the errors of the demand system have constant variances and covariances, then it is impossible for the errors of the cost function, the v_t, to have constant variance or constant covariances with the u_{it}. Thus while the n+1 dimensional system (3.5) is in some sense degenerate, in that any n dimensional subset implies the (theoretical and statistical) structure of the remaining equation, this degeneracy is data specific, and hence the usual arguments for the invariance of parameter estimates to the deleted equation do not carry through. Estimates of systems of demand equations, such as the NQ model (2.6), with or without the cost function, are based on an internal logical inconsistency! For want of a better term, systems with stochastic specifications like (3.5) will be referred to as "technically inadmissible".

An analogy with consumer demand systems is illuminating in a number of respects. While demand equations for goods are of fundamental interest in the theory of consumer demand, and are the basic building blocks of derived measures such as income and price elasticities, empirical work in consumer demand rarely (if ever?) estimates demand equations directly. Typically estimation is of expenditure systems, such as the Linear Expenditure system, or share systems, such as the Almost Ideal Demand System of Deaton and Muellbauer (1980) or the Translog Demand System of Christensen, Jorgenson and Lau (1975). In each case this consists of the use of exogenous variables to transform one set of endogenous variables, the demands, to derived sets of endogenous variables, either to expenditures by multiplying individual demands by their corresponding prices, or further to shares by then dividing expenditures by total expenditure. Individual *demands* are subject to a data-dependent degeneracy; the sum of the n equations of consumer demands multiplied by respective prices is an exogenous variable expenditure. Estimating a system of n demands would be analogous to estimating the n dimensional subsystem of (3.5), and in the terminology introduced there would be inadmissible. This is presumably the reason why systems of demand equations are rarely estimated, although it is hard to find a succinct argument based on the above justification in the literature. In contrast, individual *expenditures* add identically to (exogenous, in the case of consumer demand) expenditure, and *shares* add identically to unity, and in both cases the error in any one equation can be expressed as a (not data-dependent) linear combination of the remaining n-1 errors, and estimates are invariant to the equation deleted. (See, for example, Barten (1969), Powell (1969), Bewley (1986), McLaren (1990).)

Based on the insights from this analogy, consider first the case of transforming input demands to expenditures. Then system (3.5) is translated to

$$
\begin{aligned}
c_t &= C(w_t, y_t, z_t; \theta) + v_t \\
w_{it} x_{it} &= w_{it} X_i(w_t, y_t, z_t; \theta) + u_{it}, \quad i = 1, \ldots, n
\end{aligned}
\tag{3.8}
$$

where it may be noted that all left-hand side variables, and hence right-hand side errors, now have common units of measurement. (As is the case with the notation for parameters, the same notation will be used for errors with quite different statistical properties in different specifications. This avoids a proliferation of notation, and is meant to reinforce the assumption that any errors should have "reasonable" statistical properties.) The restriction on the error terms that is the analog of (3.7) is now

$$
v_t \equiv \sum_{i=1}^{n} u_{it} \tag{3.9}
$$

which is in the form of a "standard" adding-up condition. In this form the implicit assumption of SUR or ML type estimation that the *n*+1 vector of errors $\left(u_{1t}, u_{2t}, \ldots, u_{nt}, v_t\right)$ be i.i.d with constant (parameterized) variances and covariances is mathematically possible. In other words, at the very least, internal consistency requires that NQ demands be estimated in terms of expenditures on the *n* inputs, not their quantities, or else (equivalently, by invariance) by appending the cost function to the *n*-1 dimensional system of input expenditures. A subset of *n*-1 input quantity demands is economically incomplete and statistically inadmissible. By comparison, the system of equations in (3.8) is economically complete, statistically admissible and invariant to the deleted equation. (Although the question of empirical legitimacy remains, since the errors have units of measurement of current dollars, and hence are unlikely to have constant variances.)

Consider now the estimation in terms of shares. In this case, the analogy with consumer demand is quite misleading (or enlightening?). In consumer demand systems, transformation to shares involves division of the endogenous variables (individual commodity expenditures) by an exogenous variable, (predetermined total expenditure). In maximizing utility subject to a budget constraint, total expenditure is given. But in minimizing cost subject to a production technology, cost is a decision variable and hence endogenous. In particular, transforming the demands to cost share form by dividing by either *c* (the data) or $C(w, y, z)$ (the modelled expenditures) is a conceptually different process.

One way to think about this is to return to the set of functions that result from the optimization problem. Logically prior to any issues of estimation, these can be written as the (theoretical and degenerate) system:

$$\begin{aligned} &C(w, y, z) \\ &w_i X_i(w, y, z) \quad i = 1, \dots, n. \end{aligned} \tag{3.10}$$

This system can be nonlinearly transformed to a related but mathematically equivalent system by dividing each of the last *n* equations by the first equation (but maintaining the first equation):

$$\begin{aligned} &C(w, y, z) \\ &S_i(w, y, z) \quad i = 1, \dots, n. \end{aligned} \tag{3.11}$$

(The complete sequence of transformations of the demand equations, which consists of the two steps: multiply by w_i; divide by $C(w, y, z)$, is mathematically equivalent to applying Shephard's Lemma in logarithmic form instead of Shephard's Lemma in normal form.)

The corresponding empirical form of system (3.11) is

$$
\begin{aligned}
c_t &= C(w_t, y_t, z_t; \theta) + v_t \\
s_{it} &= S_i(w_t, y_t, z_t; \theta) + u_{it}, \quad i = 1, \ldots, n.
\end{aligned}
\tag{3.12}
$$

In this form, the data will satisfy the identity $\sum_{i=1}^{n} s_{it} \equiv 1$ by construction, and this identity together with the functional identity $\sum_{i=1}^{n} S_i\left(w_t, y_t, z_t\right) \equiv 1$ produces an identity involving the last *n* of the error terms:

$$
\sum_{i=1}^{n} u_{it} \equiv 0 . \tag{3.13}
$$

It is well-known that estimation of the *n*-1 share sub-system of (3.12) will be invariant to the deletion of any one of the *n* share equations. Similarly, estimation of the complete system (3.12) by including the cost equation will also be invariant to the deletion of any one of the *n* share equations. However, while this complete system (3.12) may be mathematically logical, it is unlikely to be empirically admissible. The reason for estimating in share form is to transform to a system where the errors are likely to be homoscedastic, by expressing the left hand variables as shares, bounded between 0 and 1, rather than as variables measured in current dollars. But one equation of the complete system, the cost equation, is still measured in current dollars. It is empirically unsustainable to assume that the errors of the cost function, the v_t, have constant variance or constant covariances with the u_{it}. Estimation of any of the *n*-1 shares as a system on their own may be admissible, though incomplete, but appending the cost function (either in levels form or in logarithmic form) in order to recover estimates of the additional parameters seems unadvisable, and this problem will be referred to as “empirical inadmissibility”.

Returning to the analogy with consumer demand systems, if what is sought is a logically complete system of estimating equations for which an assumption of a constant variance-covariance matrix for the vector of errors may be empirically admissible, scaling of data should be carried out using an exogenous variable. For the cost function model, obvious candidates are the exogenous variables *w*, *y* and *z*. In fact, given the structure of the Normalized Quadratic cost function, one obvious candidate is to deflate all expenditures by the price of the input that is used as the numeraire in the definition of normalized costs. Thus the first *n*-1 inputs could be measured as expenditures normalized by w_n (i.e. in units of the n^{th} input), input *n* simply expressed in quantity form, and cost measured as normalized cost, to give the system

$$\begin{aligned} \tilde{c}_t &= \frac{c_t}{w_{nt}} = \frac{C(w_t, y_t, z_t; \theta)}{w_{nt}} + v_t \\ \tilde{w}_{it} x_{it} &= \frac{w_{it} x_{it}}{w_{nt}} = \frac{w_{it} X_i(w_t, y_t, z_t; \theta)}{w_{nt}} + u_{it} \\ &= \tilde{w}_{it} X_i(w_t', y_t, z_t; \theta) + u_{it}, \quad i = 1, \ldots, n \end{aligned} \tag{3.14}$$

On the other hand, data for *w* are usually not firm or scale specific and would seem unsuitable on their own. And again, it would appear odd to use a particular transformation of data to achieve a certain statistical property simply because of the particular choice of functional form for the mean equation. Variables in *y* and some *z*, such as fixed factors, are observation and scale specific; however these variables are real, whereas the variables in the logically complete system (3.8) are nominal. One possible option would be to remove price effects by first dividing all of the theoretical equations by an index of the *w*, or some form of generally available price index, such as the CPI or a PPI, and then by some measure of scale, such as an index of the elements of *y* and/or those elements of *z* that are observation specific. However, if the prices of outputs are available, a more obvious procedure is the following. If the prices of outputs are given by the *n* vector *p*, define (observed) revenue as $r_t \equiv \sum_{j=1}^{m} p_{jt} y_{jt}$. This variable has the advantage of being measured in nominal dollars, and being a natural measure of the size of the particular firm (and perhaps the most obvious analogy to total expenditure in demand systems).

This logic leads to the system (3.8) being modified to

$$\begin{aligned} \frac{c_t}{r_t} &= \frac{C(w_t, y_t, z_t; \theta)}{r_t} + v_t \\ \frac{w_{it} x_{it}}{r_t} &= \frac{w_{it} X_i(w_t, y_t, z_t; \theta)}{r_t} + u_{it}, \quad i = 1, \ldots, n. \end{aligned} \tag{3.15}$$

This system of *n*+1 equations contains all of the information from the paradigm of cost minimization, and embodies a set of errors that are logically consistent and likely to empirically satisfy the implicit assumptions involved in ML or SUR estimation. Again, the errors satisfy the identity $v_t \equiv \sum_{i=1}^{n} u_{it}$ and estimation can proceed by estimating any *n* dimensional subset of equations. Parameter estimates will be invariant to the equation deleted. Note that the equation corresponding to the cost function has no specific role in system (3.15), such as allowing the estimation of parameters that are unavailable from the other equations. This is obvious since *any* equation in (3.15) can be expressed as a linear combination of the other *n* equations. The fact that appending the Translog cost function (2.2) to the share system (2.3), or appending the NQ cost function (2.5) to the demand system (2.6), allows the estimation of additional parameters such as the $\alpha_0, \beta_k, \beta_{kl}, \phi_{kg}, \psi_g, \psi_{gh}$ simply reflects the loss of information that occurs when estimating incomplete systems such as (2.3) or (2.6).

4. Further Issues with the Estimation of NQ Systems

The above issues are further confused in the case of the NQ system, because of the naturally asymmetric treatment of the n^{th} factor. In fact, there are always n possible specific NQ specifications, according to which of the n inputs is treated as the numeraire. Applying the reasoning above, an econometrically compatible system of n+1 equations is given by the cost function in (2.7) plus the input expenditure equations:

$$
\begin{aligned}
w_i x_i &= \alpha_i w_i + \sum_{j=1}^{n-1} \alpha_{ij} w_i \frac{w_j}{w_n} + \sum_{k=1}^{m} \delta_{ik} y_k w_i + \sum_{g=1}^{v} \rho_{ig} z_g w_i \qquad i = 1, \ldots, n-1 \\
w_n x_n &= \alpha_n w_n + \sum_{k=1}^{m} \beta_k y_k w_n - \tfrac{1}{2} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \alpha_{ij} w_i \frac{w_j}{w_n} + \tfrac{1}{2} \sum_{k=1}^{m} \sum_{l=1}^{m} \beta_{kl} y_k y_l w_n \\
&\quad + \sum_{k=1}^{m} \sum_{g=1}^{v} \varphi_{kg} y_k z_g w_n + \sum_{g=1}^{v} \psi_g z_g w_n + \tfrac{1}{2} \sum_{g=1}^{v} \sum_{h=1}^{v} \psi_{gh} z_g z_h w_n .
\end{aligned} \tag{4.1}
$$

The above system highlights a more fundamental asymmetry in the treatment of inputs in the NQ model[4]. Comparing the first n-1 expenditure equations with the expenditure equation for input n, it is clear that all higher order interaction terms in the cost function have in fact been loaded on the n^{th} factor. This may explain why attempts to estimate systems of n-1 demand equations plus the cost function

---

[4] Shumway and Gottret (1991) discussed this asymmetry. Symmetric treatments of the NQ system do exist, but they introduce a number of unidentified additional parameters. See Diewert and Wales (1987).

sometimes fail. There appears no logical reason why the n^{th} factor should be singled out for this responsibility. One possible response is to note that while the standard specification of the NQ cost function appears general in regard to its specification in terms of the variables *w*, *y*, and *z*, it is in fact highly constrained in regard to its treatment of the technical characteristics of the n^{th} factor relative to its treatment of all other factors. More logically, given the structure of the first *n*-1 expenditure equations it would seem that a reasonable set of *n* input demand equations, still using the n^{th} price as a numeraire, would be to specify the system:

$$\begin{aligned} w_i x_i &= \alpha_i w_i + \sum_{j=1}^{n-1} \alpha_{ij} w_i \frac{w_j}{w_n} + \sum_{k=1}^{m} \delta_{ik} y_k w_i + \sum_{g=1}^{v} \rho_{ig} z_g w_i \qquad i = 1, \ldots, n-1 \\ w_n x_n &= \alpha_n w_n - \tfrac{1}{2} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \alpha_{ij} w_i \frac{w_j}{w_n} + \sum_{k=1}^{m} \beta_k y_k w_n + \sum_{g=1}^{v} \psi_g z_g w_n \end{aligned} \tag{4.2}$$

with the implied cost function

$$\begin{aligned} C(w_t, y_t, z_t; \beta) = \alpha_0 w_n &+ \sum_{i=1}^{n-1} \alpha_i w_i + \sum_{k=1}^{m} \beta_k y_k w_n + \tfrac{1}{2} \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} \alpha_{ij} w_i \frac{w_j}{w_n} \\ &+ \sum_{i=1}^{n-1} \sum_{k=1}^{m} \delta_{ik} w_i y_k + \sum_{i=1}^{n-1} \sum_{g=1}^{v} \rho_{ig} w_i z_g + \sum_{g=1}^{v} \psi_g z_g w_n . \end{aligned} \tag{4.3}$$

This is a complete *n*+1 equation system which is both economically and statistically degenerate – any *n* dimensional subset could be estimated, and estimates would be invariant to the equation deleted. Of course, the units of measurement of all equations are now (current) dollars, and homoscedasticity may require scaling all equations by a common measure of price (such as w_n, or perhaps a measure of revenue *r*). This system would be a parsimonious system that may be preferable to a completely symmetric treatment, such as using a generalized Barnett or generalized McFadden system. It is also interesting to note the potential simplification in notation that is suggested by this form. (β_k becomes δ_{nk}, ψ_g becomes ρ_{ng})

5. An Empirical Illustration

In order to illustrate the issues raised in the previous sections, and to assess their empirical significance, estimation of the various models is carried out for a substantive and unique quasi-micro level dataset. The dataset involves broadacre farms across Australia over the period from 1990 to 2005 provided by the Australian Bureau of Agricultural and Resource Economics (ABARE). It is drawn from the Australian Agricultural and Grazing Industries Survey (ABARE 2009) which collects detailed input costs, output receipts and quantities, and values and quantities of invested capital of farm businesses that have an estimated annual value of agricultural operation of $22,500 or more. The surveyed farms fall into 32 production regions. Within each production region, they are first categorized into one of two industries, cropping or livestock, based on their relative cash receipts from the two production activities. For each industry, farms are further categorized into one of three operation sizes, based on total cash receipts for the survey year, being greater than $400,000; between $200,000 and $400,000; and less than $200,000. Farms within the same region, same industry and of the same size form an observational unit or cell. The average of each variable across farms within each cell is then used provided that the number of farms in the cell is equal to or greater than five (for confidentiality reasons).

The variables used are as follows. Five inputs: x_1 Contracts, Materials and Services (CMS) for Livestock; x_2 Fertilizers and Chemicals; x_3 Other CMS; x_4 Fuel, Oil and Grease; x_5 Livestock Trading; with corresponding prices $w_1, w_2, \ldots w_5$. Four outputs:

y_1 Wheat and Other Grains; y_2 Sheep; y_3 Beef and Other Livestock; y_4 Wool; with corresponding prices $p_1, p_2, \ldots, p_4$. Two fixed inputs: z_1 Capital; z_2 Fixed Labor. In total, there are 1054 observations.

Estimation was carried out using the Full Information Maximum Likelihood system estimation method in EViews. "Invariance" is a theoretical statistical concept. Practically, it was determined that if the EViews convergence criterion was set to one in one billion (1.0E-9), then theoretically invariant systems would generate parameter estimates and maximized log likelihoods identical up to the precision reported by default in EViews, typically seven significant digits.

Because most of the economic or statistical models are not directly nested within each other, direct model comparison via parameter tests is difficult. Even with just Translog and Normalized Quadratic specifications for the cost function, the number of possible empirical specifications is overwhelming. Thus no attempt will be made to report individual parameter estimates. Initially, the issues addressed are: (i) Invariance to the deleted equation; (ii) Completeness; (ii) Admissibility. By 'complete' it is meant that the system has 5 equations and that all coefficients in the underlying cost function can be estimated. By "technically admissible" it is meant that the assumption of a constant variance-covariance matrix is at least consistent with the adding up conditions implied by the data and the model. By "empirically admissible" is meant that the assumption of a constant variance-covariance matrix is at least potentially empirically realistic; for example, in time series data the errors are not measured in units of current dollars.

For the Translog specification the following possible models were identified and estimated:

TL1: Any 4 shares of the 5 equation share system in Eq. (2.3). Invariant but incomplete.

TL2: Any 4 shares of the 5 equation share system (Eq. (2.3)), plus the cost function in log form (Eq. (2.2)) defined using the endogenous variable lcost=ln(cost). Invariant, and complete. Technically admissible, but empirically questionable. LL=7768.853. Note that writing the cost function in log form using the mathematical form ln(cost) gives identical parameter estimates, but a different log-likelihood (LL= -4996.569), reflecting the different choice of an endogenous variable (lcost vs cost). These two models are referred to as TL2a and TL2b.

TL3: Any 4 shares of the 5 equation share system in Eq. (2.3), plus the cost function in level form, defined using the exponential of the translog function. Invariant, and complete. Technically admissible, but empirically inadmissible.

TL4: Any 5 equations of the 6 equation system consisting of the 5 equations for expenditures on the 5 inputs as in Eq. (3.8), plus the cost equation in level form as in TL3. Invariant, and complete. Technically admissible, but empirically questionable.

TL5: Any 5 equations of the 6 equation system consisting of the 5 equations of expenditures on the 5 inputs as in Eq. (3.8), plus the cost equation in level form as in TL3, all normalized by the price of input 5. Invariant, complete, technically and empirically admissible.

TL6: Any 5 equations of the 6 equation system consisting of the 5 equations of expenditures on the 5 inputs as in Eq. (3.8), plus the cost equation in level form as in

TL3, all normalized by revenue. Invariant, complete, technically and empirically admissible.

Note that only TL4 to TL6 are truly invariant, in the sense that any subset of 5 of the 6 equations contains all of the economic and statistical information. In addition, in each of these three systems all endogenous variables are measured in common units of measurement.

For the Normalized Quadratic specification, there is an essential asymmetry in that input 5 plays a special role, and in fact there are as many alternative cost function specifications as there are inputs. However, conditional on normalizing on input 5 price, a number of corresponding models can be identified:

NQ1: Input quantities 1 to 4 (Eq. (2.6)). No invariance to deleted equation. Incomplete and inadmissible.

NQ2: Input quantities 1 to 5 in Eq. (2.8). Invariance is not an issue. Complete but technically inadmissible.

NQ3: Input quantities 1 to 4 in Eq. (2.6) plus the normalized cost function in Eq. (2.5). No invariance. Complete but technically inadmissible.

NQ4: Input quantities 1 to 5 in Eq. (2.8) plus the normalized cost function in Eq. (2.5). Even though this system is economically degenerate, in the sense that the sum of the input quantities weighted by normalized input prices reproduces the normalized cost function, both in terms of the data and in terms of functional forms, when estimated using standard error specifications the 6 equation system can be estimated.

There is no error degeneracy, because the normalized input prices vary over the sample. Invariance is not an issue. Complete but technically inadmissible.

NQ5: Any 5 equations of the 6 equation system consisting of the 5 input expenditures normalized by w_5 (Eq. (3.14)), plus the normalized cost function in Eq. (2.5). Invariant, complete and technically and empirically admissible. (Note that the 6-equation system consisting of expenditures 1 to 5 normalized by w_5, plus normalized cost, is singular as expected.)

NQ6: Any 5 equations of the 6 equation system consisting of the 5 input expenditures, unnormalized as in Eq. **Error! Reference source not found.**, plus cost in (2.7). Invariant, complete and technically admissible but empirically inadmissible. (Note that the system of expenditures 1 to 5, plus cost, is singular as expected.)

NQ7: Any 5 equations of the 6 equation system consisting of the 5 input expenditures normalized by revenue, plus cost normalized by revenue (Eq. (3.15)). Invariant, complete and technically and empirically admissible. (Note that the system of expenditures 1 to 5 normalized by revenue, plus cost normalized by revenue, is singular as expected.)

NQ8: Using the simplified form of the normalized cost function, any 5 equations of the 6 equation system consisting of the 5 input expenditures in Eq. (4.2), plus cost in Eq. (4.3), all normalized by w_5. Invariant, complete and technically and empirically admissible.

NQ9: Using the simplified form of the normalized cost function, any 5 equations of the 6 equation system consisting of the 5 input expenditures, plus cost, as in Equations

(4.2) and (4.3), but all normalized by revenue. Invariant, complete and technically and empirically admissible.

A summary of comparison across the six Translog and nine Normalized Quadratic system models is given in Table 1. Only systems TL5-6, NQ5, and NQ7-9 satisfy our required theoretical and empirical properties in terms of non-degeneracy, invariance, completeness, and technical and empirical admissibility. In terms of statistical model selection, only those systems where the endogenous variables coincide can be compared. As shown in Table 1, four groups of models are compared using an AIC criterion. In general, Translog models are preferred to the Normalized Quadratic counterparts on statistical grounds. For TL2 versus TL3, TL2 is preferred by AIC. In addition, between the full version and the parsimonious version of the NQ models, NQ5 versus NQ8 and NQ7 versus NQ9, the full specifications (NQ5 and NQ7) are slightly favoured by statistical tests. In addition, the simplified NQ cost function is a nested special case of the standard NQ function, so standard chi-square likelihood ratio tests can also be applied to test NQ5 against NQ8, and NQ7 against NQ9. These reach the same model comparison conclusions as those from AIC.

While comparison of the estimated coefficients are mostly not possible across the TL and NQ models, a comparison of estimates of economic measures of interest across the various specifications is revealing. One economic measure of significance that is commonly reported is the elasticity of input substitution. While this is not well-defined for more than two inputs, for simplicity we will concentrate on the widely used Allen-Uzawa Elasticity of Substitution (AES), derived from the cost function according to the formula:

$$AES_{ij}(w,y,z)=\frac{CC_{w_i w_j}}{C_{w_i}C_{w_j}} \qquad (5.1)$$

where the notation indicates that the AES is not a parameter, but varies across the sample of exogenous variables. Summary measures will report this statistic evaluated at the arithmetic mean of the arguments (w, y, z). Because of the structure of equation (5.1) it would appear that all of the parameters of the cost function would be needed to construct an estimate of the AES. However, it is a characteristic of the Translog cost function (due to the exponential form when expressed as cost rather than log cost) that the AES can be constructed using only the estimates derived from the *n*-1 input share equations. This is not the case with the NQ, for which estimates of *n*-1 demand equations alone is not sufficient, which emphasises the importance of estimating a truly invariant system in the case of the NQ.

The application of (5.1) to the Translog functional form (2.2) gives

$$AES_{ij}(w,y,z)=1+\frac{\alpha_{ij}}{S_i S_j}, i \neq j; \qquad =1+\frac{\alpha_{ii}}{S_i^2}-\frac{1}{S_i}, i=j \qquad (5.2)$$

in which S_i in upper case is a shorthand for the function of parameters and data in expression (2.3).

Application to the NQ functional form (2.7) gives

$$AES_{ij}(w,y,z)=\frac{\alpha_{ij}C'}{X_i X_j} \qquad (5.3)$$

for $i, j = 1, \ldots, n-1,$ where C' and X_i are shorthand for the functions of parameters and data in expressions (2.5) and (2.6), respectively. Note that there is insufficient information to estimate AES for the case of NQ1.

Two other economic measures of interest are the Output Elasticity of input *i* with respect to output *k*:

$$OE_{ik}(w, y, z) = \frac{\partial \ln X_i}{\partial \ln y_k} \tag{5.4}$$

and the shadow price of fixed input z_g:

$$SP_g(w, y, z) = \frac{\partial C}{\partial z_g}. \tag{5.5}$$

For the Translog these can be expressed as

$$\begin{aligned} OE_{ik}(w, y, z) &= \frac{\partial \ln S_i}{\partial \ln y_k} + \frac{\partial \ln C}{\partial \ln y_k} \\ &= \frac{\delta_{ik}}{S_i} + \beta_k + \sum_{l=1}^{m} \beta_{kl} \ln y_l + \sum_{i=1}^{n} \delta_{ik} \ln w_i + \sum_{g=1}^{v} \varphi_{kg} \ln z_g \end{aligned} \tag{5.6}$$

and

$$\begin{aligned} SP_g(w, y, z) &= \frac{\partial C}{\partial z_g} = \frac{\partial C}{\partial \ln C} \frac{\partial \ln C}{\partial \ln z_g} \frac{\partial \ln z_g}{\partial z_g} \\ &= \frac{C}{z_g} \left(\psi_g + \sum_{i=1}^{n} \rho_{ig} \ln w_i + \sum_{k=1}^{m} \varphi_{kg} \ln y_k + \sum_{h=1}^{v} \psi_{gh} \ln z_h \right). \end{aligned} \tag{5.7}$$

For the NQ, the corresponding expressions are

$$OE_{ik}(w,y,z) = \frac{\partial \ln X_i}{\partial \ln y_k} = \frac{y_k}{X_i}\frac{\partial X_i}{\partial y_k} = \frac{\delta_{ik} y_k}{X_i} \tag{5.8}$$

and

$$SP_g(w,y,z) = \sum_{i=1}^{n-1} \rho_{ig} w_i + \psi_g w_n + \sum_{k=1}^{m} \varphi_{kg} y_k w_n + \sum_{h=1}^{v} \psi_{gh} z_h w_n \tag{5.9}$$

for $i = 1,\ldots,n-1$.

Each of these derived functions, in their general forms (5.1), (5.4) and (5.5), are clearly derived using combinations of the functional form and derivatives of the underlying cost function, and hence are functions of the same arguments as the cost function, and depend on the same parameter values. The question then arises as to how to convert these functions into estimates, and how to summarize their values using appropriate summary statistics. To introduce ideas, consider first the common Hicksian elasticities

$$H_{ij}\left(w_t, y_t, z_t; \theta\right) = \frac{\partial \ln X_i\left(w_t, y_t, z_t; \theta\right)}{\partial \ln p_j} = \frac{\partial X_i\left(w_t, y_t, z_t; \theta\right)}{\partial p_j}\frac{p_j}{X_i\left(w_t, y_t, z_t; \theta\right)} \tag{5.10}$$

which, being a (log) derivative of the input demands, emphasises its dependence on exogenous variables and parameters, and that it varies with the sample data on exogenous variables. A point estimate could be constructed by replacing the parameter values by their estimates, and then averaging, say,

$$\bar{H}_{ij}\left(w_t, y_t, z_t; \hat{\theta}\right). \tag{5.11}$$

But since these functions are usually nonlinear functions of the data, these sample means will not correspond to the evaluation of different measures at any comparable representative point in the data. A reasonable alternative would be to evaluate (5.10) at the parameter estimates and at the mean of the sample data

$$H_{ij}\left(\overline{w},\overline{y},\overline{z};\hat{\theta}\right)=\frac{\partial X_i\left(\overline{w},\overline{y},\overline{z};\hat{\theta}\right)}{\partial p_j}\frac{p_j}{X_i\left(\overline{w},\overline{y},\overline{z};\hat{\theta}\right)}. \tag{5.12}$$

This method will be used in the empirical application below, but other obvious alternatives would be to evaluate at the medians of the sample data, or at some sample point of particular interest. Note that in either case there is no role for the data on the endogenous variables, the *x*, other than their role in estimating $\hat{\theta}$. What would appear to be a very common mistake in the applied literature is to replace any endogenous variable that appears separately in a formula by its sample mean, such as

$$\frac{\partial X_i\left(\overline{w},\overline{y},\overline{z};\hat{\theta}\right)}{\partial p_j}\frac{p_j}{\overline{x}_i}, \tag{5.12}$$

a mistake that seems to follow from the common notation of using a symbol like *x* both for the theoretical demand, and for the observed data on demand. Such an inconsistent treatment of a function, on one hand, and its derivative, on the other, seems impossible to justify. Writing an elasticity explicitly as a log derivative makes the same point even more convincingly, since the levels do not appear at all.

Applying this reasoning to the economic measures of interest identified above, (5.1) for example would be written more explicitly as

$$AES_{ij}(w,y,z;\theta)=\frac{C(w,y,z;\theta)C_{w_iw_j}(w,y,z;\theta)}{C_{w_i}(w,y,z;\theta)C_{w_j}(w,y,z;\theta)} \tag{5.13}$$

with a point estimate reported as

$$AES_{ij}(\bar{w},\bar{y},\bar{z};\hat{\theta}) = \frac{C(\bar{w},\bar{y},\bar{z};\hat{\theta})C_{w_i w_j}(\bar{w},\bar{y},\bar{z};\hat{\theta})}{C_{w_i}(\bar{w},\bar{y},\bar{z};\hat{\theta})C_{w_j}(\bar{w},\bar{y},\bar{z};\hat{\theta})}. \quad (5.14)$$

When an expression such as (5.14) is evaluated for specific models, such as (5.2) for the Translog and (5.3) for the NQ, it is true that expressions like S_i and X_i appear. But from the argument above, it is clear that these represent the functions for shares and demands, not the observed data on s_i and x_i, and are really just mathematical shorthand to simplify the representation of more complicated expressions.

A comparison of the estimated values of some selected subsets of the measures of interest across model specifications are given in Table 2. Note that some of these elasticities of interest are not available for TL1 and NQ1 models due to their lack of completeness. As seen in Table 2, there are significant differences between some of the elasticities estimated using different models. In general, results from different versions of the TL models are closer in magnitude to each other than those from different versions of the NQ models. Many elasticities even have different signs for different NQ models. The magnitudes of the elasticities and shadow prices from the TL are generally larger in magnitude than those from the NQ models.

Without formally checking monotonicity and curvature regularity conditions for all models at all sample points, a quick look at the estimated $AES_{i,j}$ in Table 2 is interesting. A necessary condition for satisfying the curvature condition is that $AES_{i,i} > 0 \ (i = 1,...,5)$. As shown in Table 2, all TL models satisfy this condition, while the NQ models violate this sign condition except for NQ5 and NQ8, which are two models possessing all desirable properties as listed in Table 1. Finally, we note the very large standard errors in Table 2 for those AES involving input 5 for Model

NQ3. As NQ3 is estimated with only the equations of input 1-4 quantities, the implied fit for input 5 quantity (of which the data are not used in the estimation) equation may be very poor. This may result in some very small or negative values for the predicted input 5 quantities and cost shares using the estimated coefficients, and thus some very large values of AES (which has input 5 cost share in the denominator) when simulating standard errors for the AES estimates. This highlights the potential problems associated with estimating models without technical admissibility.

6. Conclusion

This paper has considered a number of theoretical issues that arise in the statistical specification of commonly used econometric models based on a cost function specification of technology and optimizing behaviour, with special emphasis on the Translog and Normalized Quadratic functional forms. The issues have been illustrated by estimating all of the specifications considered for a large dataset comprising pseudo-micro level observations on Australian broadacre agriculture, and deriving estimated measures of interest such as elasticities of substitution.

Emphasis has been placed on the implications of the underlying economic model for the means of the estimating equations, and on the properties of the errors treated in the standard way as a residual. There is an alternative stream of literature attempting to integrate the error directly into the economic model, by for example specifying the cost function as $C(w_t, y_t, z_t; \theta; \varepsilon)$, where the ε explain observation to observation variation that is then manifested in the error terms of the estimating equations. See for example McElroy (1987), Brown and Walker (1989, 1995) and Kumbhakar and Tsionas (2009). But such an approach usually imposes virtually impossible structure on the error terms, and seems not to have influenced the applied literature. In terms of the models of this paper, random production models would require that the u_{it} in system (3.8) be homogeneous of degree one in *w*, and hence data dependent. See Brown and Walker (1995). But in the interpretation of this paper, the u_{it} are simply random variables that inherit the units of measurement of the left hand variables in (3.8), and automatically adjust to changes in units of measurement in prices. More recently, the multiplicative general error model of Kumbhakar and Tsionas (2009) gives an estimating system for the Translog system more like the models of this paper.

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Table 1. Summary of Different Demand Systems Models

	TL1	TL2a	TL2b	TL3	TL4	TL5	TL6	NQ1	NQ2	NQ3	NQ4	NQ5	NQ6	NQ7	NQ8	NQ9
Cost Form	n/a	Lcost	Log(cost)	cost	cost	cost/w5	cost/r	n/a	n/a	cost/w5	cost/w5	cost/w5	cost	cost/r	cost/w5	cost/r
Input Demand Form	shares	shares	shares	shares	exp	exp/w5	exp/r	quan	quan	quan	quan	exp/w5	exp	exp/r	exp/w5	exp/r
(No. of Inputs)	(4)	(4)	(4)	(4)	(4 or 5)	(4 or 5)	(4 or 5)	(4)	(5)	(4)	(5)	(4 or 5)	(4 or 5)	(4 or 5)	(4 or 5)	(4 or 5)
No Degeneracy	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes
Invariance	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	n/a	No	n/a	Yes	Yes	Yes	Yes	Yes
Completeness	No	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Technical Admissibility	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No	Yes	Yes	Yes	Yes	Yes
Empirical Admissibility	Yes	No	No	No	No	Yes	Yes	No	No	No	No	Yes	No	Yes	Yes	Yes
MaxLL	7622	7769	-4997	-5712	-59572	-34359	8157	-31021	-37278	-38810	-45201	-34678	-60076	7718	-34821	7603
AIC - group1 (cost&exp)					113.2*								114.1			
AIC - group2 ((C/w5)&(Exp/w5))						65.3*						65.9			66.2	
AIC - group3 ((C/r)&(Exp/r))							-15.4*							-14.5		-14.3
AIC - group4 (cost&share)			9.61*	11.0												

Notes: AIC = (2*K* -2LL) /*N* , where *K* is the number of coefficients, *N* is the sample size, and LL is the maximised log-likelihood. * indicates the model selected by AIC.

Table 2. Estimated AES for All Models (Standard Errors in Italic)

AES	TL1	TL2	TL3	TL4	TL5	TL6	NQ1	NQ2	NQ3	NQ4	NQ5	NQ6	NQ7	NQ8	NQ9
(x1,x1)	**-2.91**	**-3.06**	**-2.99**	**-1.34**	**-2.54**	**-1.69**		**0.22**	**-0.89**	**0.07**	**-0.08**	**-0.17**	**-0.12**	**-0.13**	**-0.12**
	0.24	*0.27*	*0.25*	*0.20*	*0.28*	*0.20*		*0.12*	*0.29*	*0.14*	*0.10*	*0.10*	*0.01*	*0.09*	*0.02*
(x1,x2)	**0.92**	**1.12**	**0.76**	**-0.37**	**0.80**	**-0.42**		**-0.06**	**0.46**	**0.37**	**-0.01**	**0.15**	**0.07**	**0.03**	**0.06**
	0.24	*0.26*	*0.25*	*0.25*	*0.27*	*0.22*		*0.26*	*0.28*	*0.25*	*0.19*	*0.21*	*0.06*	*0.18*	*0.06*
(x1,x3)	**1.13**	**1.15**	**1.17**	**0.85**	**1.06**	**1.05**		**-0.15**	**0.15**	**-0.16**	**0.06**	**0.11**	**0.07**	**0.07**	**0.06**
	0.10	*0.10*	*0.10*	*0.10*	*0.11*	*0.11*		*0.10*	*0.14*	*0.12*	*0.06*	*0.06*	*0.01*	*0.06*	*0.01*
(x1,x4)	**0.69**	**0.74**	**0.76**	**0.53**	**0.72**	**0.76**		**-0.24**	**0.20**	**-0.07**	**0.08**	**0.18**	**0.08**	**0.09**	**0.08**
	0.21	*0.23*	*0.22*	*0.19*	*0.23*	*0.20*		*0.15*	*0.21*	*0.16*	*0.09*	*0.09*	*0.04*	*0.09*	*0.04*
(x1,x5)	**1.50**	**1.30**	**1.32**	**1.46**	**1.38**	**1.72**		**0.17**	**1.53**	**0.12**	**-0.04**	**-0.25**	**-0.05**	**-0.02**	**-0.01**
	0.15	*0.16*	*0.15*	*0.16*	*0.16*	*0.14*		*0.10*	*13.69*	*0.11*	*0.10*	*0.19*	*0.06*	*0.12*	*0.06*
(x2,x2)	**-10.2**	**-11.3**	**-11.0**	**-9.8**	**-11.2**	**-10.4**		**7.06**	**5.62**	**4.86**	**-1.95**	**1.73**	**1.21**	**-2.02**	**1.31**
	1.25	*1.20*	*1.19*	*1.36*	*1.73*	*1.19*		*2.46*	*2.18*	*2.17*	*1.11*	*1.56*	*0.53*	*1.12*	*0.53*
(x2,x3)	**2.51**	**2.70**	**2.79**	**2.85**	**2.68**	**3.19**		**-2.57**	**-2.40**	**-2.37**	**-0.31**	**-1.28**	**-0.49**	**-0.29**	**-0.50**
	0.45	*0.45*	*0.44*	*0.42*	*0.59*	*0.44*		*0.72*	*0.66*	*0.66*	*0.35*	*0.43*	*0.17*	*0.34*	*0.17*
(x2,x4)	**-0.90**	**-0.49**	**-0.32**	**2.33**	**-1.75**	**2.05**		**8.91**	**9.21**	**8.36**	**4.50**	**6.78**	**2.14**	**4.55**	**2.21**
	1.13	*1.12*	*1.11*	*1.16*	*1.50*	*1.12*		*1.84*	*1.70*	*1.61*	*0.95*	*1.05*	*0.47*	*0.95*	*0.47*
(x2,x5)	**-0.44**	**-0.22**	**-0.14**	**-0.83**	**-0.32**	**0.18**		**-1.60**	**-3.98**	**-1.41**	**0.52**	**-1.34**	**-0.87**	**0.40**	**-0.92**
	0.50	*0.48*	*0.46*	*0.51*	*0.61*	*0.50*		*0.52*	*58.10*	*0.46*	*0.51*	*0.72*	*0.21*	*0.61*	*0.19*
(x3,x3)	**-1.79**	**-1.84**	**-1.87**	**-1.74**	**-1.77**	**-1.97**		**0.61**	**0.18**	**0.61**	**-0.07**	**0.08**	**0.00**	**-0.08**	**0.00**
	0.18	*0.19*	*0.18*	*0.15*	*0.22*	*0.17*		*0.22*	*0.23*	*0.22*	*0.12*	*0.14*	*0.06*	*0.12*	*0.06*
(x3,x4)	**1.76**	**1.63**	**1.61**	**1.08**	**2.09**	**1.10**		**-1.23**	**-1.95**	**-1.39**	**-0.49**	**-1.02**	**-0.23**	**-0.53**	**-0.26**
	0.43	*0.43*	*0.43*	*0.43*	*0.52*	*0.47*		*0.57*	*0.54*	*0.53*	*0.28*	*0.30*	*0.15*	*0.27*	*0.16*
(x3,x5)	**0.53**	**0.55**	**0.56**	**0.39**	**0.31**	**-0.31**		**1.06**	**3.40**	**1.05**	**0.69**	**1.40**	**0.56**	**0.75**	**0.56**
	0.17	*0.17*	*0.17*	*0.17*	*0.20*	*0.18*		*0.16*	*33.21*	*0.15*	*0.16*	*0.19*	*0.07*	*0.17*	*0.06*
(x4,x4)	**-15.1**	**-15.1**	**-15.5**	**-15.6**	**-16.8**	**-16.2**		**-0.10**	**-0.53**	**0.14**	**-2.92**	**-2.19**	**-1.68**	**-2.90**	**-1.68**
	1.07	*1.12*	*1.11*	*1.10*	*1.35*	*1.27*		*1.47*	*1.46*	*1.39*	*0.58*	*0.68*	*0.43*	*0.56*	*0.44*
(x4,x5)	**2.05**	**1.86**	**1.98**	**0.59**	**1.92**	**0.02**		**-2.45**	**-1.78**	**-2.26**	**-1.31**	**-2.24**	**-0.78**	**-1.36**	**-0.68**
	0.42	*0.42*	*0.40*	*0.44*	*0.50*	*0.46*		*0.40*	*30.6*	*0.35*	*0.45*	*0.44*	*0.19*	*0.51*	*0.17*
(x5,x5)	**-8.19**	**-7.51**	**-7.58**	**-7.15**	**-6.52**	**-5.00**		**-1.60**	**-17.6**	**-1.50**	**-2.05**	**-2.28**	**-0.89**	**-2.34**	**-0.99**
	0.36	*0.31*	*0.30*	*0.35*	*0.31*	*0.33*		*0.36*	*166421*	*0.29*	*0.37*	*0.49*	*0.21*	*0.53*	*0.16*

Table 3. Estimated OE and SP for All Models (Standard Errors in Italic)

OE	TL1	TL2	TL3	TL4	TL5	TL6	NQ1	NQ2	NQ3	NQ4	NQ5	NQ6	NQ7	NQ8	NQ9
(x1,y1)		**0.055**	**0.070**	**0.188**	**0.063**	**0.174**	**0.010**	**-0.012**	**0.011**	**-0.048**	**0.012**	**0.007**	**0.006**	**0.013**	**0.007**
		0.013	*0.011*	*0.011*	*0.016*	*0.012*	*0.039*	*0.041*	*0.024*	*0.021*	*0.021*	*0.020*	*0.028*	*0.020*	*0.029*
(x1,y2)		**0.161**	**0.126**	**0.006**	**0.138**	**0.041**	**0.255**	**0.225**	**0.266**	**0.293**	**0.252**	**0.180**	**0.215**	**0.233**	**0.215**
		0.027	*0.029*	*0.025*	*0.029*	*0.027*	*0.040*	*0.045*	*0.042*	*0.033*	*0.032*	*0.030*	*0.026*	*0.026*	*0.026*
(x1,y3)		**0.434**	**0.496**	**0.681**	**0.472**	**0.639**	**0.734**	**0.720**	**0.840**	**0.790**	**0.823**	**0.803**	**0.691**	**0.801**	**0.698**
		0.015	*0.013*	*0.025*	*0.021*	*0.023*	*0.052*	*0.057*	*0.050*	*0.030*	*0.034*	*0.036*	*0.022*	*0.032*	*0.022*
(x1,y4)		**0.107**	**0.143**	**0.118**	**0.088**	**0.089**	**-0.085**	**-0.086**	**-0.087**	**-0.178**	**-0.051**	**-0.029**	**-0.007**	**-0.015**	**0.005**
		0.032	*0.030*	*0.031*	*0.032*	*0.034*	*0.040*	*0.046*	*0.040*	*0.030*	*0.037*	*0.037*	*0.026*	*0.028*	*0.026*
(x2,y1)		**0.276**	**0.296**	**0.306**	**0.269**	**0.313**	**0.576**	**0.586**	**0.588**	**0.575**	**0.502**	**0.580**	**0.605**	**0.504**	**0.604**
		0.012	*0.011*	*0.017*	*0.012*	*0.017*	*0.027*	*0.028*	*0.031*	*0.025*	*0.027*	*0.031*	*0.023*	*0.027*	*0.022*
(x2,y2)		**0.258**	**0.227**	**0.073**	**0.246**	**0.165**	**0.292**	**0.308**	**0.303**	**0.273**	**0.335**	**0.355**	**0.278**	**0.339**	**0.277**
		0.043	*0.038*	*0.030*	*0.042*	*0.033*	*0.032*	*0.039*	*0.036*	*0.036*	*0.039*	*0.038*	*0.031*	*0.035*	*0.029*
(x2,y3)		**0.013**	**0.070**	**-0.209**	**-0.107**	**-0.181**	**-0.092**	**-0.083**	**-0.083**	**-0.080**	**-0.085**	**-0.090**	**-0.054**	**-0.084**	**-0.060**
		0.025	*0.020*	*0.031*	*0.035*	*0.031*	*0.022*	*0.028*	*0.024*	*0.025*	*0.028*	*0.027*	*0.040*	*0.025*	*0.039*
(x2,y4)		**-0.086**	**-0.044**	**0.151**	**-0.103**	**0.044**	**-0.086**	**-0.094**	**-0.099**	**-0.058**	**-0.183**	**-0.144**	**-0.147**	**-0.188**	**-0.149**
		0.043	*0.038*	*0.030*	*0.041*	*0.033*	*0.034*	*0.040*	*0.037*	*0.037*	*0.042*	*0.041*	*0.037*	*0.040*	*0.036*
(x3,y1)		**0.156**	**0.173**	**0.187**	**0.142**	**0.190**	**0.197**	**0.174**	**0.200**	**0.168**	**0.162**	**0.142**	**0.213**	**0.165**	**0.214**
		0.009	*0.008*	*0.008*	*0.010*	*0.007*	*0.009*	*0.010*	*0.010*	*0.008*	*0.007*	*0.007*	*0.009*	*0.007*	*0.009*
(x3,y2)		**0.083**	**0.051**	**0.047**	**0.061**	**0.070**	**-0.053**	**-0.096**	**-0.060**	**-0.098**	**0.048**	**0.002**	**0.072**	**0.057**	**0.073**
		0.024	*0.020*	*0.017*	*0.024*	*0.016*	*0.016*	*0.020*	*0.017*	*0.016*	*0.016*	*0.017*	*0.023*	*0.015*	*0.022*
(x3,y3)		**0.221**	**0.280**	**0.224**	**0.170**	**0.228**	**0.164**	**0.135**	**0.172**	**0.120**	**0.162**	**0.143**	**0.183**	**0.168**	**0.177**
		0.013	*0.010*	*0.012*	*0.014*	*0.011*	*0.007*	*0.009*	*0.008*	*0.007*	*0.008*	*0.008*	*0.017*	*0.008*	*0.017*
(x3,y4)		**0.071**	**0.105**	**0.122**	**0.058**	**0.088**	**0.193**	**0.212**	**0.199**	**0.223**	**0.079**	**0.117**	**0.074**	**0.067**	**0.069**
		0.026	*0.019*	*0.018*	*0.024*	*0.016*	*0.014*	*0.020*	*0.016*	*0.014*	*0.017*	*0.017*	*0.024*	*0.015*	*0.024*
(x4,y1)		**0.190**	**0.208**	**0.212**	**0.175**	**0.217**	**0.276**	**0.249**	**0.274**	**0.246**	**0.175**	**0.162**	**0.297**	**0.176**	**0.301**
		0.011	*0.009*	*0.010*	*0.012*	*0.009*	*0.012*	*0.013*	*0.015*	*0.012*	*0.010*	*0.009*	*0.017*	*0.010*	*0.017*
(x4,y2)		**0.050**	**0.020**	**0.025**	**0.018**	**0.043**	**0.018**	**-0.033**	**0.007**	**-0.049**	**-0.041**	**-0.062**	**0.024**	**-0.034**	**0.027**
		0.031	*0.025*	*0.026*	*0.031*	*0.024*	*0.025*	*0.029*	*0.028*	*0.027*	*0.024*	*0.023*	*0.037*	*0.022*	*0.036*
(x4,y3)		**0.168**	**0.230**	**0.135**	**0.086**	**0.153**	**0.159**	**0.124**	**0.160**	**0.117**	**0.083**	**0.085**	**0.106**	**0.088**	**0.092**
		0.017	*0.014*	*0.018*	*0.024*	*0.017*	*0.011*	*0.012*	*0.012*	*0.011*	*0.011*	*0.013*	*0.034*	*0.011*	*0.036*
(x4,y4)		**-0.019**	**0.015**	**0.062**	**-0.026**	**0.034**	**0.021**	**0.047**	**0.023**	**0.071**	**0.025**	**0.071**	**0.000**	**0.017**	**-0.016**
		0.032	*0.024*	*0.025*	*0.031*	*0.023*	*0.026*	*0.031*	*0.029*	*0.027*	*0.023*	*0.021*	*0.039*	*0.020*	*0.039*
(x5,y1)		**-0.033**	**-0.011**	**-0.099**	**-0.062**	**-0.026**		**-0.019**	**-0.583**	**-0.011**	**-0.059**	**-0.077**	**-0.044**	**-0.044**	**-0.002**
		0.014	*0.013*	*0.021*	*0.020*	*0.012*		*0.044*	*5.782*	*0.041*	*0.042*	*0.038*	*0.030*	*0.027*	*0.015*
(x5,y2)		**-0.065**	**-0.091**	**-0.039**	**-0.055**	**0.025**		**-0.011**	**0.370**	**-0.025**	**0.044**	**0.152**	**0.108**	**0.120**	**0.144**
		0.030	*0.028*	*0.026*	*0.031*	*0.019*		*0.058*	*6.024*	*0.056*	*0.056*	*0.057*	*0.046*	*0.023*	*0.017*
(x5,y3)		**0.436**	**0.493**	**0.426**	**0.437**	**0.426**		**0.816**	**0.674**	**0.791**	**0.805**	**0.826**	**0.938**	**0.823**	**0.850**
		0.018	*0.017*	*0.022*	*0.022*	*0.014*		*0.051*	*12.146*	*0.041*	*0.043*	*0.040*	*0.031*	*0.039*	*0.015*
(x5,y4)		**0.222**	**0.247**	**0.220**	**0.213**	**0.150**		**0.202**	**-0.347**	**0.226**	**0.142**	**0.009**	**0.141**	**0.081**	**0.047**
		0.032	*0.030*	*0.030*	*0.032*	*0.019*		*0.061*	*6.305*	*0.059*	*0.062*	*0.048*	*0.043*	*0.017*	*0.015*
SP	**TL1**	**TL2**	**TL3**	**TL4**	**TL5**	**TL6**	**NQ1**	**NQ2**	**NQ3**	**NQ4**	**NQ5**	**NQ6**	**NQ7**	**NQ8**	**NQ9**
(z1)		**5.828**	**4.585**	**4.520**	**6.394**	**5.167**		**0.037**	**-0.761**	**0.090**	**0.043**	**-0.386**	**-0.368**	**-0.045**	**-0.128**
		0.301	*0.289*	*0.294*	*0.285*	*0.297*		*0.078*	*0.390*	*0.081*	*0.085*	*0.053*	*0.035*	*0.043*	*0.010*
(z2)		**50.84**	**14.59**	**55.33**	**128.46**	**-15.23**		**19.52**	**83.72**	**18.04**	**15.67**	**13.74**	**3.18**	**19.76**	**1.55**
		33.02	*29.57*	*30.99*	*31.20*	*29.28*		*9.80*	*44.96*	*9.40*	*9.56*	*6.16*	*4.62*	*6.77*	*1.13*